Metric Temporal Logic and Faulty Turing Machines

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AVOCS 2005

Automated Verification

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  - *Qualitative* model of time (ordered sequences of states/actions).
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- Real-time theory:
  - Timed automata, quantitative temporal logic, model checking, ...
  - *Quantitative* and dense model of time.
  - Theoretical foundations under active development.

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  - "Every request is eventually granted."

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- A *time-bounded* response property:
  - "Every request is eventually granted *within 5 time units*."

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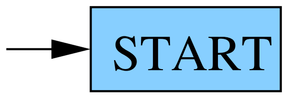
Example: A Login Protocol



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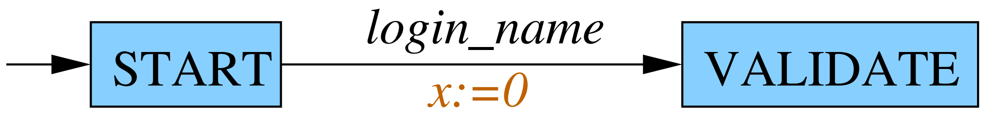

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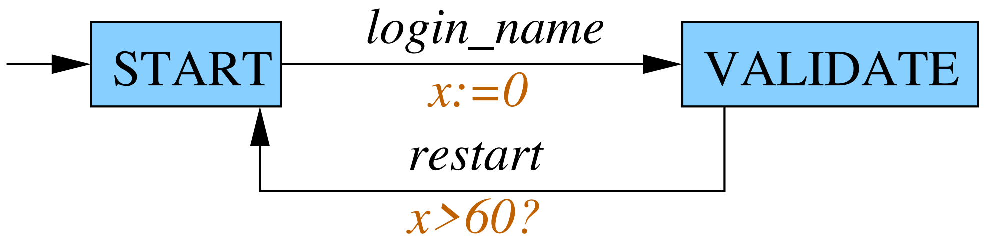

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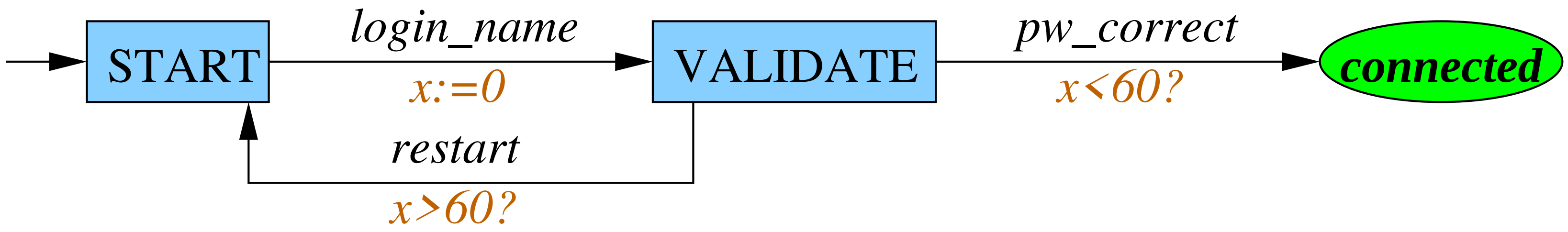

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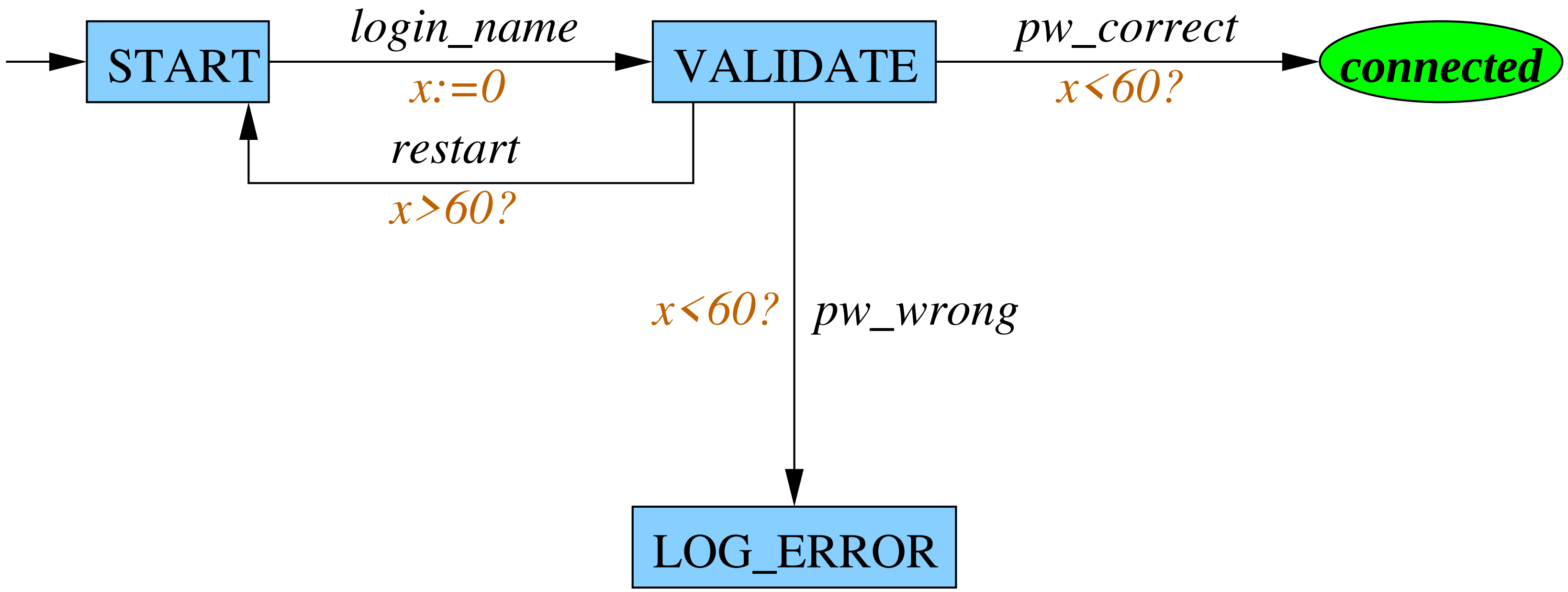

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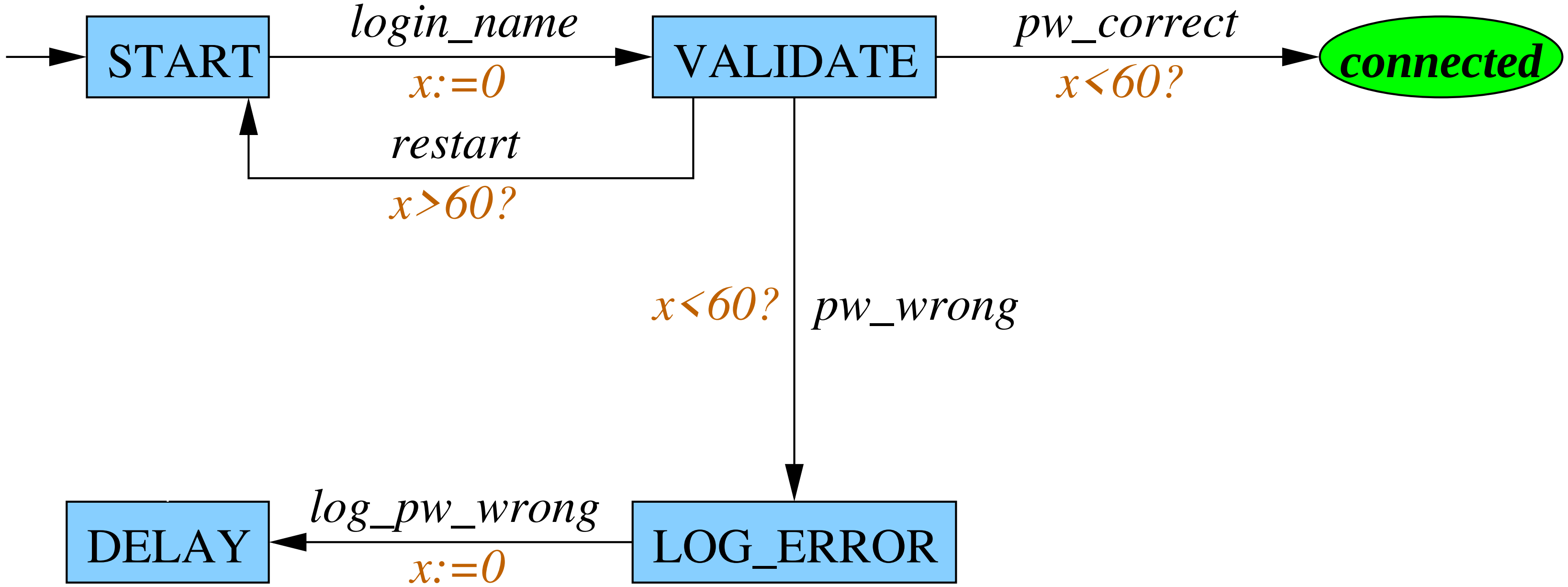

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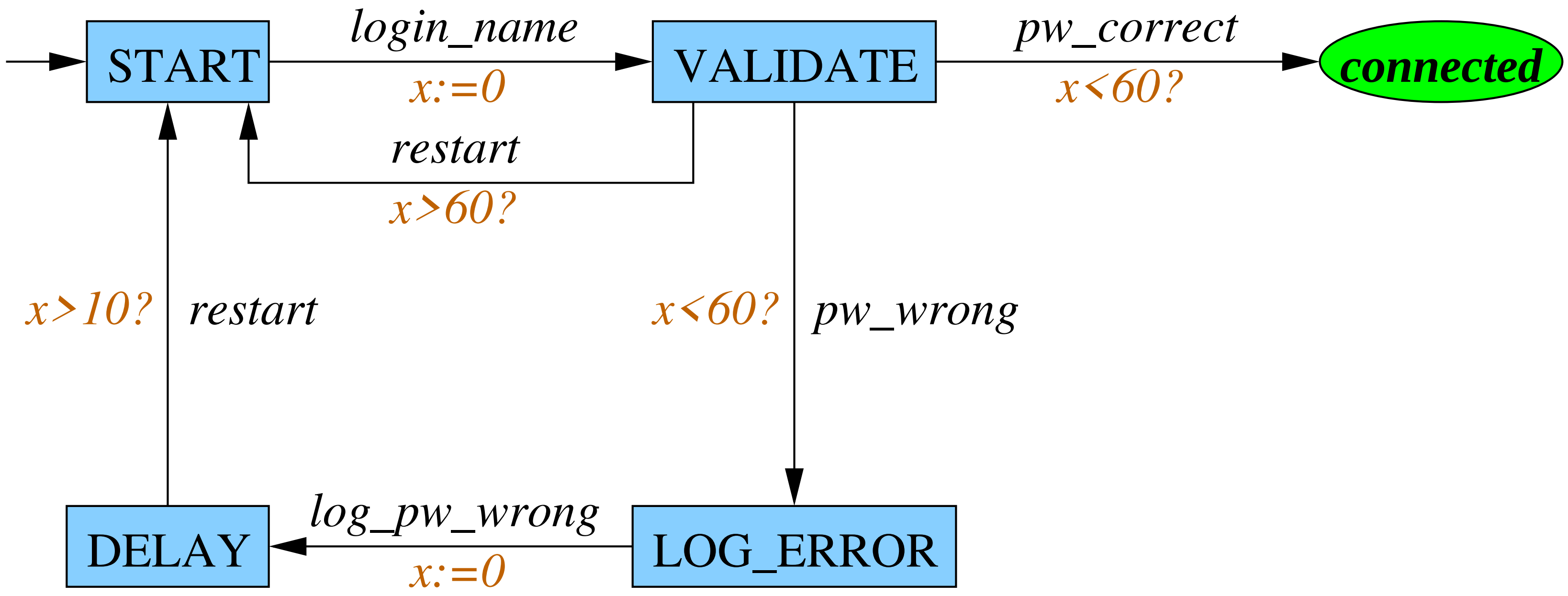

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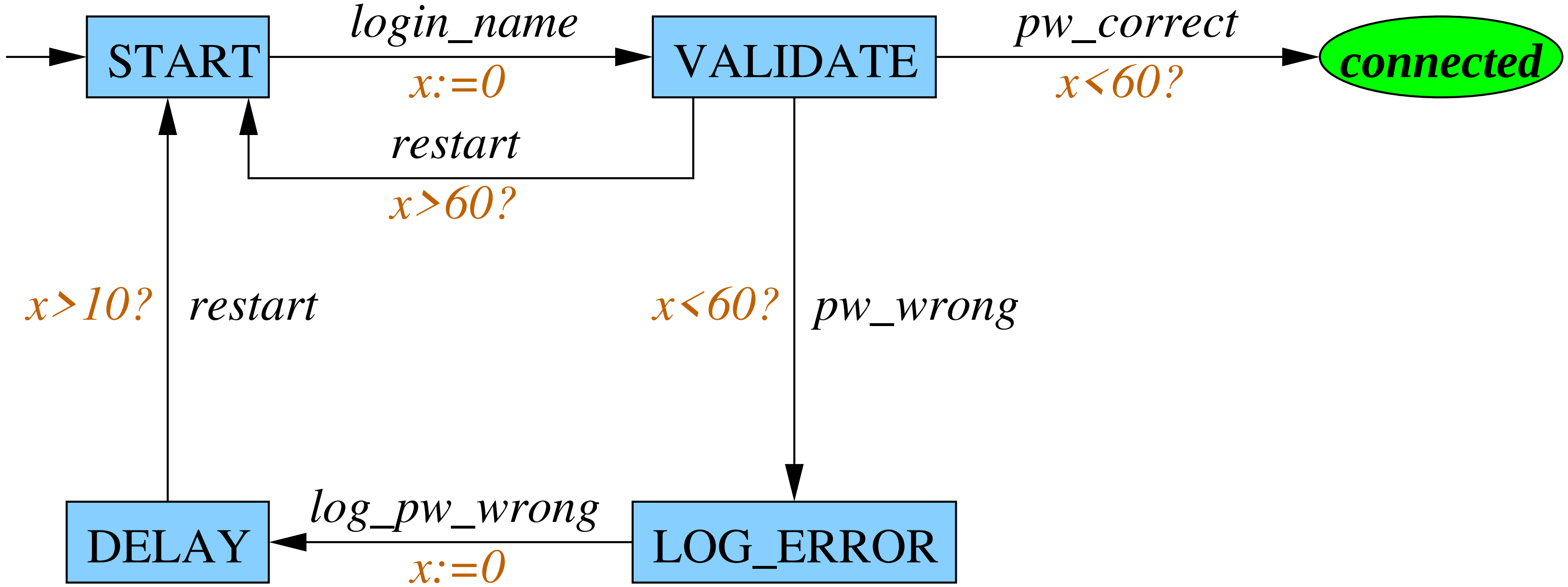

SPECIFICATION: $\square(pw\_wrong \longrightarrow \square_{[0,10]} \neg restart)$

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- **Metric Temporal Logic (MTL)** is a prominent such dense-time specification formalism.
  - Over two hundred and fifty papers according to `scholar.google.com` !

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  where a is atomic and $I \subseteq \mathbb{R}$ is an interval with integer (or unbounded) endpoints.
- Modal operators $\Box_I$ and $\Diamond_I$ are derived from $\mathcal{U}_I$ in the usual way.

