Fuzzy OWL: Uncertainty and the Semantic Web

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Abstract. In the Semantic Web context information would be retrieved, processed, shared, reused and aligned in the maximum automatic way possible. Our experience with such applications in the Semantic Web has shown that these are rarely a matter of true or false but rather procedures that require degrees of relatedness, similarity, or ranking. Apart from the wealth of applications that are inherently imprecise, information itself is many times imprecise or vague. For example, the concepts of a "hot" place, an "expensive" item, a "fast" car, a "near" city, are examples of such concepts. Dealing with such type of information would yield more realistic, intelligent and effective applications. In the current paper we extend the OWL web ontology language, with fuzzy set theory, in order to be able to capture, represent and reason with such type of information.

1 Introduction

In the Semantic Web vision [1], information and knowledge would be structured in a machine understandable and processable way. To this extend Semantic Web agents would be able to (semi)automatically carry out complex tasks assigned by humans in a meaningful (semantic) way. For example, they would be able to carry out a "holiday organization", an "item purchase", "doctor appointment" [1] and many more. Such tasks reflect every day procedures, which contain a wealth of imprecise and vague information. For example a task of "holiday organization" could look something like: a "hot" place, with "many" attractions, or a "doctor appointment" could involve concepts like "close enough", "not too early" and many more. In order to accurately represent such type of information current ontology languages need to be extended with proper mathematical frameworks that intend in capturing such kind of information.

Such types of assignments (tasks) should be carried out in the maximum automatic way possible, where information and knowledge would be retrieved from databases, processed, shared and exchanged. In order to fulfill such preconditions, semantic web applications should reach a high level of interoperability, scalability and modularity. To achieve such goals special procedures like information retrieval, alignment or ontology partitioning are used in the context of semantic web. A more careful look to these procedures would reveal that they involve a high level of uncertainty and imprecision. This is a result of both the facts that information is sometimes imprecise or vague but also of the nature of these applications. For example it is almost impossible to automatically match two concepts to degrees 1 or 0 or even in a semi-automatic alignment processes, two concepts might not be completely compatible, i.e. to a degree of 1, thus speaking of *confidence degrees* [2]. The need for covering uncertainty in the Semantic Web context has been stressed out in literature many times the last years [3–5]. It has been pointed out that dealing with such information would improve Semantic Web applications like, portals [6], multimedia application in the semantic web [7, 8], e-commerce applications [9], situation awareness and information fusion [4], rule languages [5, 3], medicine and diagnosis [10], geospatial applications [11] and many more.

Knowledge in the SW is usually structured in the form of ontologies [12]. This has led to considerable efforts to develop a suitable ontology language, culminating in the design of the OWL Web Ontology Language [13]. The OWL language consists of three sub-languages of increasing expressive power, namely OWL Lite, OWL DL and OWL Full. *OWL Lite* and *OWL DL* are, basically very expressive description logics; they are almost equivalent to the $\mathcal{SHIF}(\mathbf{D}^+)$ and $\mathcal{SHOIN}(\mathbf{D}^+)$ DLs. OWL Full is clearly undecidable because it does not impose restrictions on the use of transitive properties. Although the above DL languages are very expressive, they feature expressive limitations regarding their ability to represent vague and imprecise knowledge.

In the current paper we will extend the OWL web ontology language with fuzzy set theory, which is a mathematical framework for covering vagueness [14], thus getting fuzzy OWL (f-OWL). We will also investigate several issues that arise from such an extension. More precisely, we will also extend the DL $\mathcal{SHOIN}$, in order to provide reasoning for f-OWL, present a mapping from f-OWL entailment to f-$\mathcal{SHOIN}$ satisfiability and at last provide a preliminary investigation on querying capabilities for f-$\mathcal{SHOIN}$ *ABoxes*.