Timed Words

- A **timed word** is a finite or infinite sequence of **timed events**:

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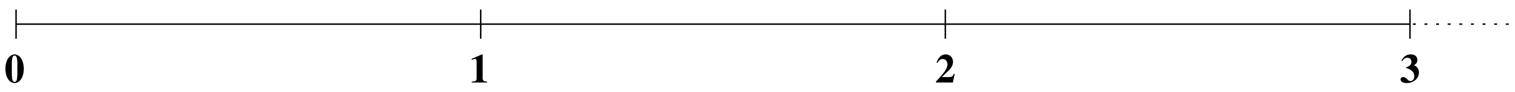

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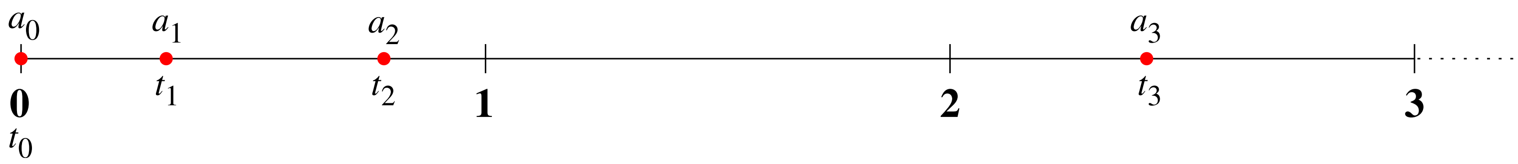

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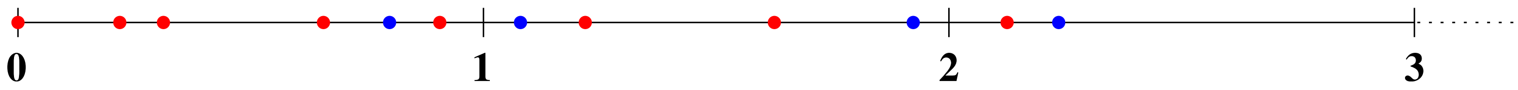

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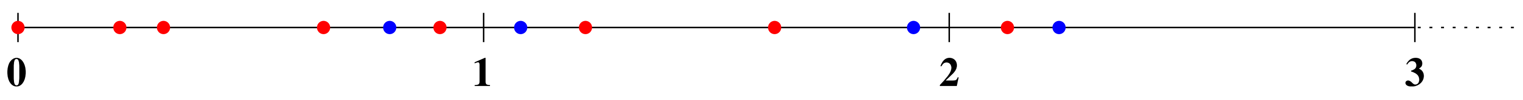

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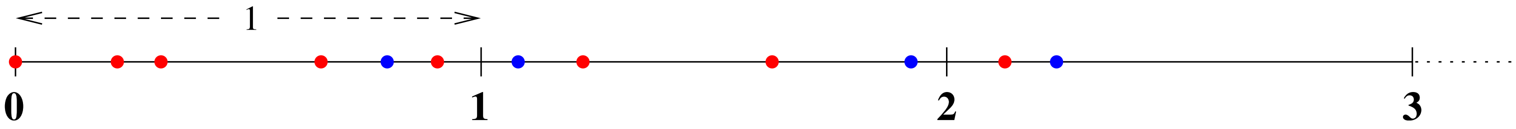

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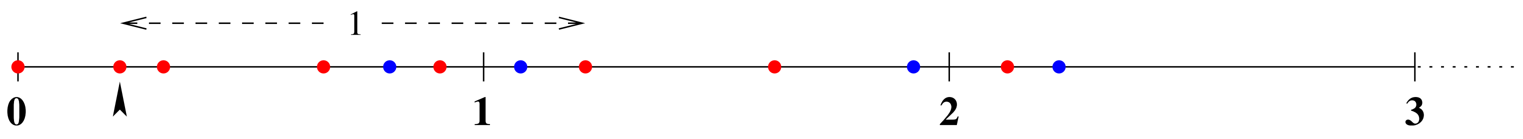

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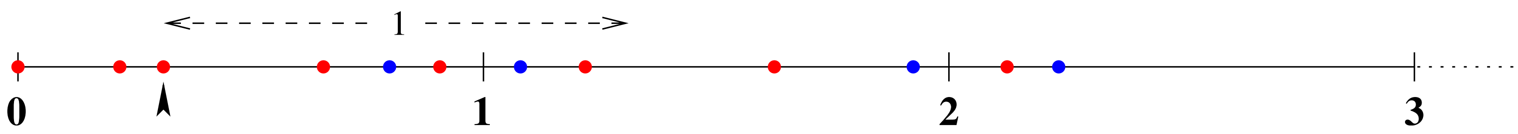

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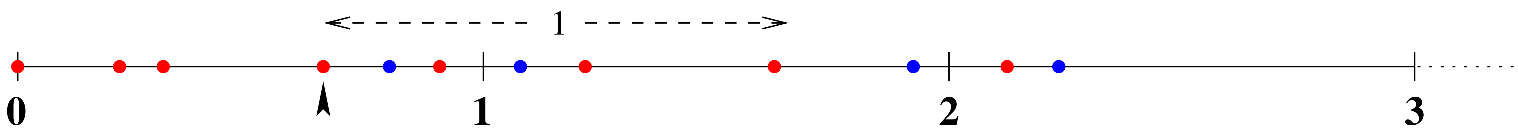

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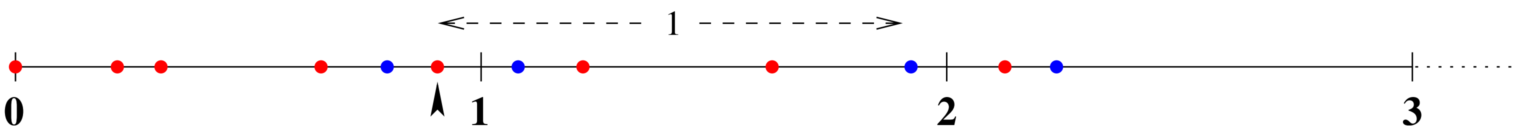

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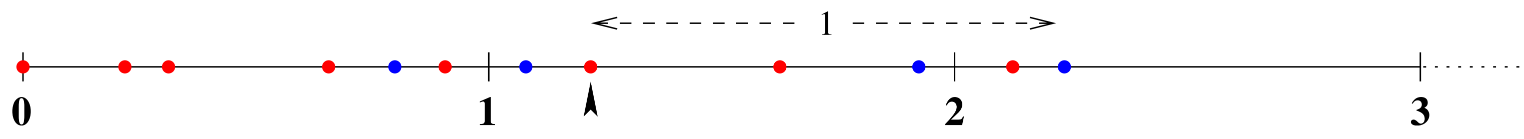

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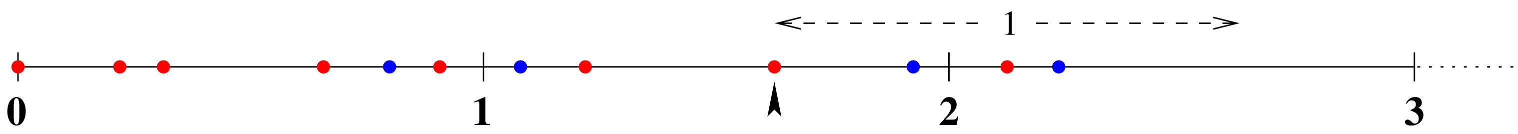

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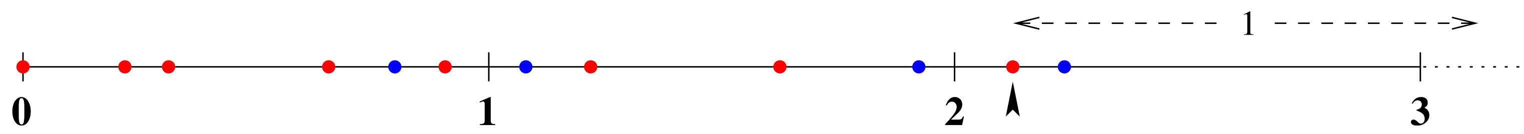

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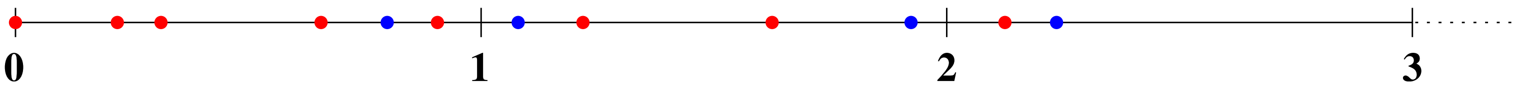

Indeed, $w \models \varphi$.

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- The dichotomy ‘point-based vs. interval semantics’ can be read as ‘event-based vs. state-based semantics’.

Satisfiability and Model Checking

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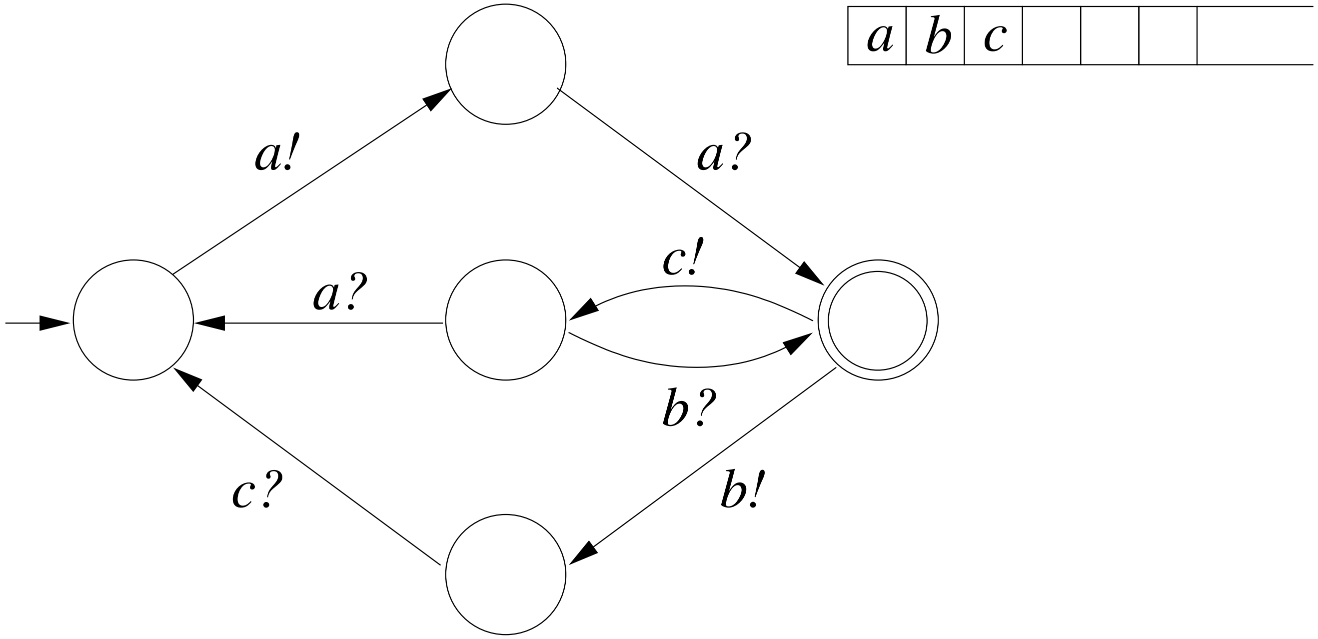
- **Model checking:** Given φ and a timed automaton A, does $w \models \varphi$ for all $w \in L(A)$?
  - For simplicity we mainly focus on satisfiability in this presentation.

Interlude: Channel Machines

Our undecidability results use **channel machines**.



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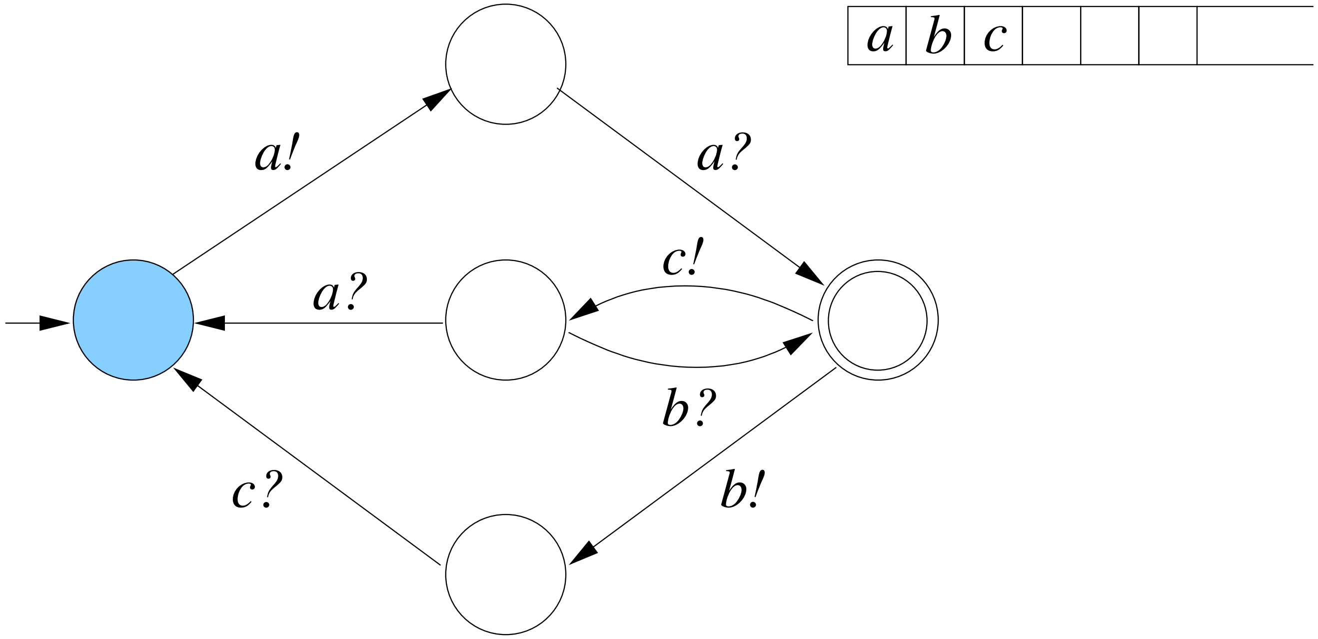

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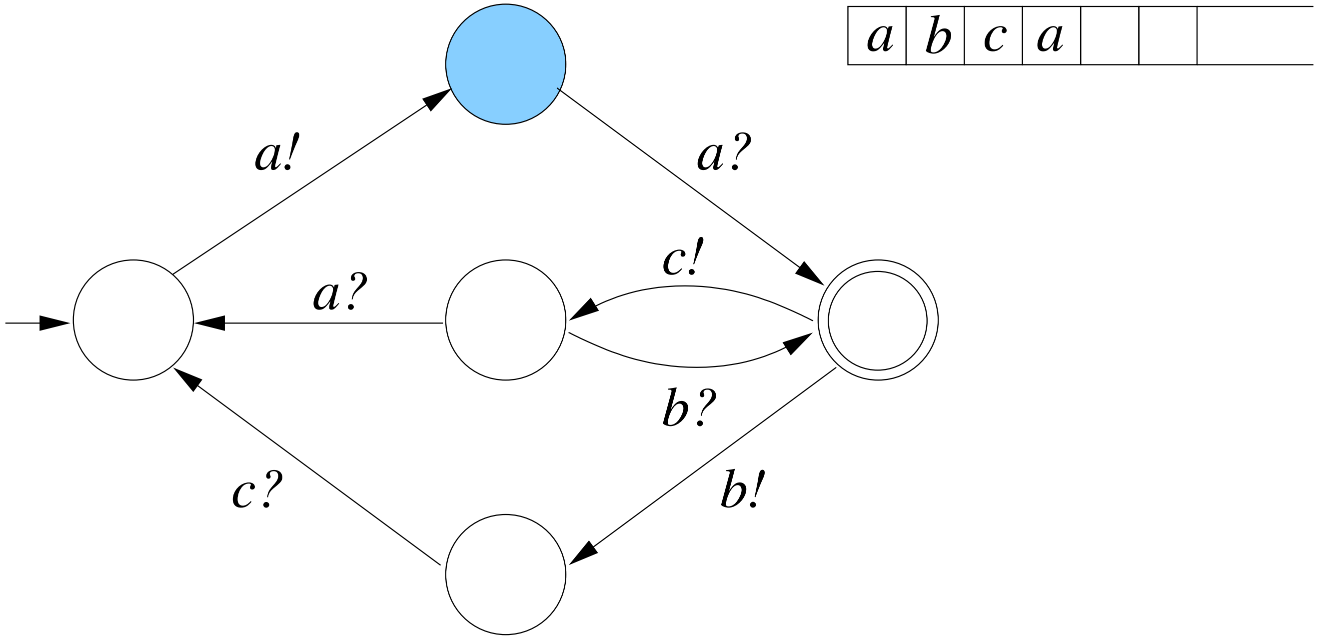

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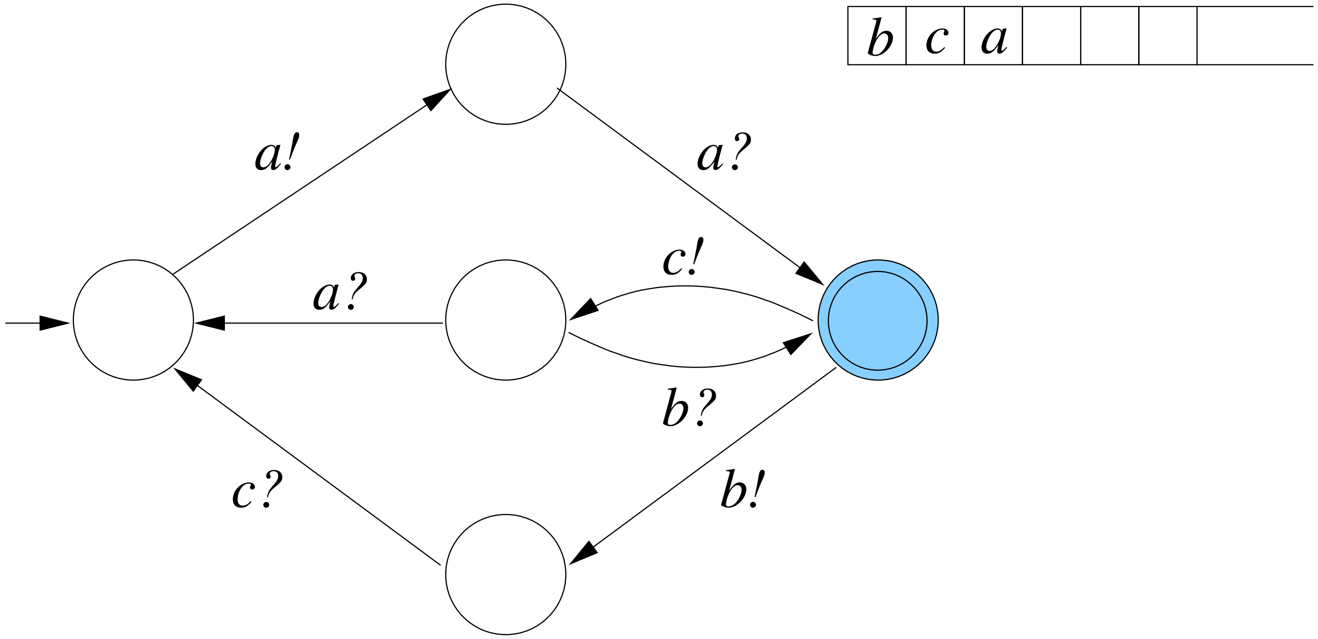

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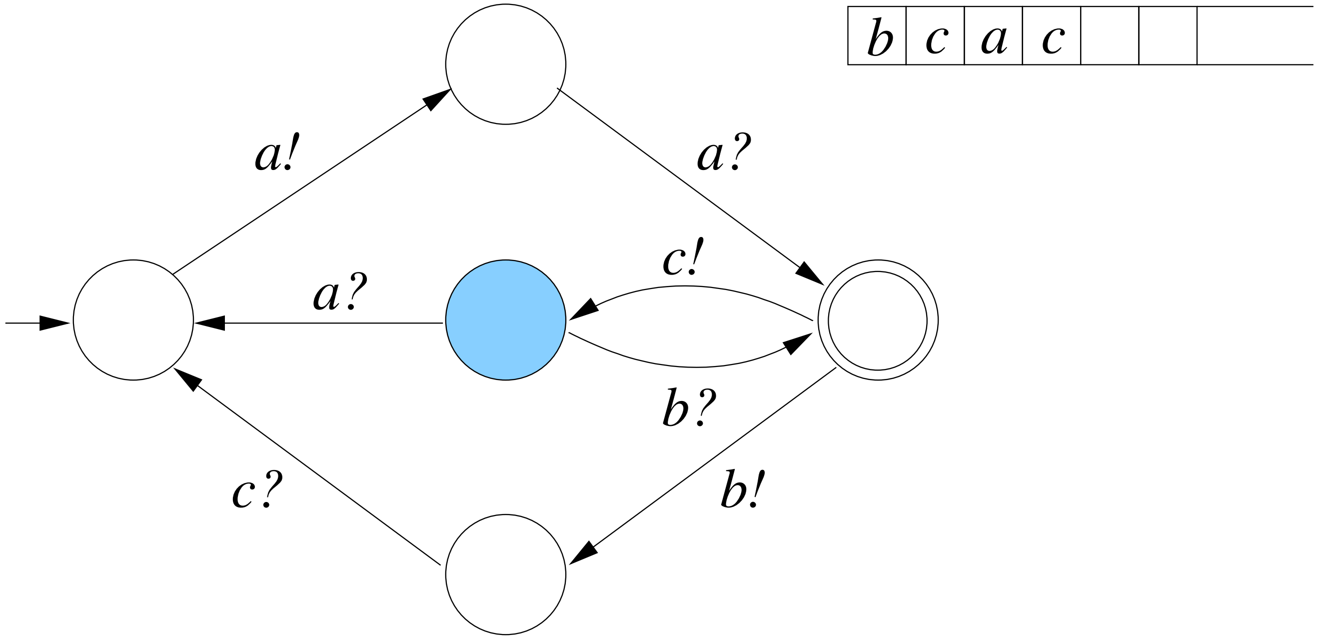

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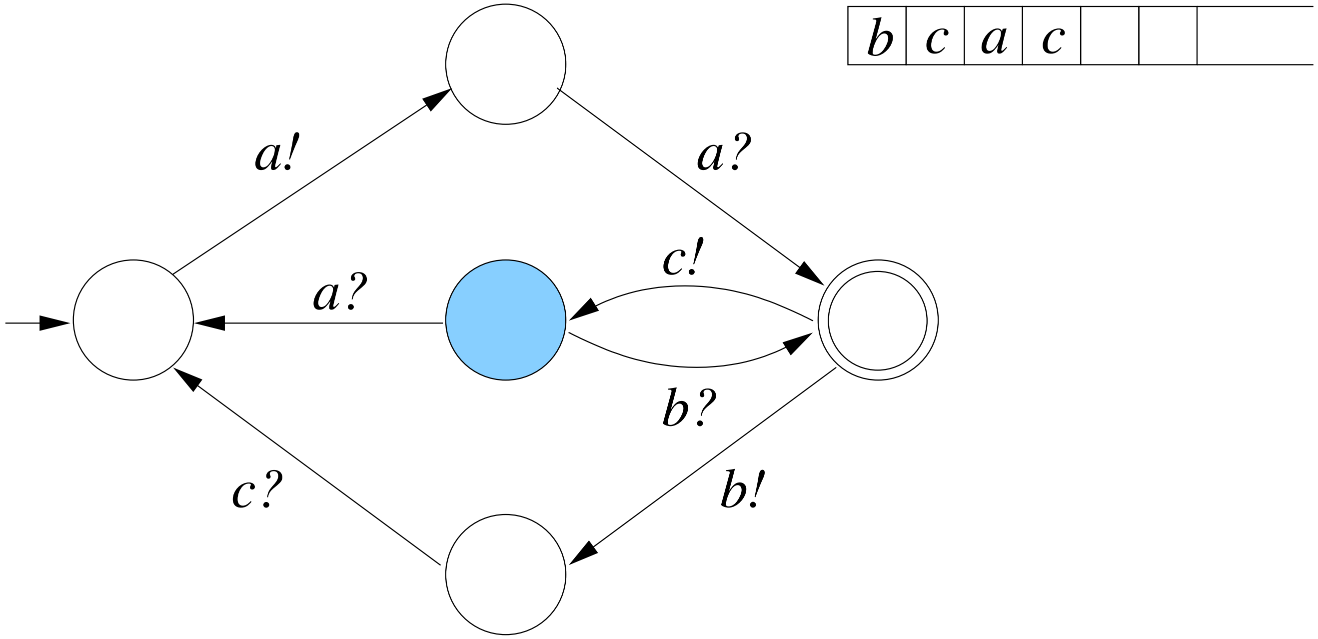

Thm. The halting problem for channel machines is undecidable.

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Write down an MTL formula φ that captures precisely those timed words corresponding to halting computations of C.

Then C halts iff φ is satisfiable.

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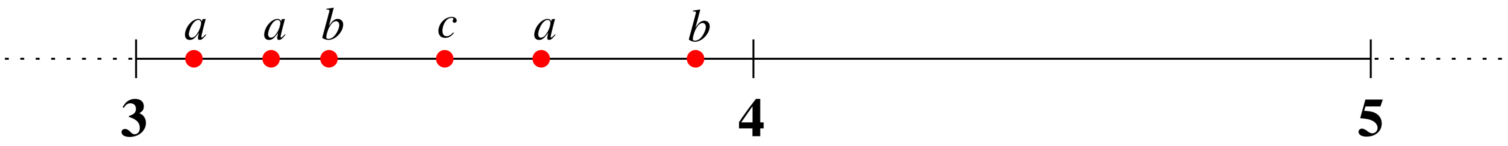

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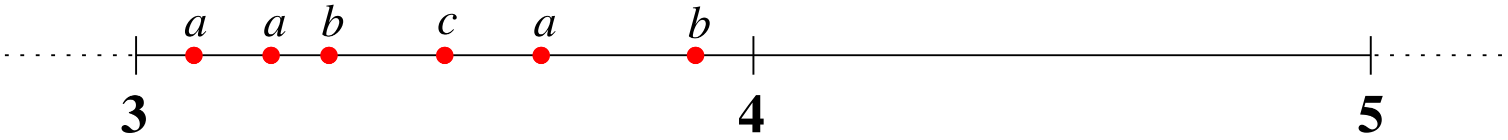

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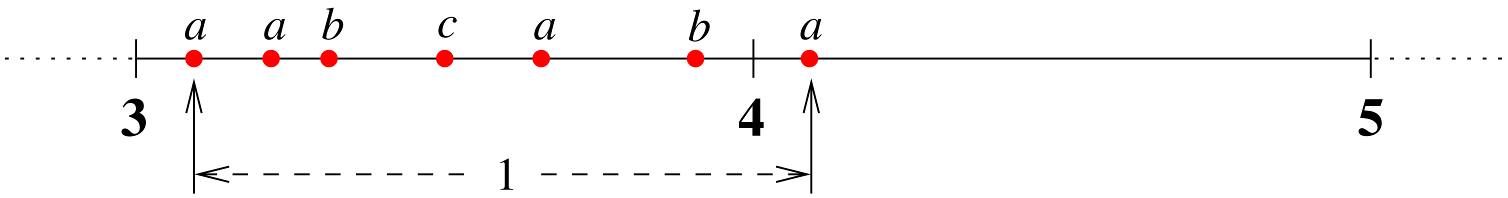

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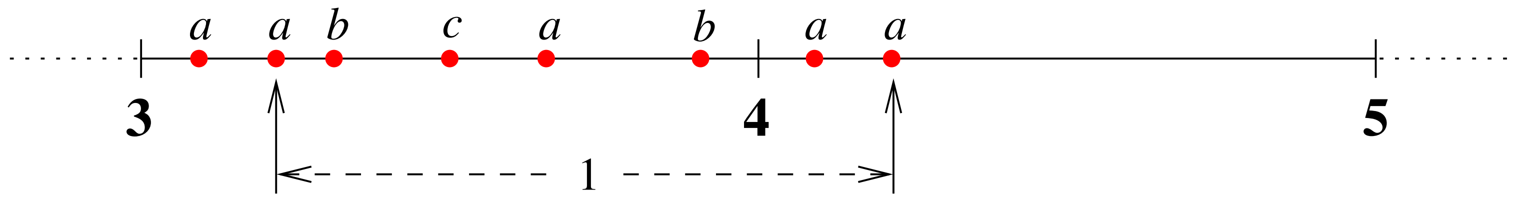

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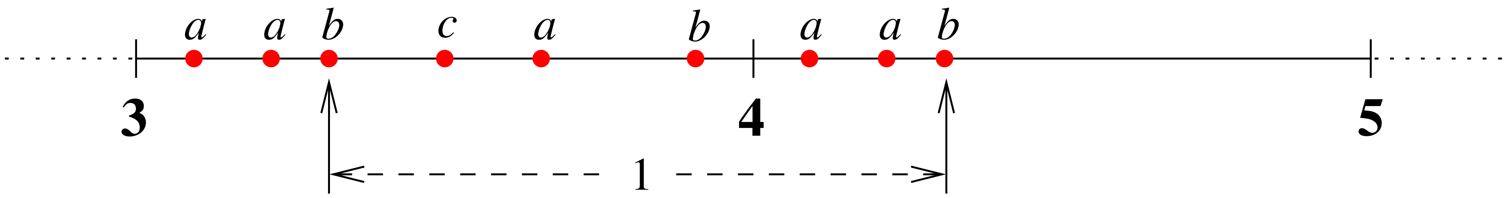

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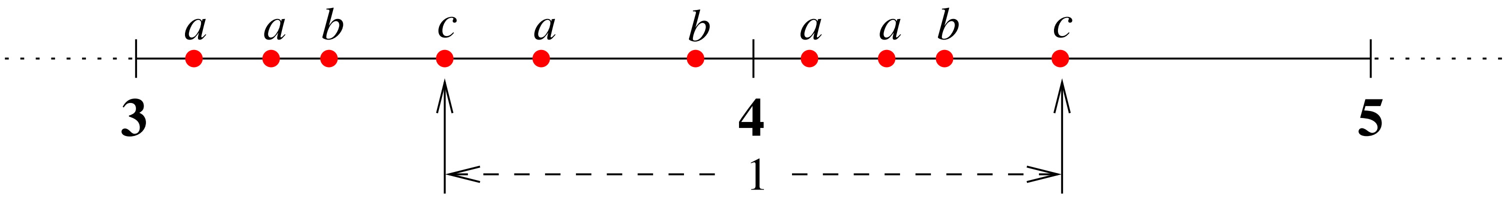

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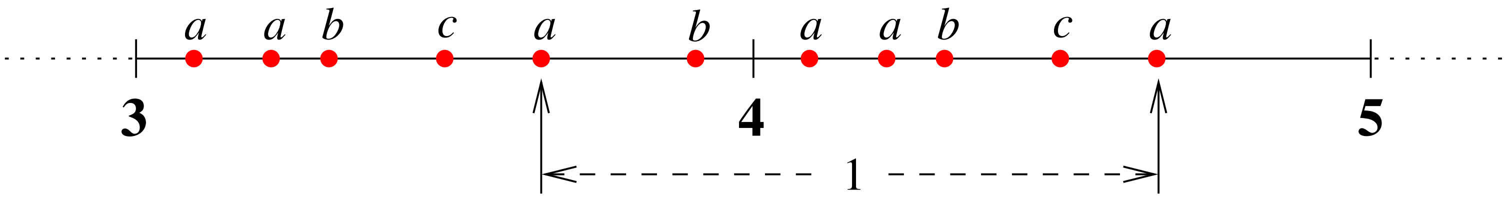

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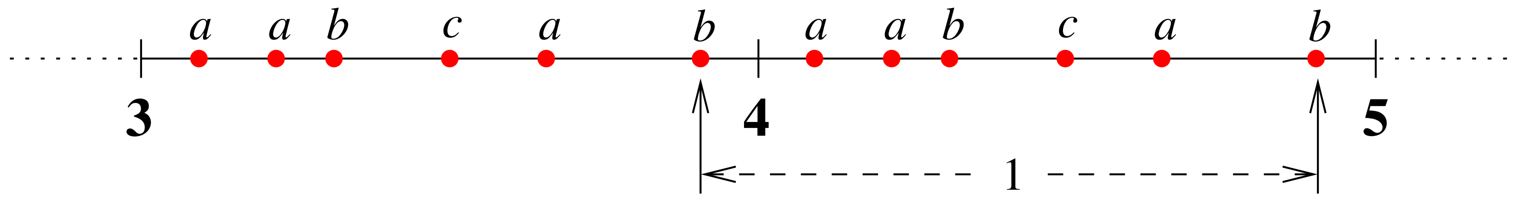

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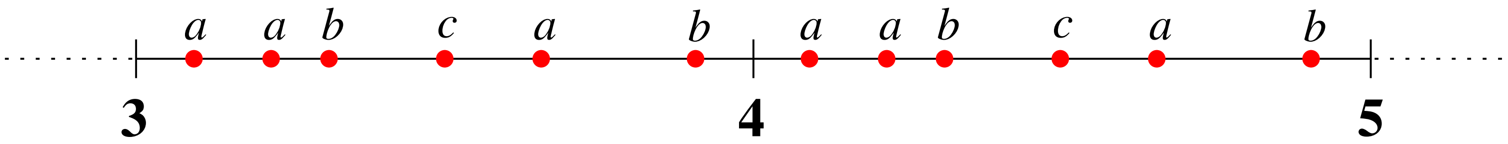

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Insertion Errors!