2 The Fuzzy $\mathcal{SHOIN}$ DL

In this section we introduce the DL f-$\mathcal{SHOIN}$ (we will discard datatypes, as it is considered an ongoing research effort for fuzzy DLs). As usual we have an alphabet of distinct concept names ($\mathbf{C}$), role names ($\mathbf{R}$) and individual names ($\mathbf{I}$). f-$\mathcal{SHOIN}$-roles and f-$\mathcal{SHOIN}$-concepts are defined as follows:

Definition 1. *Let $RN \in \mathbf{R}$ be a role name and R an f-$\mathcal{SHOIN}$-role. f-$\mathcal{SHOIN}$-roles are defined by the abstract syntax: $R ::= RN \mid R^-$. The inverse relation of roles is symmetric, and to avoid considering roles such as R^{--}, we define a function* Inv *which returns the inverse of a role, more precisely* $\mathsf{Inv}(R) := RN^-$ *if $R = RN$ and $\mathsf{Inv}(R) := RN$ if $R = RN^-$. The set of f-$\mathcal{SHOIN}$ concepts is the smallest set such that*

1. *every concept name $C \in CN$ is an f-$\mathcal{SHOIN}$-concept,*

2. *if* $o \in \mathbf{I}$ *then* $\{o\}$ *is an f-*$\mathcal{SHOIN}$*-concept,*
3. *if* C *and* D *are f-*$\mathcal{SHOIN}$*-concepts,* R *an f-*$\mathcal{SHOIN}$*-role,* S *a* simple *f-*$\mathcal{SHOIN}$*-role* [3] *and* $p \in \mathbb{N}$*, then* $(C \sqcup D)$*,* $(C \sqcap D)$*,* $(\neg C)$*,* $(\forall R.C)$*,* $(\exists R.C)$*,* $(\geq pS)$ *and* $(\leq pS)$ *are also f-*$\mathcal{SHOIN}$ *concepts.*

A fuzzy *TBox* is a finite set of fuzzy concept axioms. Let A be a concept name, C an f-$\mathcal{SHOIN}$-concept. Fuzzy concept axioms of the form $A \sqsubseteq C$ are called *fuzzy inclusion introductions*; fuzzy concept axioms of the form $A \equiv C$ are called *fuzzy equivalence introductions*. Note that how to deal with *fuzzy general concept inclusion* axioms [16] still remains an open problem in fuzzy concept languages. A fuzzy *RBox* is a finite set of fuzzy role axioms. Fuzzy role axioms of the form $\mathsf{Trans}(RN)$, where RN is a role name, are called *fuzzy transitive role* axioms; fuzzy role axioms of the form $R \sqsubseteq S$ are called *fuzzy role inclusion* axioms. A fuzzy *ABox* is a finite set of fuzzy assertions. A *fuzzy assertion* [17] is of the form $\langle a : C \bowtie n\rangle$, $\langle (a, b) : R \bowtie n\rangle$, where $\bowtie \in \{\geq, >, \leq, <\}$, or $a \not\doteq b$, for $a, b \in \mathbf{I}$. We call assertions defined by $\geq, >$ *positive* assertions, while those defined by $\leq, <$ *negative* assertions. A fuzzy knowledge base Σ is a triple $\langle \mathcal{T}, \mathcal{R}, \mathcal{A}\rangle$, that contains a fuzzy *TBox*, *RBox* and *ABox*, respectively. A pair of assertions is called *conjugated* if they impose contradicting restrictions. For example, for ϕ a classical DL assertion, the pair of assertions $\langle \phi \geq n\rangle$ and $\langle \phi < m\rangle$, with $n \geq m$ contradict to each other. Observe that since an *ABox* can contain an unlimited number of positive assertion without forming a contradiction we need a special procedure to compute which is the best lower and upper truth-value bounds of a fuzzy assertion. A procedure for that purpose was provided in [17].