There is something missing — the formula

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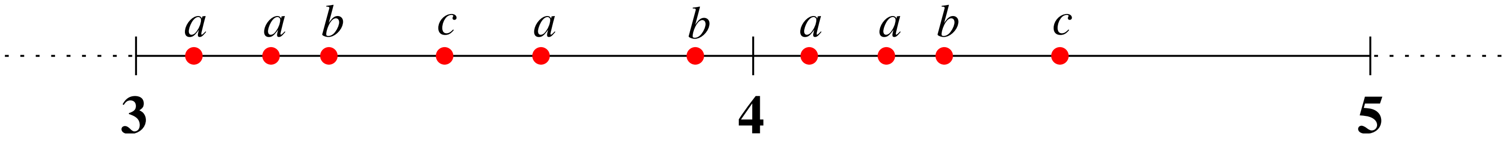
only enforces **forward** propagation of events:



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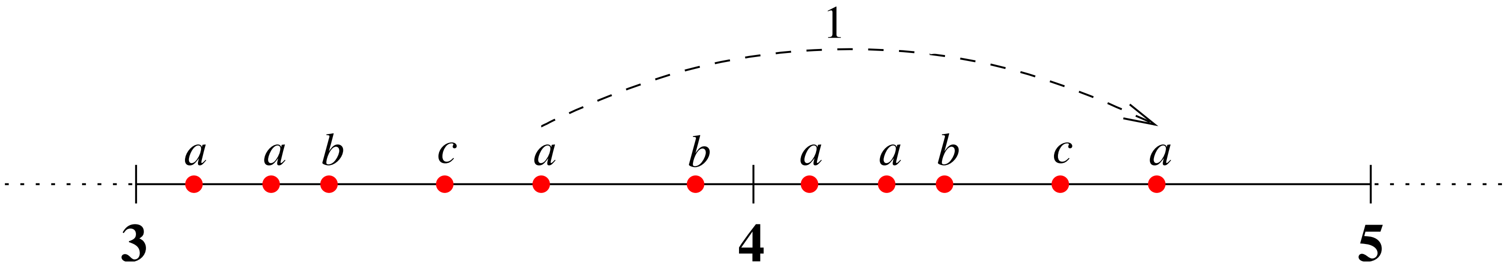

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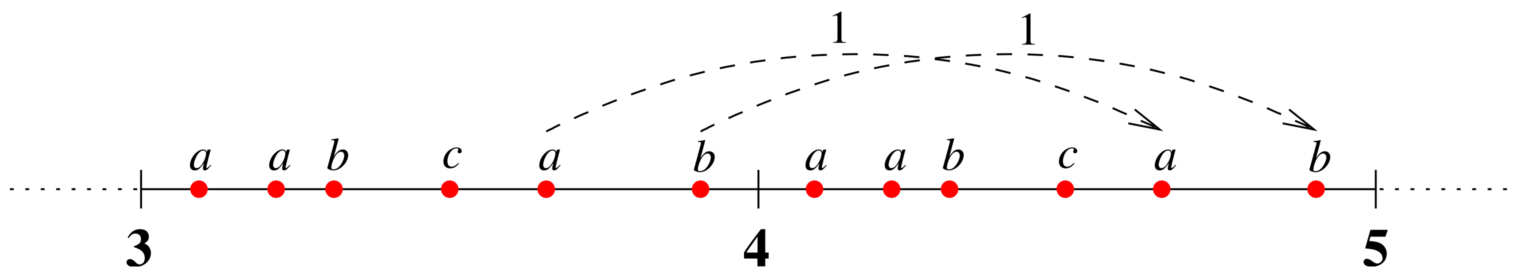

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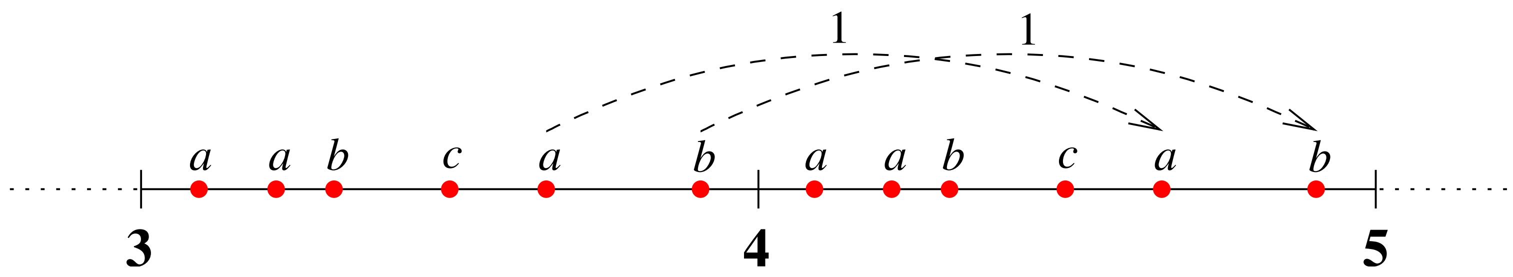

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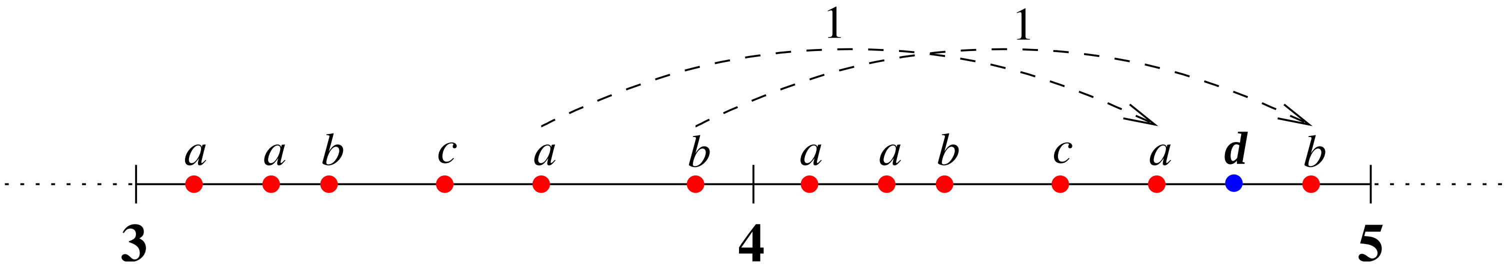

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Undecidability for MTL

- The decidability of a number of real-time logics was considered in *Real-time Logics: Complexity and Expressiveness* [Alur and Henzinger 93].
- The undecidability proofs of Alur and Henzinger rely on encoding computations of Turing machines as timed words. They only apply to richer formalisms than MTL.

Our Contribution

	Satisfiability	Model Checking
MTL (Inf.)	??	??
MTL (Fin.)	??	??
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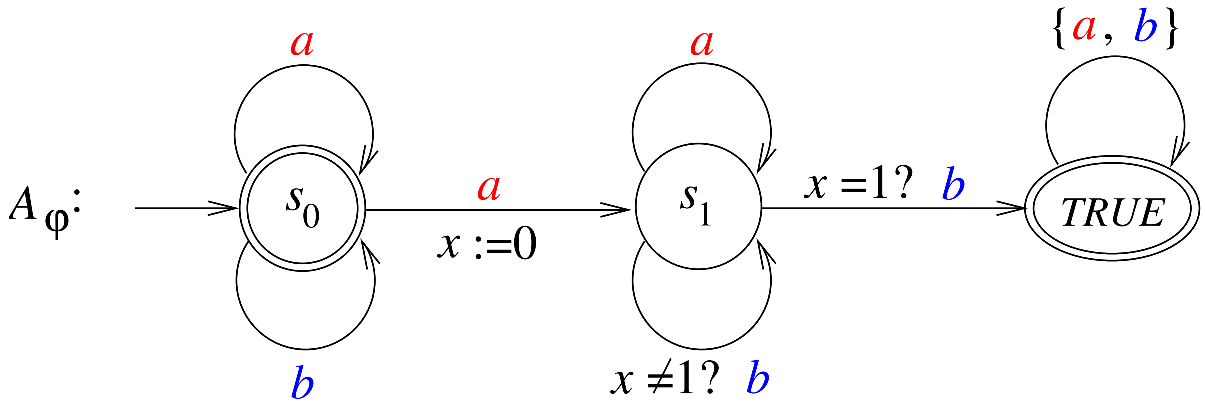

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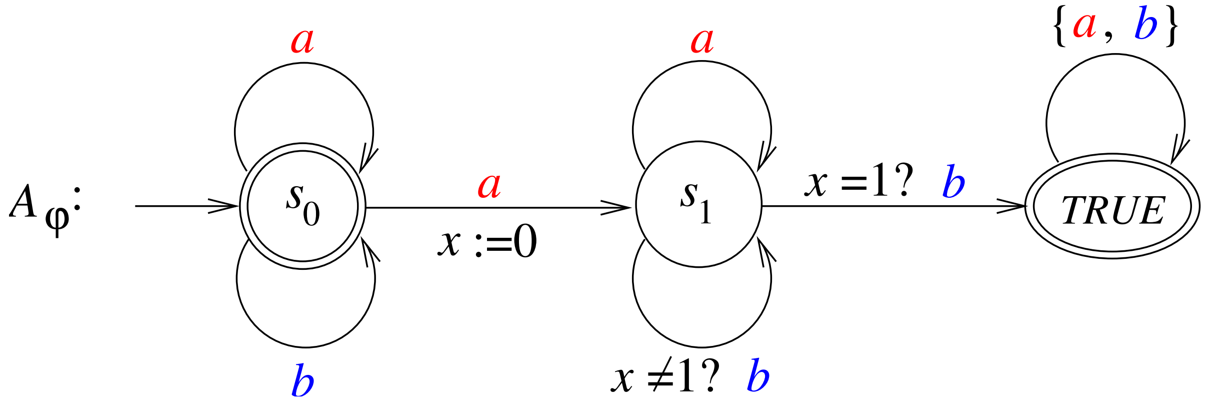

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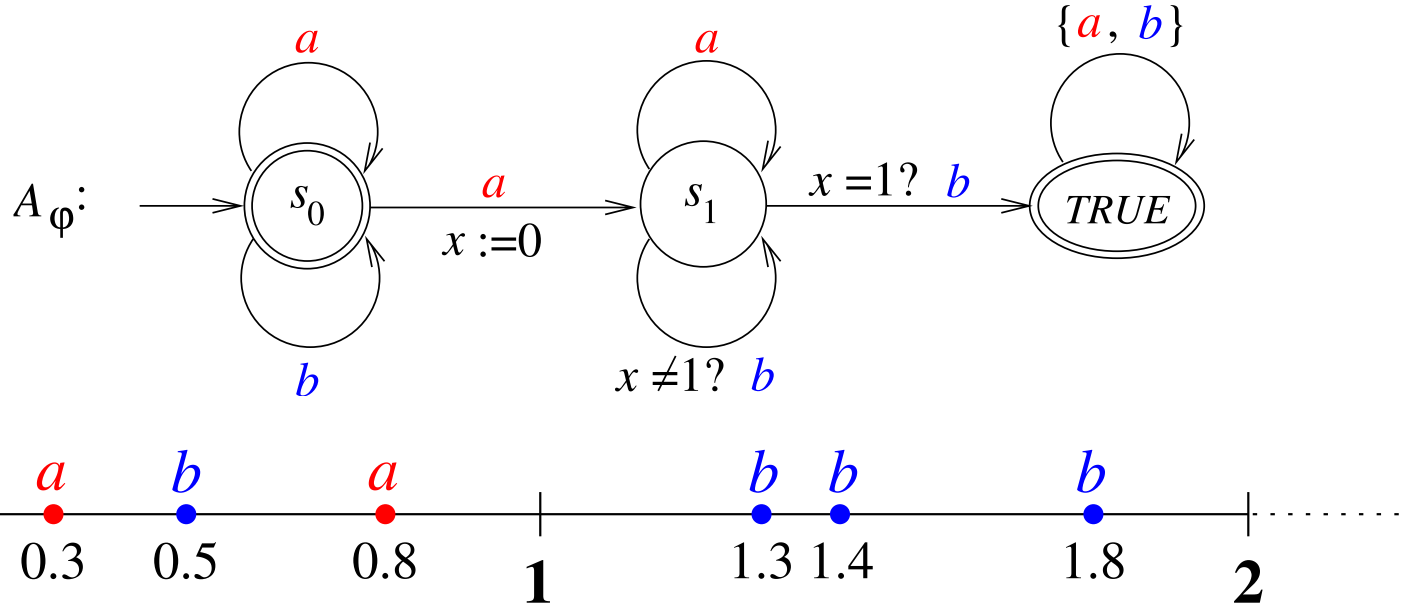