The semantics of fuzzy DLs are provided by a *fuzzy interpretation* [17]. A fuzzy interpretation is a pair $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}}\rangle$ where the domain $\Delta^{\mathcal{I}}$ is a non-empty set of objects and $\cdot^{\mathcal{I}}$ is a fuzzy interpretation function, which maps an individual name a to elements of $\mathsf{a}^{\mathcal{I}} \in \Delta^{\mathcal{I}}$ and a concept name A (role name R) to a membership function $\mathsf{A}^{\mathcal{I}} : \Delta^{\mathcal{I}} \to [0, 1]$ ($R^{\mathcal{I}} : \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \to [0, 1]$). Intuitively, an object (pair of objects) can now belong to a degree from 0 to 1 to a fuzzy concept (role). Moreover, fuzzy interpretations are extended to interpret arbitrary f-$\mathcal{SHOIN}$-concepts and roles, with the aid of the fuzzy set theoretic operations. The complete semantics are the following:

$$\{o\}^{\mathcal{I}}(a) = 1 \text{ if } a \in \{o^I\} \qquad \bot^{\mathcal{I}}(a) = 0 \qquad \top^{\mathcal{I}}(a) = 1$$
$$(C \sqcap D)^{\mathcal{I}}(a) = t(C^{\mathcal{I}}(a), D^{\mathcal{I}}(a)),\ (C \sqcup D)^{\mathcal{I}}(a) = u(C^{\mathcal{I}}(a), D^{\mathcal{I}}(a)),\ (\neg C)^{\mathcal{I}}(a) = c(C^{\mathcal{I}}(a))$$
$$(\exists R.C)^{\mathcal{I}}(a) = \sup_{b \in \Delta^{\mathcal{I}}} t(R^{\mathcal{I}}(a, b), C^{\mathcal{I}}(b))$$
$$(\forall R.C)^{\mathcal{I}}(a) = \inf_{b \in \Delta^{\mathcal{I}}} \mathcal{J}(R^{\mathcal{I}}(a, b), C^{\mathcal{I}}(b))$$
$$(\geq nR)^{\mathcal{I}}(a) = \sup_{b_1,\ldots,b_n \in \Delta^{\mathcal{I}}} t_{i=1}^{n} R^{\mathcal{I}}(a, b_i)$$
$$(\leq nR)^{\mathcal{I}}(a) = \inf_{b_1,\ldots,b_{n+1} \in \Delta^{\mathcal{I}}} u_{i=1}^{n+1} c(R^{\mathcal{I}}(a, b_i))$$

where c represent a fuzzy complement, t a fuzzy intersection, u a fuzzy union and $\mathcal{J}$ a fuzzy implication [14]. Most of these semantics appear in [18]. Some

[3] A role is called *simple* if it is neither transitive nor has any transitive sub-roles. Restricting roles that participate in number restrictions only to simple ones, is crucial in order to get a decidable logic [15].

remarks regarding nominals are in place. Note that we choose not to fuzzify nominal concepts. The reason for this choice is that concepts of the form {o} do not represent any real life concept which pertains some specific meaning. Thus, such concepts cannot represent imprecise or vague information. Please also note that at the extreme points of 0 and 1 the semantics of fuzzy interpretations coincide with those of crisp interpretations.

An f-$\mathcal{SHOIN}$-concept C is *satisfiable* iff there exists some fuzzy interpretation $\mathcal{I}$ for which there is some $a \in \Delta^{\mathcal{I}}$ such that $C^{\mathcal{I}}(a) = n$, and $n \in (0, 1]$. A fuzzy interpretation $\mathcal{I}$ satisfies a fuzzy *TBox* $\mathcal{T}$ iff $\forall a \in \Delta^{\mathcal{I}}$, $A^{\mathcal{I}}(a) \leq C^{\mathcal{I}}(a)$ for each $A \sqsubseteq C \in \mathcal{T}$ and $A^{\mathcal{I}}(a) = C^{\mathcal{I}}(a)$ for each $A \equiv C \in \mathcal{T}$. This is the usual way subsumption is defined in the context of fuzzy sets [14]. A fuzzy interpretation $\mathcal{I}$ satisfies a fuzzy *RBox* $\mathcal{R}$ iff $\forall a, c \in \Delta^{\mathcal{I}}$, $R^{\mathcal{I}}(a, c) \geq sup_{b \in \Delta^{\mathcal{I}}}\{t(R^{\mathcal{I}}(a, b), R^{\mathcal{I}}(b, c))\}$ for each $\mathsf{Trans}(R) \in \mathcal{R}$, $\forall \langle a, b\rangle \in \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$, $R^{\mathcal{I}}(a, b) \leq S^{\mathcal{I}}(a, b)$ for each $R \sqsubseteq S \in \mathcal{R}$, and $\forall \langle a, b\rangle \in \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$, $(R^-)^{\mathcal{I}}(b, a) = R^{\mathcal{I}}(a, b)$ for each $R \in \mathbf{R}$. Given a fuzzy interpretation $\mathcal{I}$, $\mathcal{I}$ satisfies $\langle a : C \geq n\rangle$ ($\langle (a, b) : R \geq n\rangle$) if $C^{\mathcal{I}}(a^{\mathcal{I}}) \geq n$ ($R^{\mathcal{I}}(a^{\mathcal{I}}, b^{\mathcal{I}}) \geq n$), while $\mathcal{I}$ satisfies $a \not\doteq b$ if $a^{\mathcal{I}} \neq b^{\mathcal{I}}$. The satisfiability of fuzzy assertions with $\leq, >$ and $<$ is defined analogously. A fuzzy interpretation satisfies a fuzzy *ABox* $\mathcal{A}$ if it satisfies all fuzzy assertions in $\mathcal{A}$. In this case, we say $\mathcal{I}$ is a *model* of $\mathcal{A}$. If $\mathcal{A}$ has a model then we say that it is *consistent*. At last, a fuzzy knowledge base Σ is satisfiable iff there exists a fuzzy interpretation $\mathcal{I}$ which satisfies all axioms in Σ. Moreover, Σ *entails* an assertion $\langle \phi \bowtie n\rangle$, written $\Sigma \models \langle \phi \bowtie n\rangle$, iff any model of Σ also satisfies the fuzzy assertion. The problems of entailment and subsumption can be reduced to fuzzy knowledge base satisfiability [17].

3 Reasoning in f-$\mathcal{SHOIN}$

Reasoning in DLs is usually performed with tableaux decision procedures [19]. Such procedures try to prove the consistency of an Abox $\mathcal{A}$, by attempting to construct a model for it. Since concepts that appear in $\mathcal{A}$ might by complex, such algorithms apply *expansion rules*, that decompose the initial concept, to sub-concepts, until no rule is applicable or an evident contradiction is reached. Proceeding that way leads to the creation of a model for $\mathcal{A}$, which has a forest or graph-like shape [16, 20]. That forest structure is a collection of *trees*, where nodes correspond to objects in the model, and edges to certain relations that connect two nodes. Each node x is labelled with the set of objects that it belongs to ($\mathcal{L}(x)$), and each edge $\langle x, y\rangle$ with a set of roles that connect two nodes x, y ($\mathcal{L}(\langle x, y\rangle)$). In the fuzzy case, since now we have fuzzy assertion, we extend these mappings to also include the membership degree that a node belongs to a concept as well as the type of inequality that holds for the fuzzy assertion, thus speaking of *membership triples*. For example a fuzzy assertion of the form $\langle a : C \geq n\rangle$ is represented as $\mathcal{L}(a) = \{\langle C, \geq, n\rangle\}$ in the tree.

In [21] a tableaux decision procedure for deciding consistency of f$_{KD}$-$\mathcal{SHIN}$ ABoxes is presented. The f$_{KD}$-$\mathcal{SHIN}$ language is obtained from f-$\mathcal{SHIN}$ by using specific operators for performing the fuzzy set theoretic operations. More precisely fuzzy complement is performed by the equation, c(a)=1-a, fuzzy intersection by, t(a,b)=min(a,b), fuzzy union by, u(a,b)=max(a,b) and fuzzy implication by $\mathcal{J}(a,b) = max(1-a,b)$. Since we argue that nominals should not be fuzzyfied, this algorithm, together with the results obtained in [20] for crisp $\mathcal{SHOIN}$, can be extended to provide a tableaux procedure for f$_{KD}$-$\mathcal{SHOIN}$. The only additional rules that are needed are those which would ensure that positive assertion with concepts {o} would be equal to one, and negative ones equal to zero. Such types of rules are depicted in Table 1. Regarding implementation issues we have implemented a fuzzy reasoner for the f$_{KD}$-$\mathcal{SI}$ language, based on the direct tableaux rules in [8], and we started the extension of the algorithm to cover the f$_{KD}$-$\mathcal{SHIN}$ language. Please note that how to reason with other norm operations in fuzzy DLs remains an open problem.