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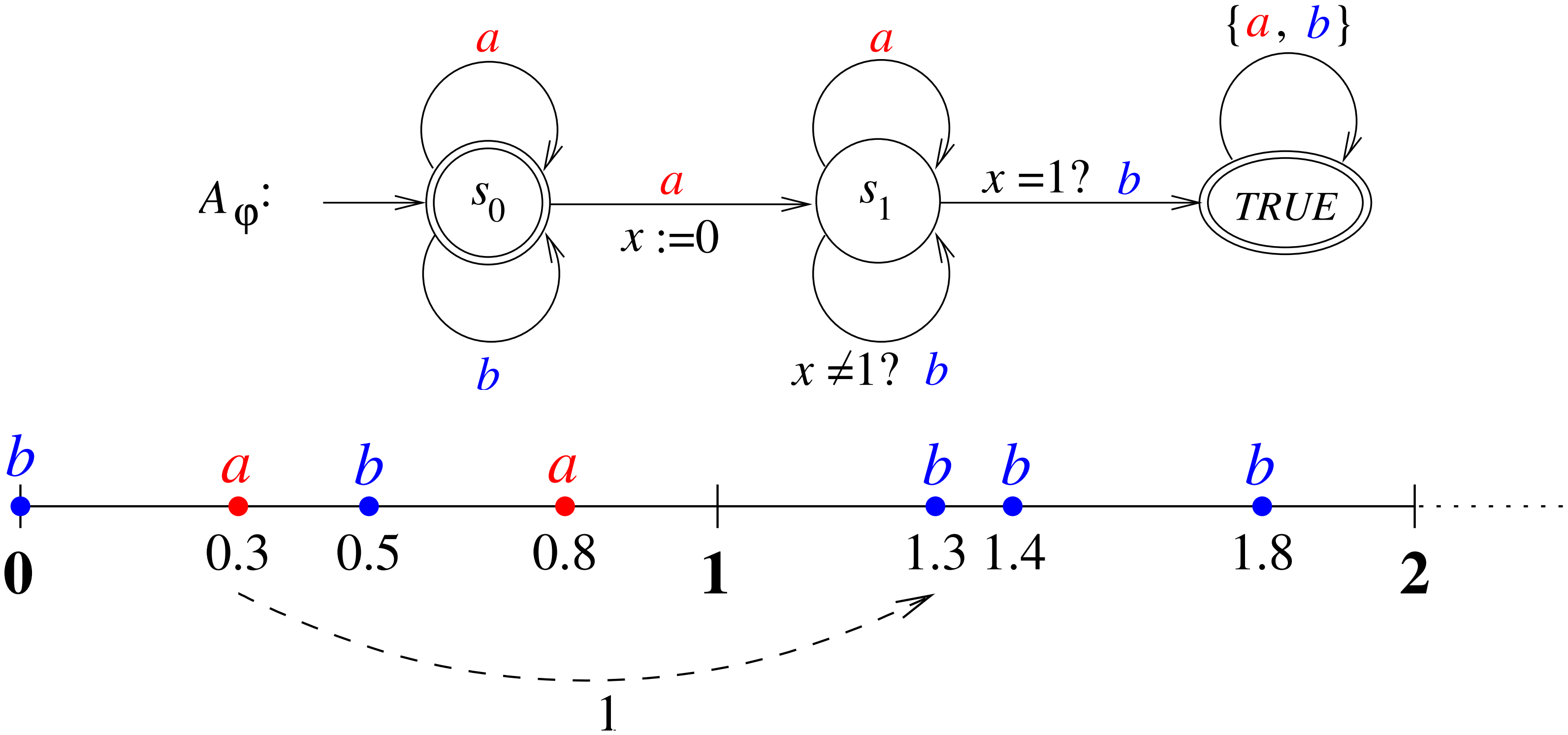

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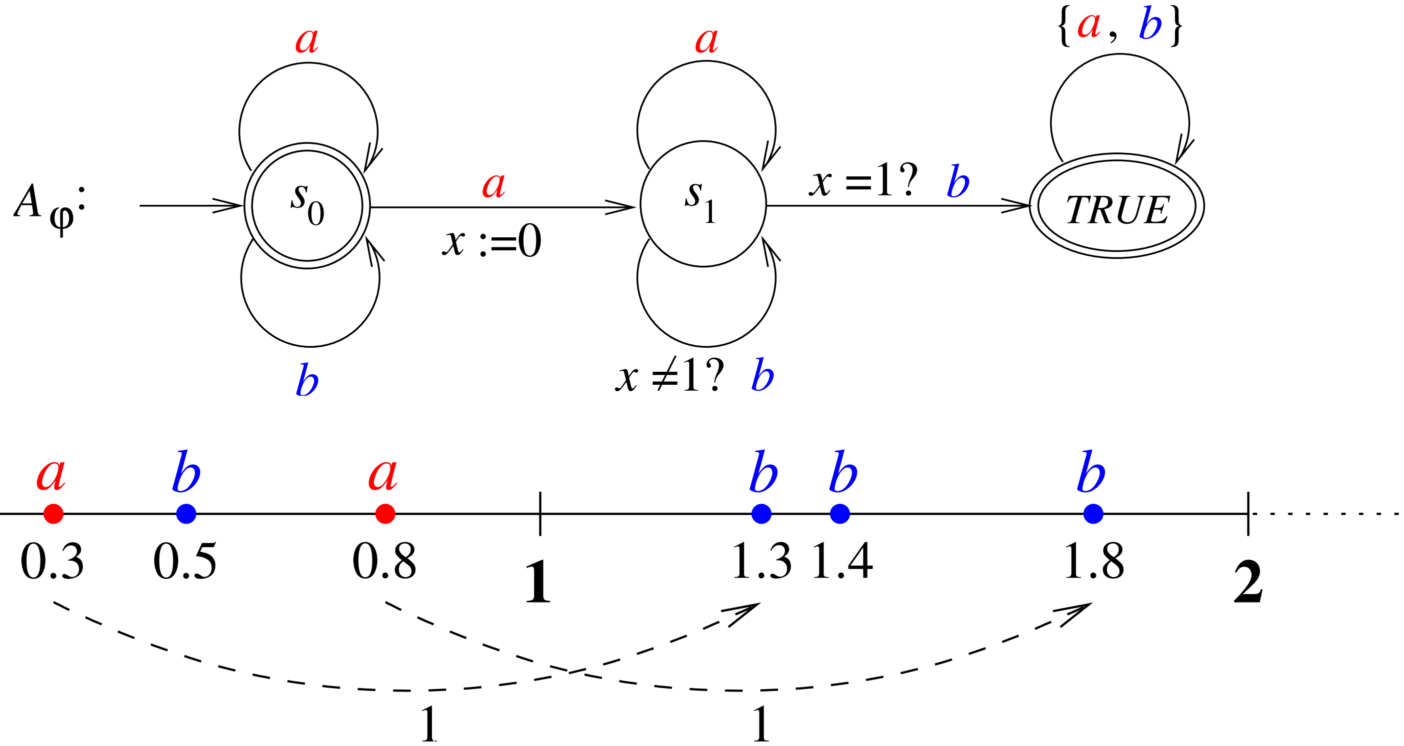

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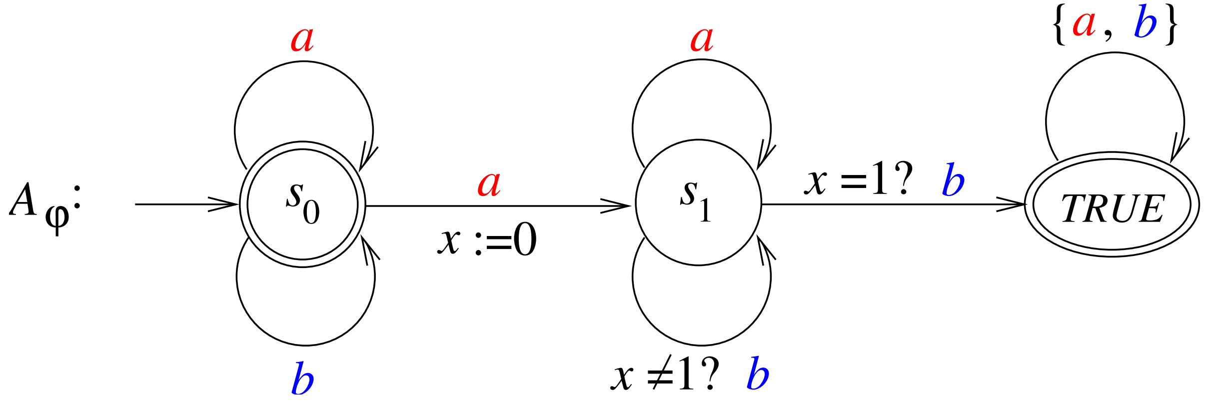

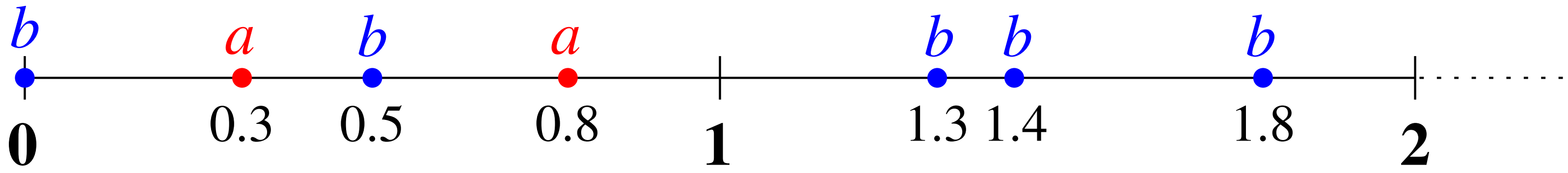

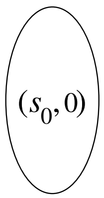

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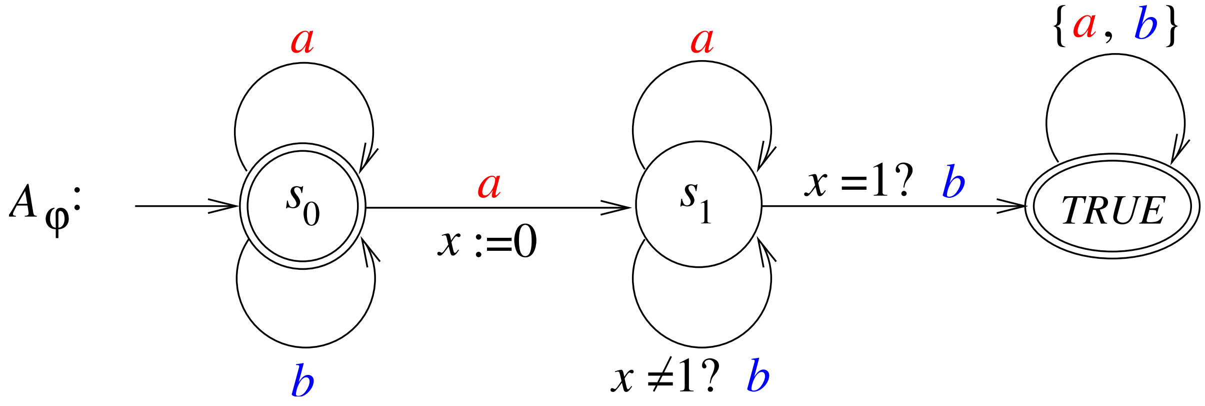

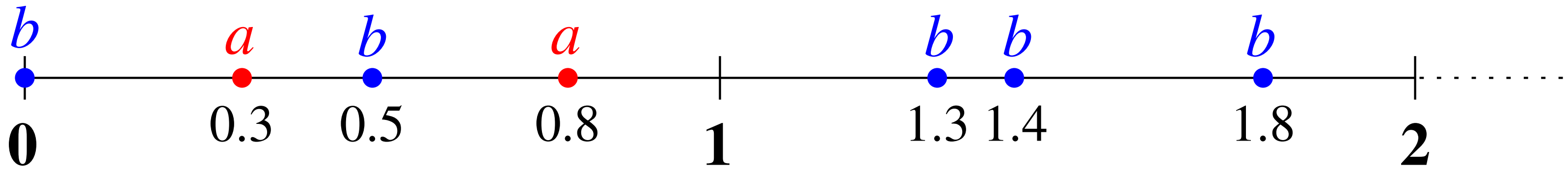

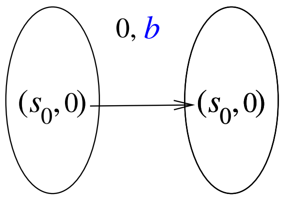

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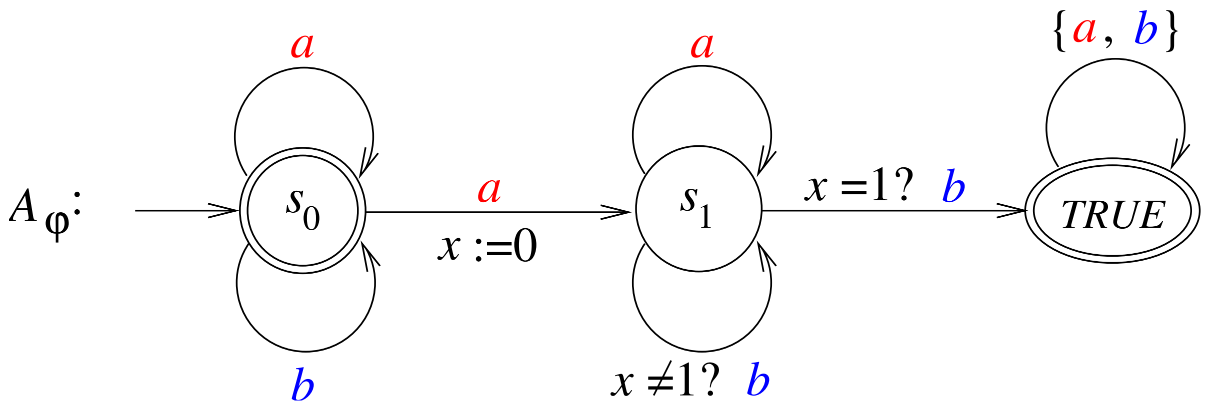

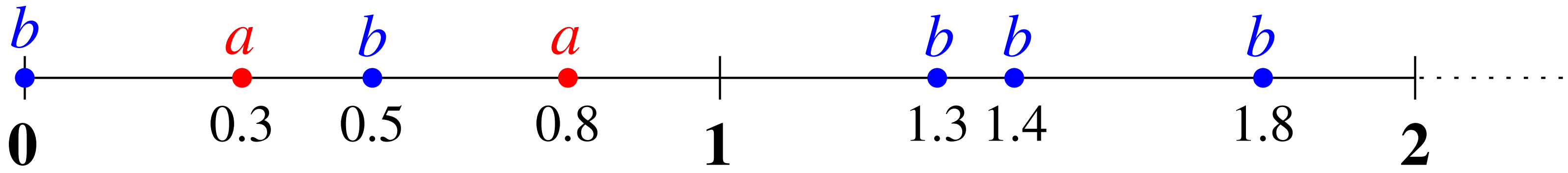

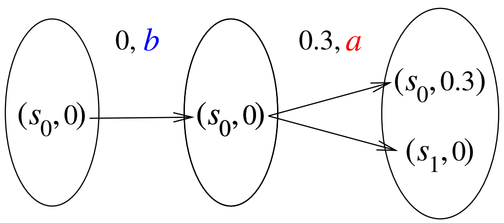

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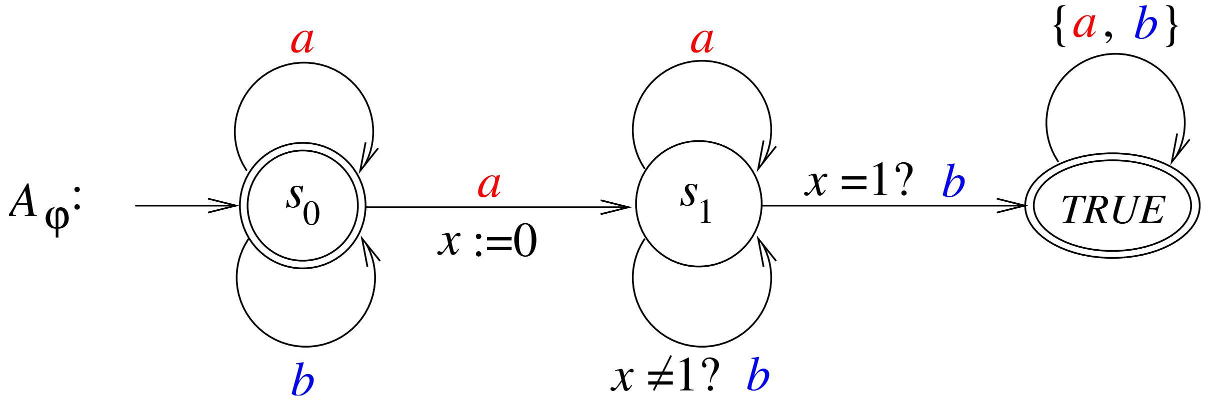