Table 1. The new expansion rules for f$_{KD}$-$\mathcal{SHOIN}$

Rule	Description	Rule	Description
$\{o\}_{\triangleright}$	if 1. $\langle\{o\},\triangleright,n\rangle \in \mathcal{L}(x)$, and 2. $\langle\{o\},\triangleright,1\rangle \notin \mathcal{L}(x)$ then $\mathcal{L}(x) \cup \{\langle\{o\},\triangleright,1\rangle\}$	$\{o\}_{\triangleleft}$	if 1. $\langle\{o\},\triangleleft,n\rangle \in \mathcal{L}(x)$, and 2. $\langle\{o\},\triangleleft,0\rangle \notin \mathcal{L}(x)$ then $\mathcal{L}(x) \cup \{\langle\{o\},\triangleleft,0\rangle\}$
$\triangleright \in \{\geq,>\}$, $\triangleleft \in \{\leq,<\}$			

4 Fuzzy OWL

In this section, we present a fuzzy extension of OWL DL by adding degrees to OWL facts; we call our extension f-OWL.

As mentioned in section 1, OWL is an ontology language that has recently been a W3C recommendation. As with f-DLs, also in f-OWL, the extension is focused on OWL facts, resulting in fuzzy facts, and the semantics of the extended language. The extension of the direct model-theoretic semantics of f-OWL are provided by a fuzzy interpretation, which in the absence of data types and the concrete domain is the same as the one introduced in section 2. An f-OWL interpretation can be extended to give semantics to fuzzy concept and *individual-valued* Property descriptions. Since the equivalence of f-OWL class descriptions [13] with f-$\mathcal{SHOIN}$ is evident, we don't address them here again. The reader is referred to [22] and to section 2 for the semantics of them.

Now we would like to focus more on the semantics of f-OWL Axioms. These are summarized in Table 2. We would like to point out that omitting a degree from an individual axiom is equivalent to specifying a value of 1. From a semantics point of view, an f-OWL axiom of the first column of Table 2, *is satisfied* by a fuzzy interpretation $\mathcal{I}$ iff the respective equation of the third column is satisfied. A *fuzzy ontology* O, is a set of f-OWL axioms. We say that a fuzzy interpretation $\mathcal{I}$ is a model of O iff it satisfies all axioms in O.

There are some remarks regarding Table 2. The semantics of domain and range restrictions have been carefully selected in order to give an intuitive ac-

Table 2. Fuzzy OWL Axioms

Abstract Syntax	DL Syntax	Semantics
(Class A partial $C_1 \ \ldots\ C_n$)	$A \sqsubseteq C_1 \sqcap \ldots \sqcap C_n$	$A^{\mathcal{I}}(a) \leq t(C_1^{\mathcal{I}}(a), \ldots, C_n^{\mathcal{I}}(a))$
(Class A complete $C_1 \ \ldots\ C_n$)	$A \equiv C_1 \sqcap \ldots \sqcap C_n$	$A^{\mathcal{I}}(a) = t(C_1^{\mathcal{I}}(a), \ldots, C_n^{\mathcal{I}}(a))$
(EnumeratedClass $A\ o_1 \ \ldots\ o_n$)	$A \equiv o_1 \sqcup \ldots \sqcup o_n$	$A^{\mathcal{I}}(a) = 1$ if $a \in \{o_1^I, \ldots, o_n^I\}$, $A^{\mathcal{I}}(a)$=0 otherwise
(SubClassOf C_1, C_2)	$C_1 \sqsubseteq C_2$	$C_1^{\mathcal{I}}(a) \leq C_2^{\mathcal{I}}(a)$
(EquivalentClasses $C_1 \ \ldots\ C_n$)	$C_1 \equiv \ldots \equiv C_n$	$C_1^{\mathcal{I}}(a) = \ldots = C_n^{\mathcal{I}}(a)$
(DisjointClasses $C_1 \ \ldots\ C_n$)	$C_i \not\equiv C_j,\ 1 \leq i < j \leq n$	$t(C_1^{\mathcal{I}}(a), C_j^{\mathcal{I}}(a)) = 0\ 1 \leq i < j \leq n$
(SubPropertyOf R_1, R_2)	$R_1 \sqsubseteq R_2$	$R_1^{\mathcal{I}}(a,b) \leq R_2^{\mathcal{I}}(a,b)$
(EquivalentProperties $R_1 \ \ldots\ R_n$)	$R_1 \equiv \ldots \equiv R_n$	$R_1^{\mathcal{I}}(a,b) = \ldots = R_n^{\mathcal{I}}(a,b)$
ObjectProperty(R super(R_1) ... super(R_n)	$R \sqsubseteq R_i$	$R^{\mathcal{I}}(a,b) \leq R_i^{\mathcal{I}}(a,b)$
domain(C_1) ... domain(C_k)	$\exists R.\top \sqsubseteq C_i$	$R^{\mathcal{I}}(a,b) \leq C_i^{\mathcal{I}}(a)$
range(C_1) ... range(C_h)	$\top \sqsubseteq \forall R.C_i$	$R^{\mathcal{I}}(a,b) \leq C_i^{\mathcal{I}}(b)$
[Symmetric]	$R \equiv R^-$	$R^{\mathcal{I}}(a,b) = (R^-)^{\mathcal{I}}(a,b)$
[Functional]	$\top \sqsubseteq \leqslant 1R$	$\forall a \in \Delta^{\mathcal{I}}\ \inf_{b_1,b_2 \in \Delta^{\mathcal{I}}} u(R(a,b_1), R(a,b_2)) \geq 1$
[InverseFunctional]	$\top \sqsubseteq \leqslant 1R^-$	$\forall a \in \Delta^{\mathcal{I}}\ \inf_{b_1,b_2 \in \Delta^{\mathcal{I}}} u(R^-(a,b_1), R^-(a,b_2)) \geq 1$
[Transitive])	$\mathsf{Trans}(R)$	$\sup_{b \in \Delta^{\mathcal{I}}} t(R^{\mathcal{I}}(a,b), R^{\mathcal{I}}(b,c)) \leq R^{\mathcal{I}}(a,c)$
Individual(o type(C_1) [$\bowtie$ degree(m_1)] ...type(C_n) [$\bowtie$ degree(m_n)]	$o : C_i \bowtie m_i, 1 \leq i \leq n$	$C_i^{\mathcal{I}}(o^{\mathcal{I}}) \bowtie m_i, m_i \in [0,1], 1 \leq i \leq n$
value(R_1, o_1) [$\bowtie$ degree(k_1)] ...value(R_ℓ, o_ℓ)) [$\bowtie$ degree(k_ℓ)]	$(o, o_i) : R_i \bowtie k_i$,$1 \leq i \leq \ell$	$R_i^{\mathcal{I}}(o^{\mathcal{I}}, o_i^{\mathcal{I}}) \bowtie k_i, k_i \in [0,1], 1 \leq i \leq \ell$
Sameindividual($o_1 \ \ldots\ o_n$)	$o_1 = \ldots = o_n$	$o_1^{\mathcal{I}} = \ldots = o_n^{\mathcal{I}}$
DifferentIndividuals($o_1 \ \ldots\ o_n$)	$o_i \neq o_j, 1 \leq i < j \leq n$	$o_i^{\mathcal{I}} \neq o_j^{\mathcal{I}}, 1 \leq i < j \leq n$

count on fuzzy domain and range restrictions. Finally, the semantics of disjoint classes result by the fact that the axiom $C \not\equiv D$ is equivalent to the subsumption relation, $C \sqcap D \sqsubseteq \bot$. Fuzzyfing this last equation we get that $\forall a \in \Delta^{\mathcal{I}}.t(C^{\mathcal{I}}(a), D^{\mathcal{I}}(a)) \leq \bot^{\mathcal{I}}(a) = 0$. This equation reflects our intuition behind disjointness, which says that two concepts are disjoint if and only if they have no common object in any interpretation. Keep in mind that the translation to the subsumption axiom $C \sqsubseteq \neg D$ does not *always* hold in the fuzzy case.