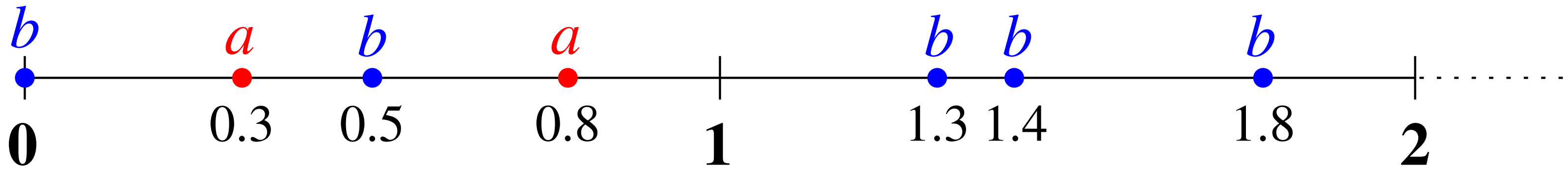

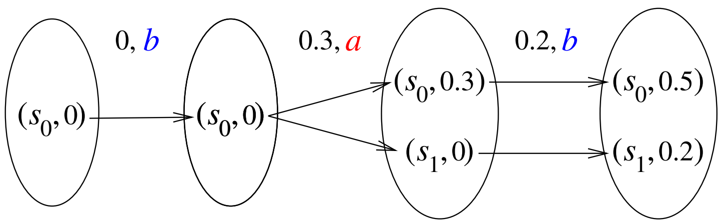

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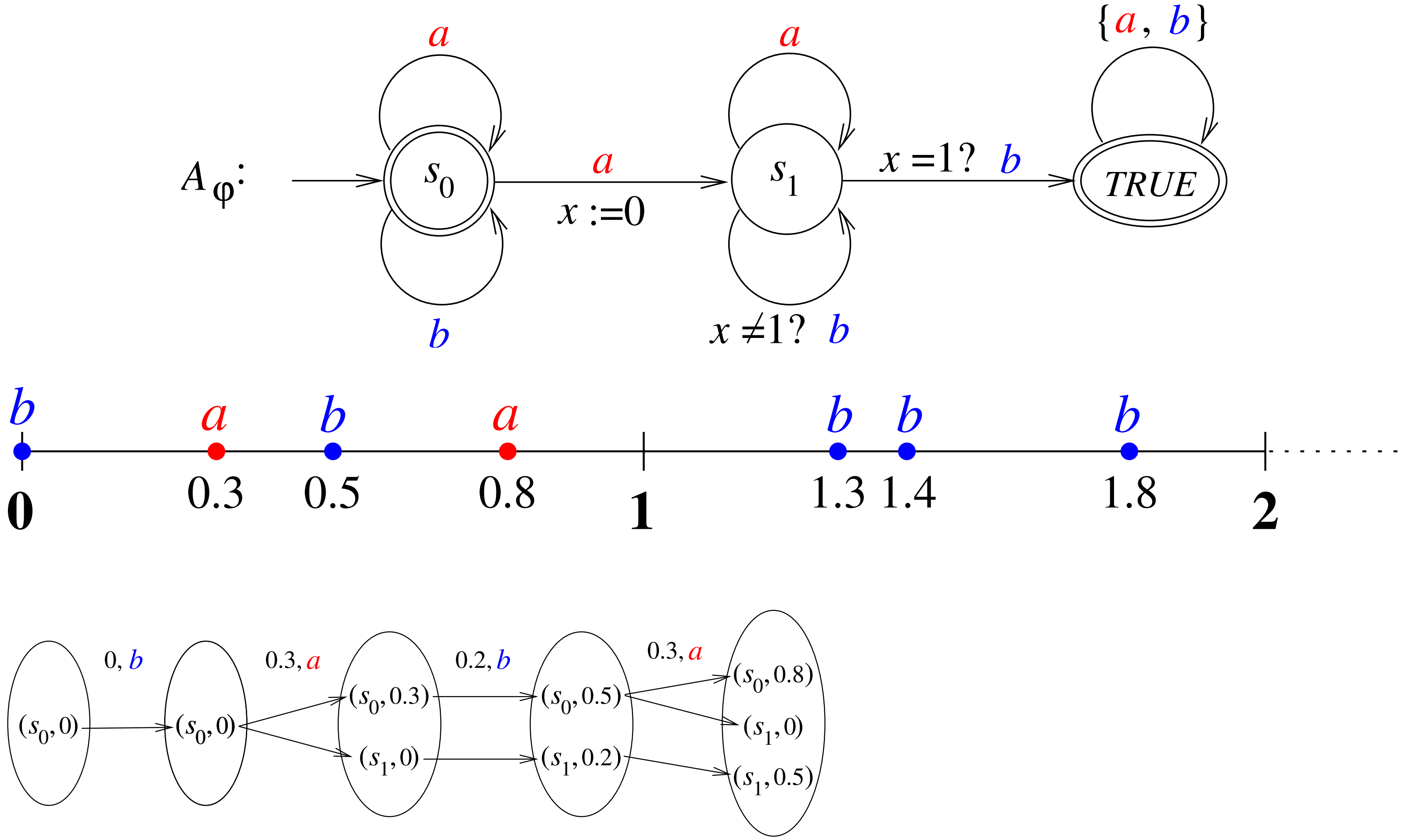

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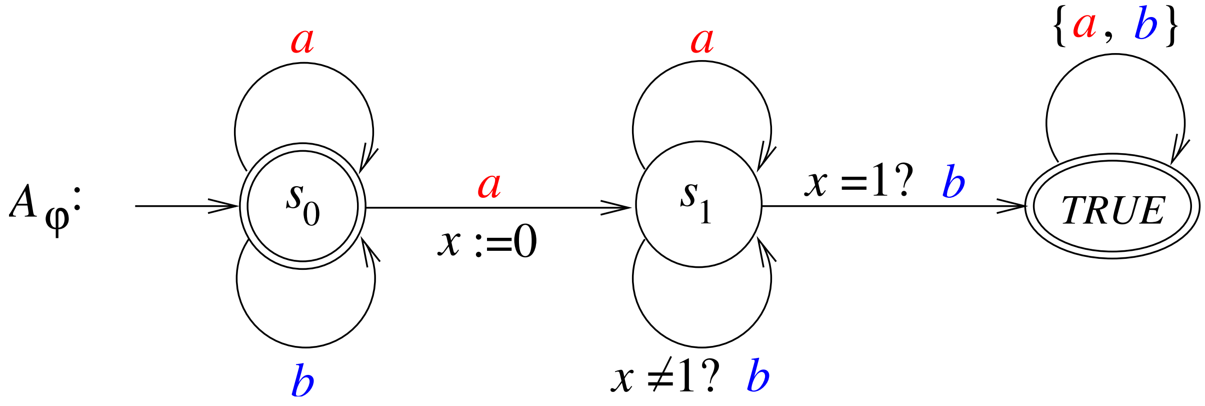

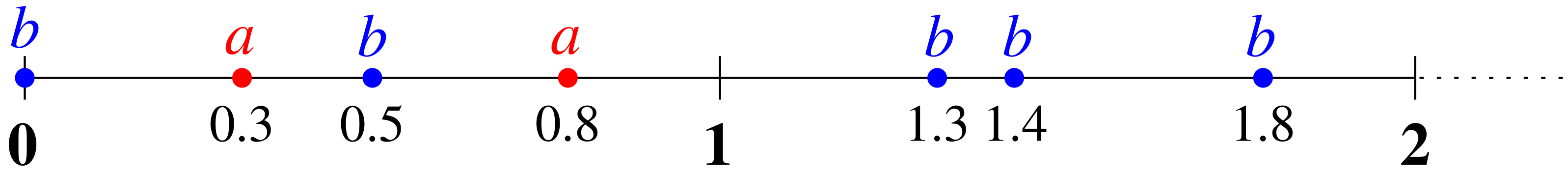

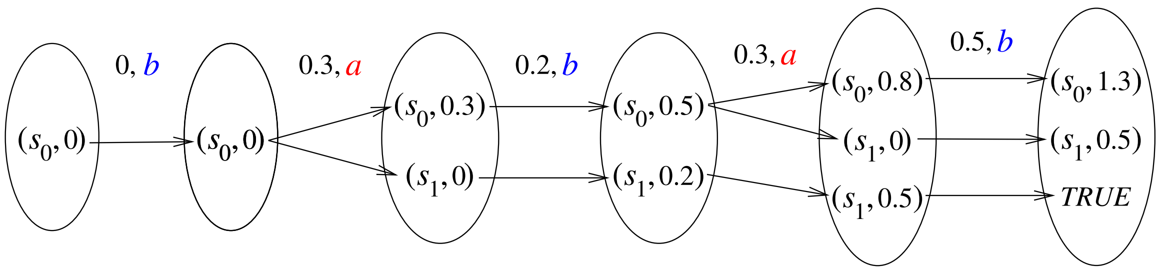

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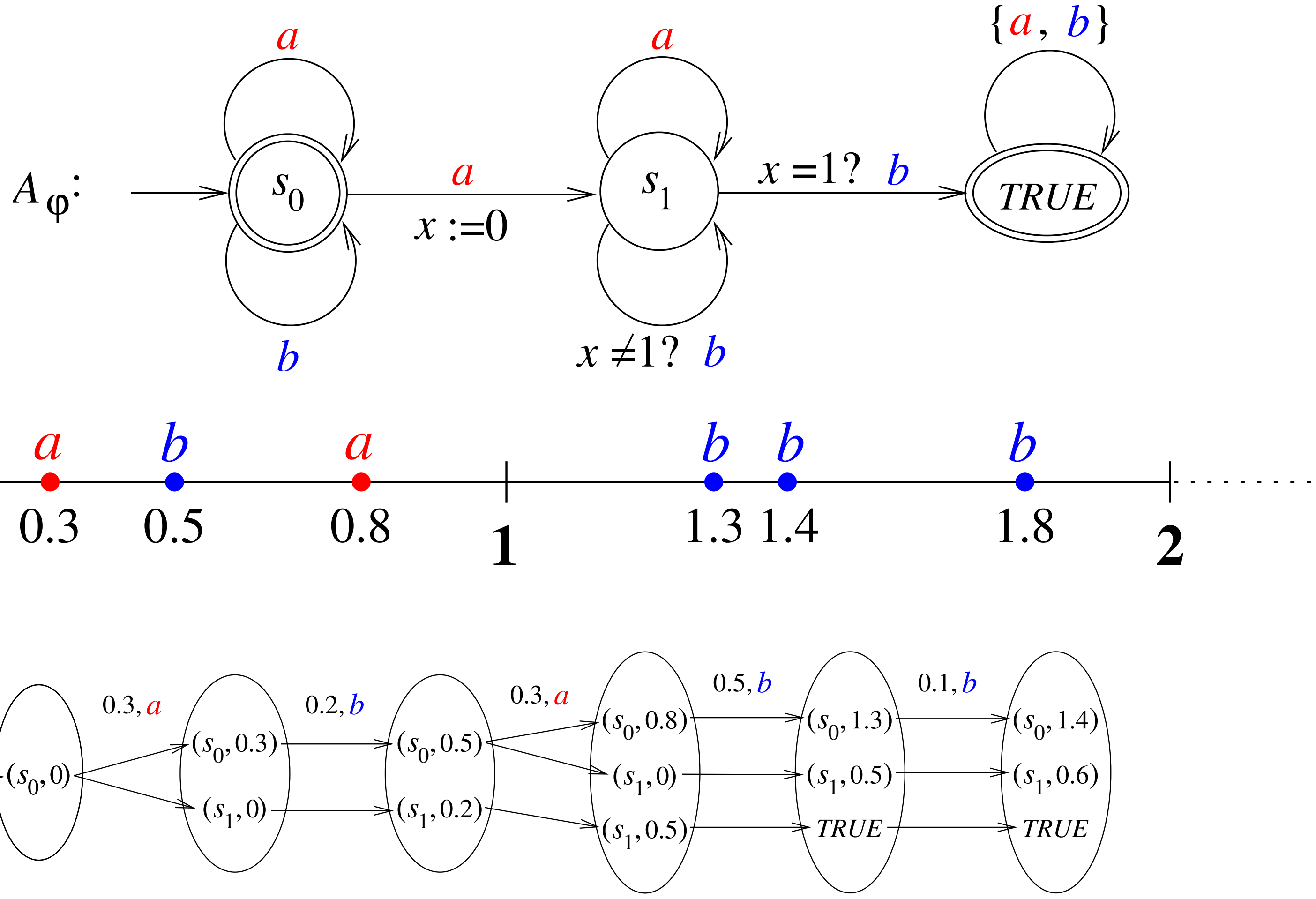

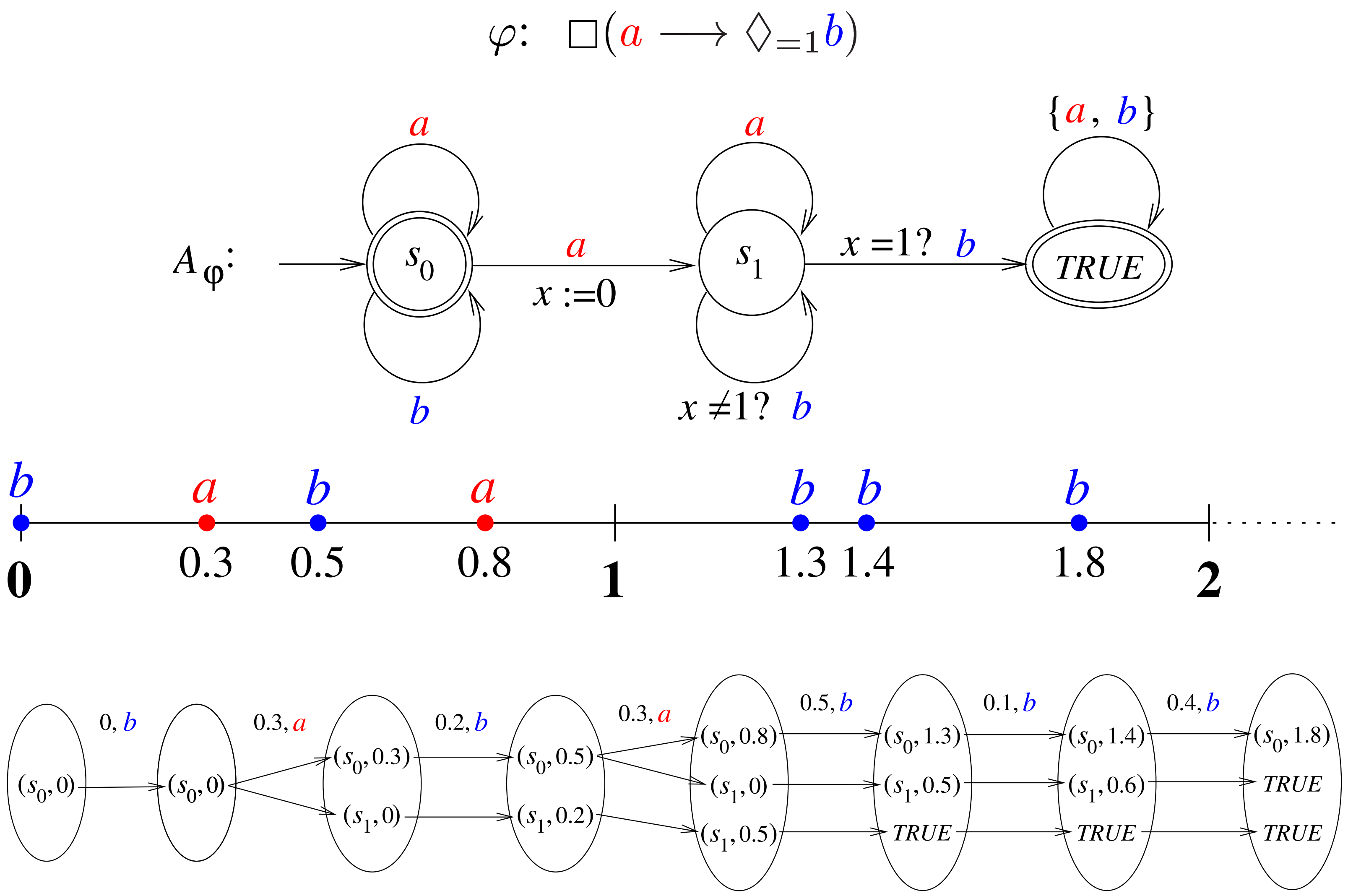

φ: □(a ⟶ ◊=1 b)
Aφ:
s0
s1
TRUE
a
b
x :=0
x =1? b
x ≠1? b
{a, b}
0
0.3
0.5
0.8
1
1.3
1.4
1.8
2
(s0, 0)
0, b
0.3, a
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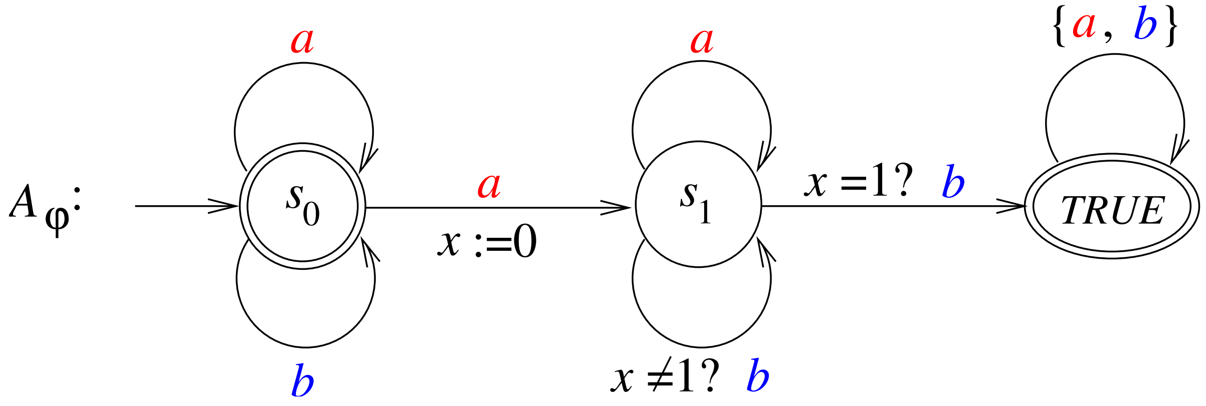