Table 3. Abstract Syntax of f-OWL

individual ::= 'Individual(' [individualID] {annotation}
{'type'(type ')' [membership]} {value [membership] } ')'
membership ::= ineqType degree
ineqType ::= '>=' | '>' | '<=' | '<'
degree ::= 'degree(' real-number-between-0-and-1-inclusive ')'

We conclude this section by providing the syntactic changes that need to take place in the OWL language in order to assert the membership degree of an individual to a fuzzy concept to a specific degree that ranges from 0 to 1. The abstract syntax of the extended language is depicted in Table 3. Observe that the only syntactic change involves the addition of a membership degree, that ranges from 0 to 1, in the definition of f-OWL facts. If such a value is omitted then it is assumed to be equal to 1, i.e. total membership.

5 From f-OWL entailment to f-$\mathcal{SHOIN}$ satisfiability

In [22] a translation from OWL entailment to DL satisfiability was provided. In this section we will study the reduction in the case of f-OWL and f-DLs.

Translating f-OWL DL class descriptions into f-$\mathcal{SHOIN}$ is a straightforward task, since f-OWL DL class descriptions are almost identical to that of f-$\mathcal{SHOIN}$ concept descriptions. The complete translation is described in [22]. Also in the current paper we have more or less provided the DL counterpart of the OWL class axioms (see Table 2). The only difference from [22], is in the disjointness axioms, which are translated to $C \sqcap D \sqsubseteq \bot$, rather than, $C \sqsubseteq \neg D$, for reasons explained in the previous section.

Table 4. From f-OWL facts to f-DL fuzzy assertions

OWL fragment F	Translation $\mathcal{F}(F)$
Individual($x_1 \bowtie n_1 ... x_p \bowtie n_p$)	$\mathcal{F}(\mathsf{a} : x_1) \bowtie n_1, ..., \mathcal{F}(\mathsf{a} : x_n) \bowtie n_p$ for a new
a:type(C)	$\mathcal{V}(C)$
a:value(R x)	$(\mathsf{a}, \mathsf{b}) : R$, $\mathcal{F}(\mathsf{b} : x)$ for b new
a:o	$\mathsf{a} = o$

The most complex part of the translation is the translation of individual axioms because they can be stated with respect to anonymous individuals [22]. In [22] two translations where provided, one for OWL DL and one for OWL Lite. This is because the translation of OWL DL uses nominals, which OWL Lite does not support. Closely inspecting the abstract syntax of fuzzy individual axioms, from Table 2, and the translations in [22], would reveal that the OWL Lite reduction serves better our needs in f-OWL DL, too. This is because now we also have membership degrees. The translation is illustrated in Table 4, where $\mathcal{V}$ represents the mapping from f-OWL class descriptions to f-$\mathcal{SHOIN}$ concept descriptions.

Table 5. From entailment to unsatisfiability

Axiom A	Transformation $\mathcal{G}(A)$
$C \sqsubseteq D$	$\{\langle x : C \geq n \rangle, \langle x : D < n \rangle\}$
$\exists C$	$\top \sqsubseteq \neg C$
$\mathsf{Trans}(R)$	$\{\langle x : \exists R.(\exists R.\{y\}) \geq n \rangle, \langle x : \exists R.\{y\} < n \rangle\}$
$R \sqsubseteq S$	$\{\langle x : \exists R.\{y\} \geq n \rangle, \langle x : \exists S.\{y\} < n \rangle\}$
$\langle a : C \bowtie n \rangle$	$\langle a : C \neg\bowtie n \rangle$
$\langle (a, b) : R \bowtie n \rangle$	$\langle (a, b) : R \neg\bowtie n \rangle$
$\neg\bowtie$ is the contrapositive of $\bowtie$, e.g. if $\bowtie = \geq$, $\neg\bowtie = <$	