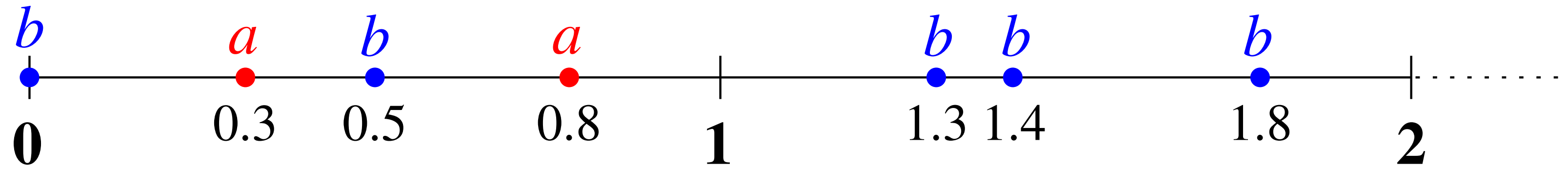

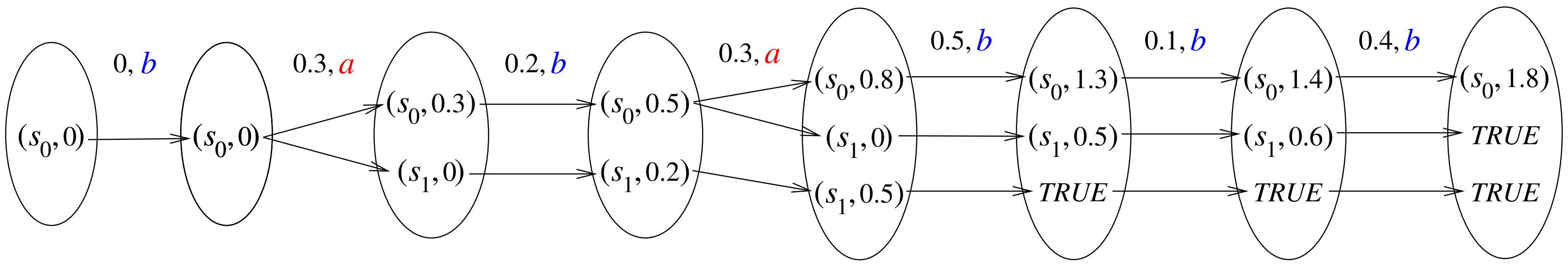

- Since all states in final slice are accepting, $w \vDash \varphi$.

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  - The subword order (e.g., $HIGMAN \preccurlyeq HIGHMOUNTAIN$) is a wqo.
- We solve the language-emptiness problem for A by a forward-reachability algorithm. Termination is guaranteed by the wqo on the configurations of A.

Our Contribution

	Satisfiability	Model Checking
MTL (Inf.)	??	??
MTL (Fin.)	Dec. / NPR	Dec. / NPR
Safety MTL (Inf.)	??	Dec. / ??

- Safety MTL: bounded timing constraints on all eventuality operators.

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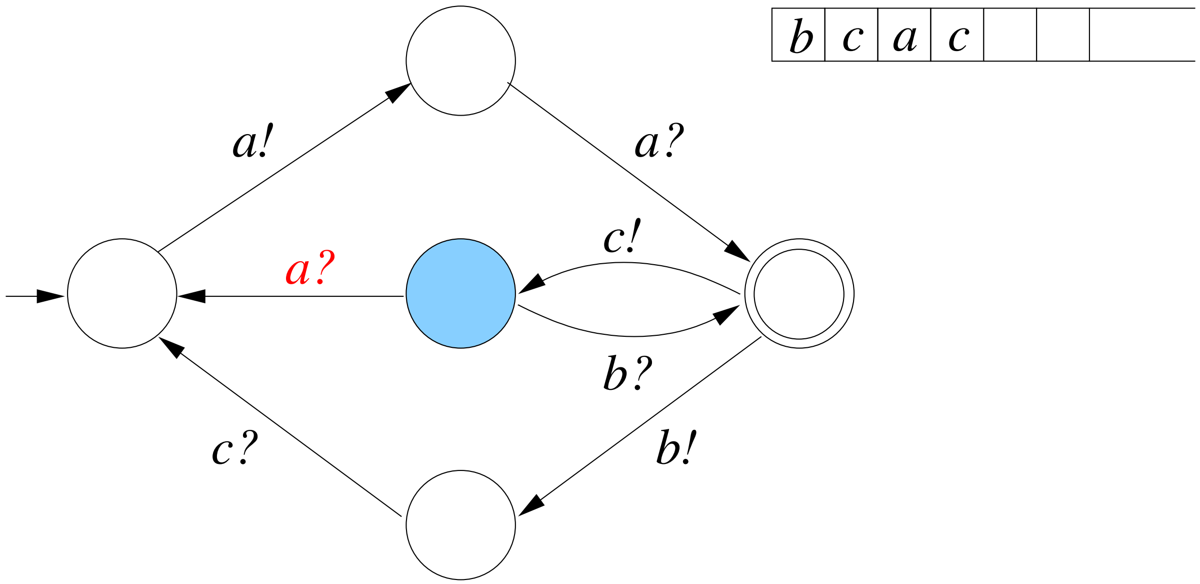

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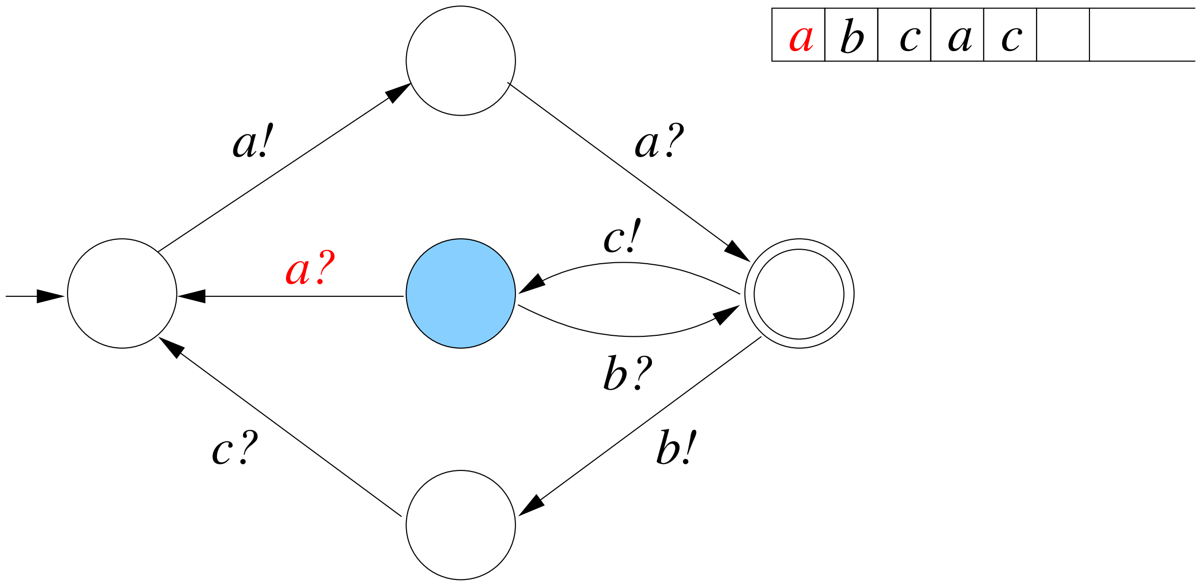

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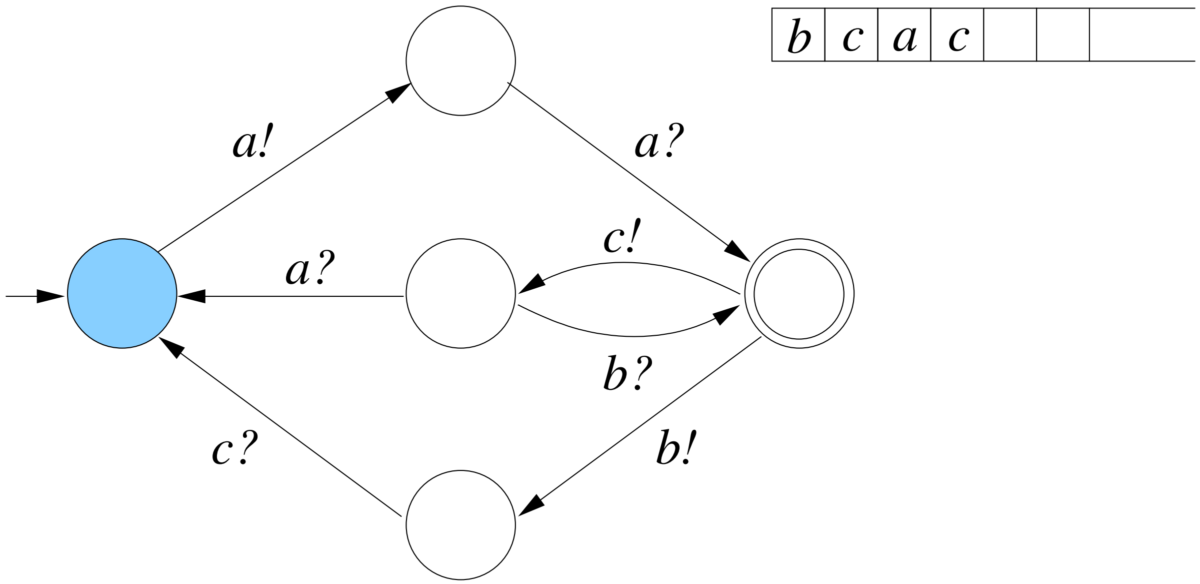

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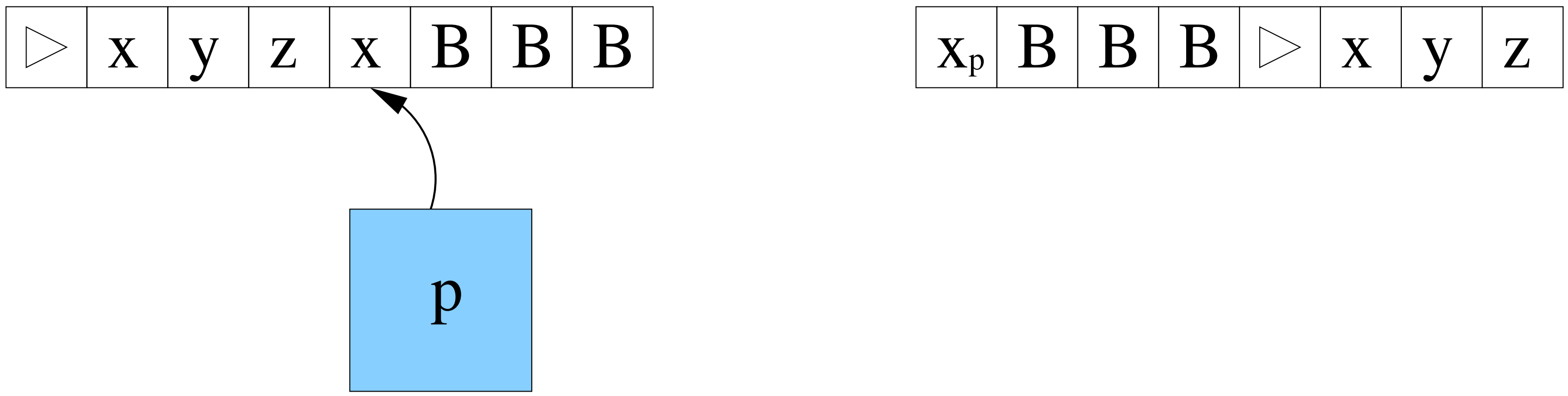
  - Simulate Turing machine using faulty channel machine. Use liveness to implement error correction mechanism.

Simulating a Turing Machine

- Simulate a Turing machine M with a faulty channel machine C.
- C has two channels c and d.
  - Store configuration of M on c.
  - Use d as a counter.
  - Both channels are subject to insertion errors.

Simulation Phase



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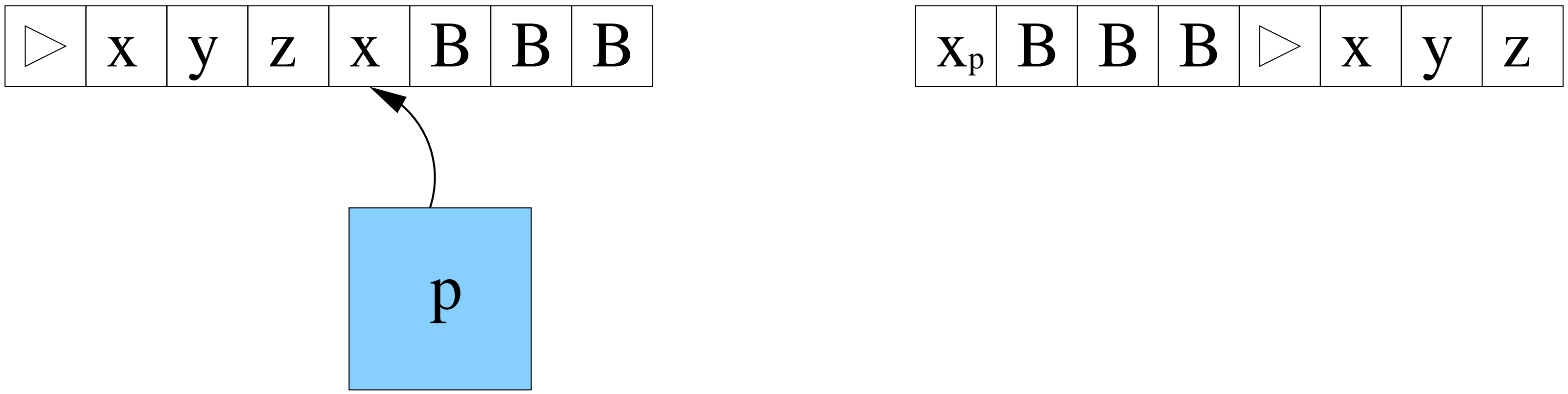

- Simulate transition $(p, x, q, y, \rightarrow)$ of M.