Finally we need to reduce f-$\mathcal{SHOIN}$ entailment to f-$\mathcal{SHOIN}$ unsatisfiability. Since we used the OWL Lite reduction for individual axioms, we additionally need to reduce two more entailment axioms. The complete reduction is depicted in Table 5. There are some remarks regarding Table 5. The notation $\exists C$, also called the non-emptiness construct, represent the satisfiability problem [22]. In the entailment problems of concept and role subsumption and transitive role axiom, where a membership degree n appears, it suffices to check for the unsatisfiability of the system for two randomly selected data values from the intervals $(0, 0.5]$ and $(0.5, 1]$, as it is shown in [17]. At last, observe how easy and straightforward is the reduction of the entailment of role assertions to unsatisfiability, in our case. In fact, since fuzzy DLs are just a generalization of crisp DLs, this reduction also holds in the crisp case, if we consider degrees 0 and 1. As for the case of f-OWL Lite, which corresponds to the DL f-$\mathcal{SHIF}(\mathbf{D}^{+})$, and does not support nominals, we can simply replace nominal concepts {y}, in Table 5, with a new concept B that does not appear elsewhere in the KB.

6 Querying the Fuzzy Semantic Web

After building an ontology, with the aid of the OWL ontology language, it would feel natural to be able to perform complex queries, or retrieval tasks, over the *ABox* defined. This is achieved by the use of so called *conjunctive queries*, which take the form of the formula $\exists x_1 ... x_n (q_1 \wedge ... \wedge q_n)$ where $q_1, ..., q_n$ are concept or role terms [23]. In classical DLs, we face the unpleasant phenomenon that if a tuple of the KB does not satisfy the exact constraints of the query, issued by the user, then it is not included in the result. This is due to the fact that the KB engineer made some specific choices about the membership or non-membership of an individual to certain concepts when building the knowledge base.

In contrast, fuzzy KBs provide us with an interesting feature by which we can overcame the above deficiency. Instead of specifing the exact membership degrees that an individual (pair of individuals) should belong to a concept (role), one could leave such constraints unspecified. The result would be to retrieve every tuple of the knowledge base that participates in the assertions to any degree. Then interpreting the conjunctions as fuzzy intersections, we can provide a ranking of tuples and present the user with an initial set of the most relevant information. Subsequently the user can choose if more results should be fetched. This is a very interesting feature of f-DLs, since ranking is considered a very crucial feature in every information retrieval application [7]. Please note that how to answer such types of queries in fuzzy KBs is an open research issue.

7 Related Work

Much work towards combining fuzzy DLs has been carried out the last decade. The initial idea was presented by Yen in [24], where a *structural subsumption* algorithm was provided in order to perform reasoning and the DL used a sublanguage of the basic DL $\mathcal{ALC}$. Several approaches extending the DL language $\mathcal{ALC}$ were latter presented in [17, 25, 26] where reasoning was based on tableaux rules. In [25, 26] an additional constructor called *membership manipulator* was added for defining new fuzzy concepts from already defined ones. Approaches towards more expressive DLs, are presented in [27], [28] and [18] where the DLs are $\mathcal{ALCQ}$, $\mathcal{ALC}(\mathcal{D})$ and $\mathcal{SHOIN}(\mathbf{D}^+)$, respectively. The former approach also includes fuzzy quantifiers. In [27] and [18], only the semantics were provided while in [28] also reasoning, based on an optimization technique. As far as we know the most expressive f-DLs presented till now, which also cover reasoning, are f_{KD}-$\mathcal{SI}$ [8] and f_{KD}-$\mathcal{SHIN}$ [21]. In the current paper we fully cover the f-OWL language and provide investigations for the translation method, querying and syntax extensions.

8 Conclusions

Representing and reasoning with uncertainty is expected to play a significant role in future ontology based applications. Uncertainty is a factor that is apparent in many real life applications and domains [11, 10, 4, 7]and dealing with it can provide means for more expressive and realistic knowledge based systems [9, 6]. To this end we have provided a fuzzy extension to the OWL web ontology language. We have provided the semantics and abstract syntax of fuzzy OWL, as well as a reduction technique from f-OWL to the f-$\mathcal{SHOIN}$ DL. The last reduction aims at providing reasoning support for f-OWL ontologies. At last we show that f-$\mathcal{SHOIN}$ can support query services that go beyond the classical ones, by providing ranking degrees to the result of a query.

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