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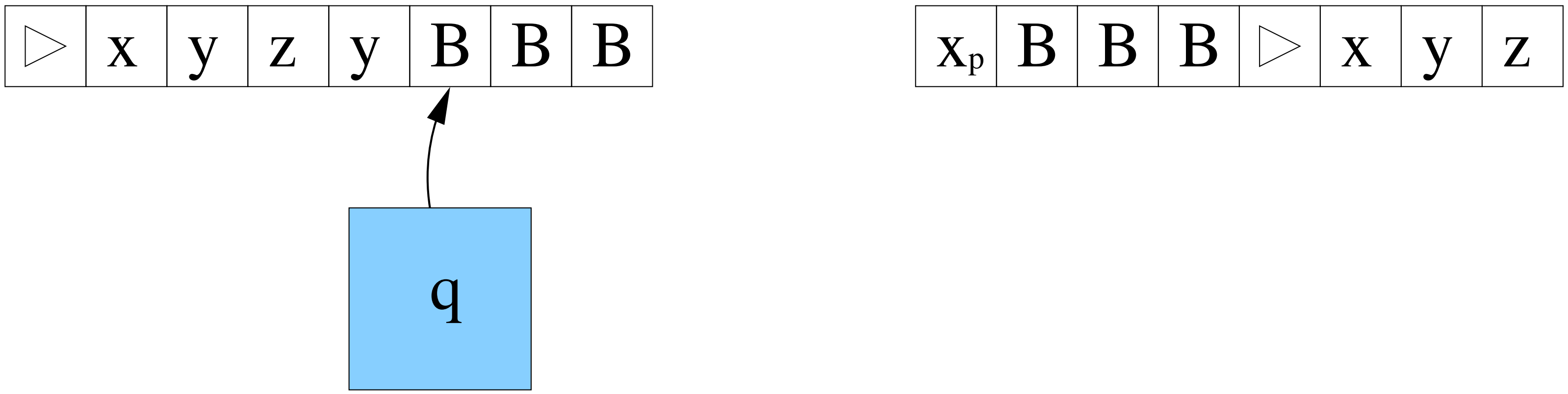

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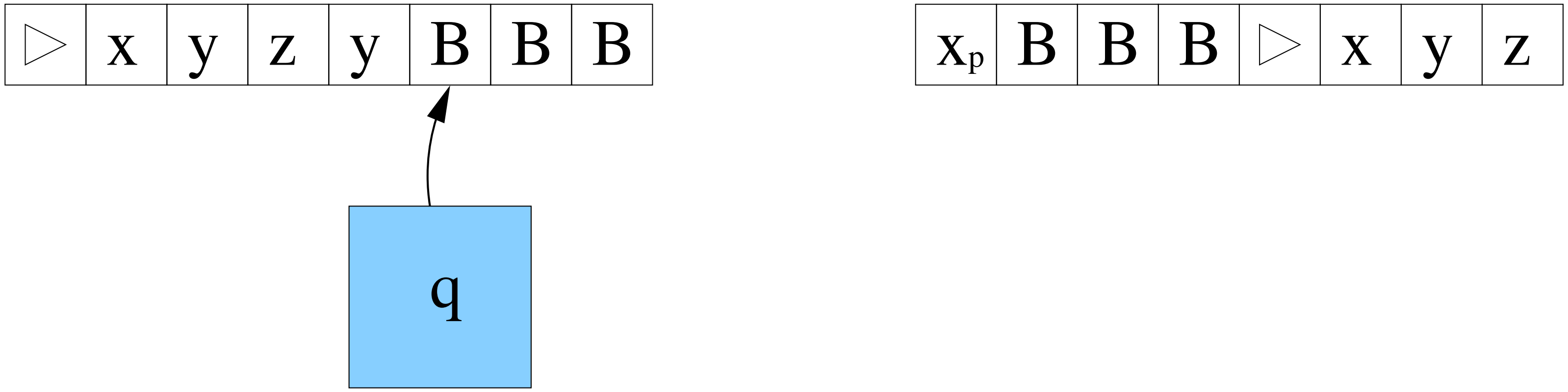

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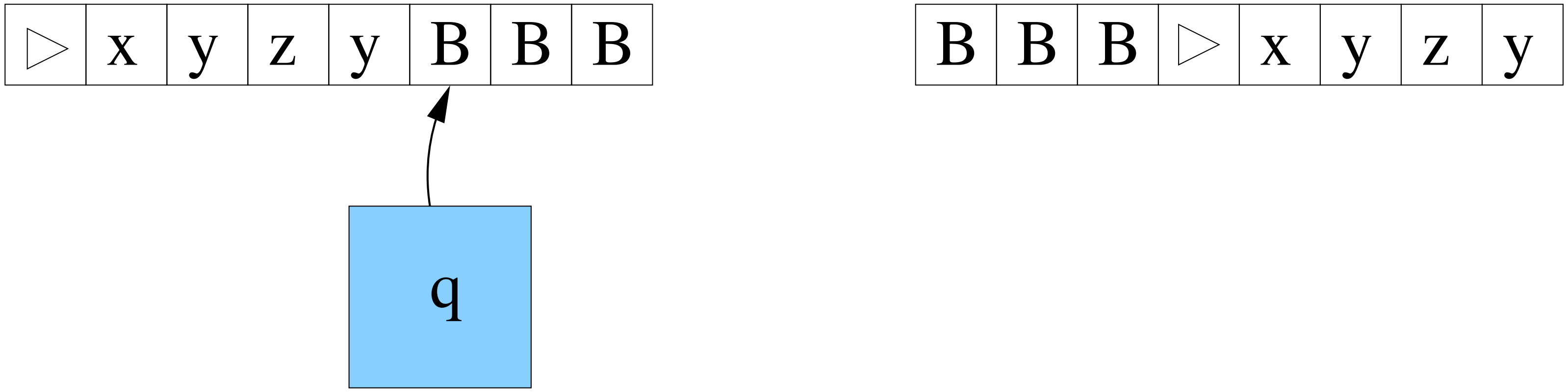

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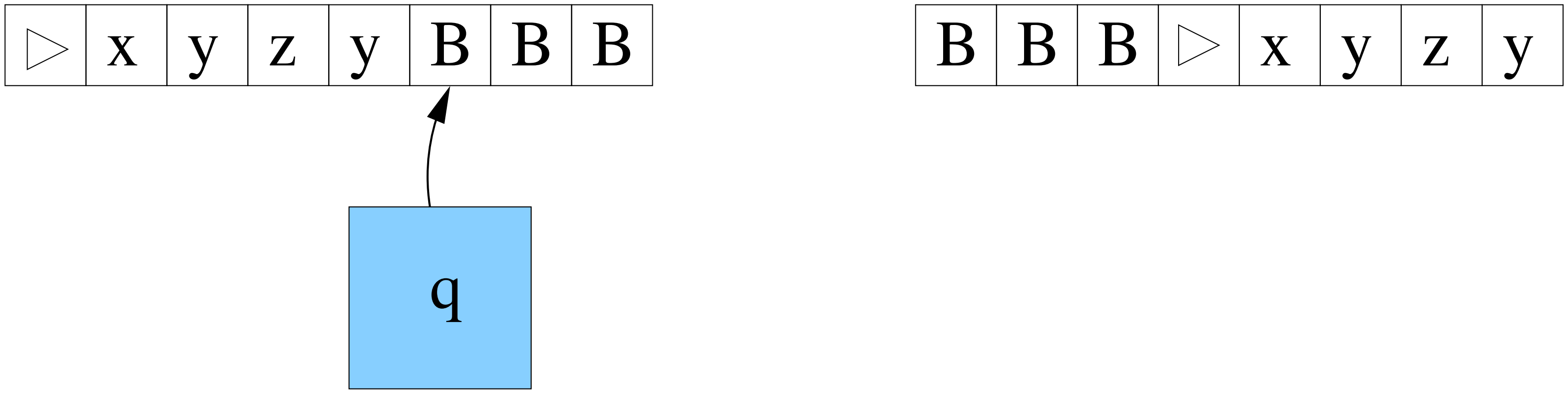

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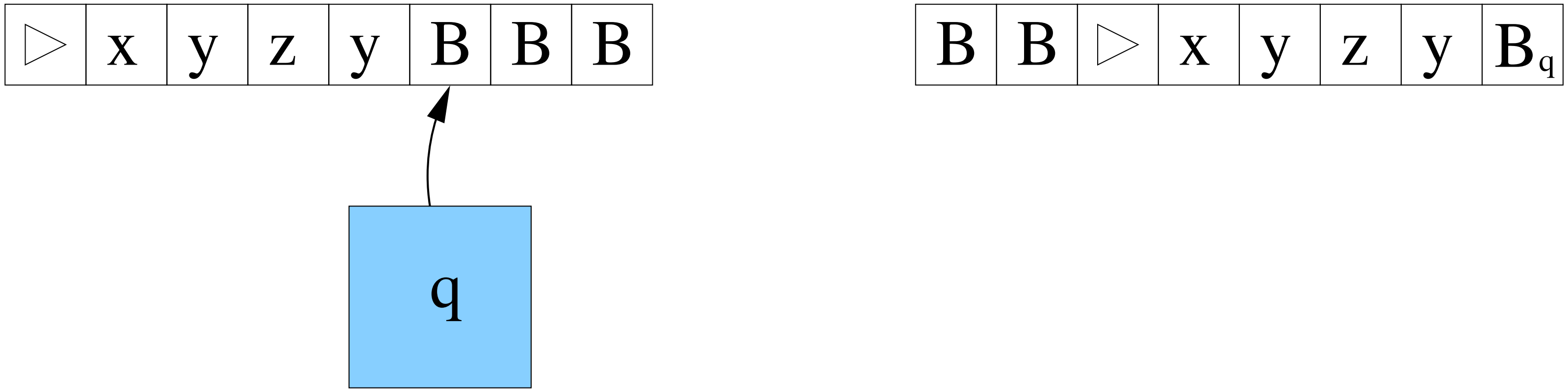

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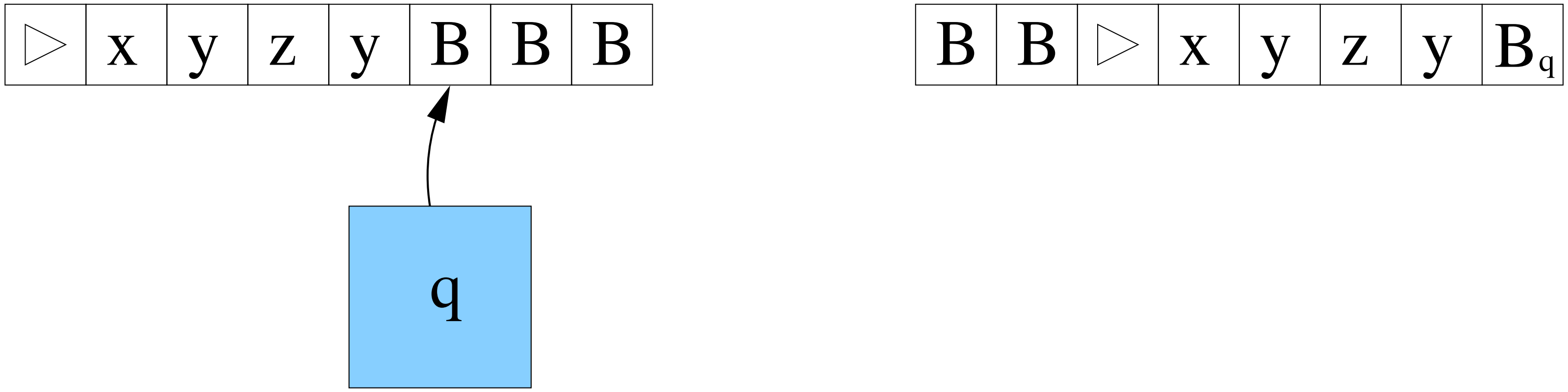
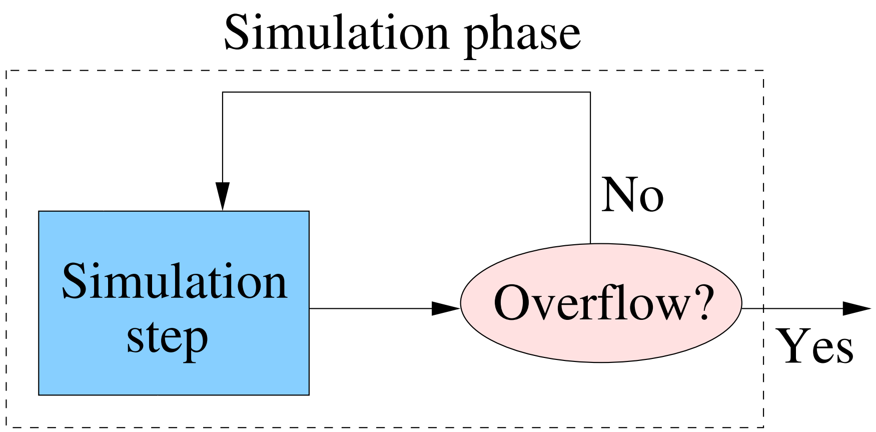


- Simulate transition $(p, x, q, y, \rightarrow)$ of M.
  - Read x_q $\Rightarrow$ Write y $\Rightarrow$ Read B $\Rightarrow$ Write B_q.
- Notice the alternation between read and write events.

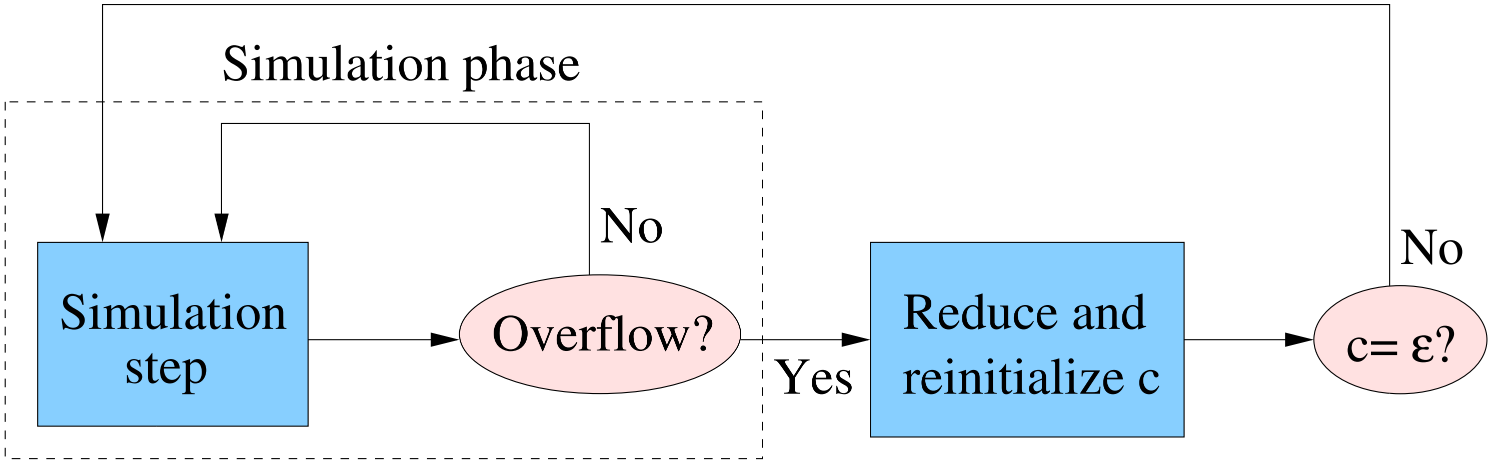
Simulation Phase

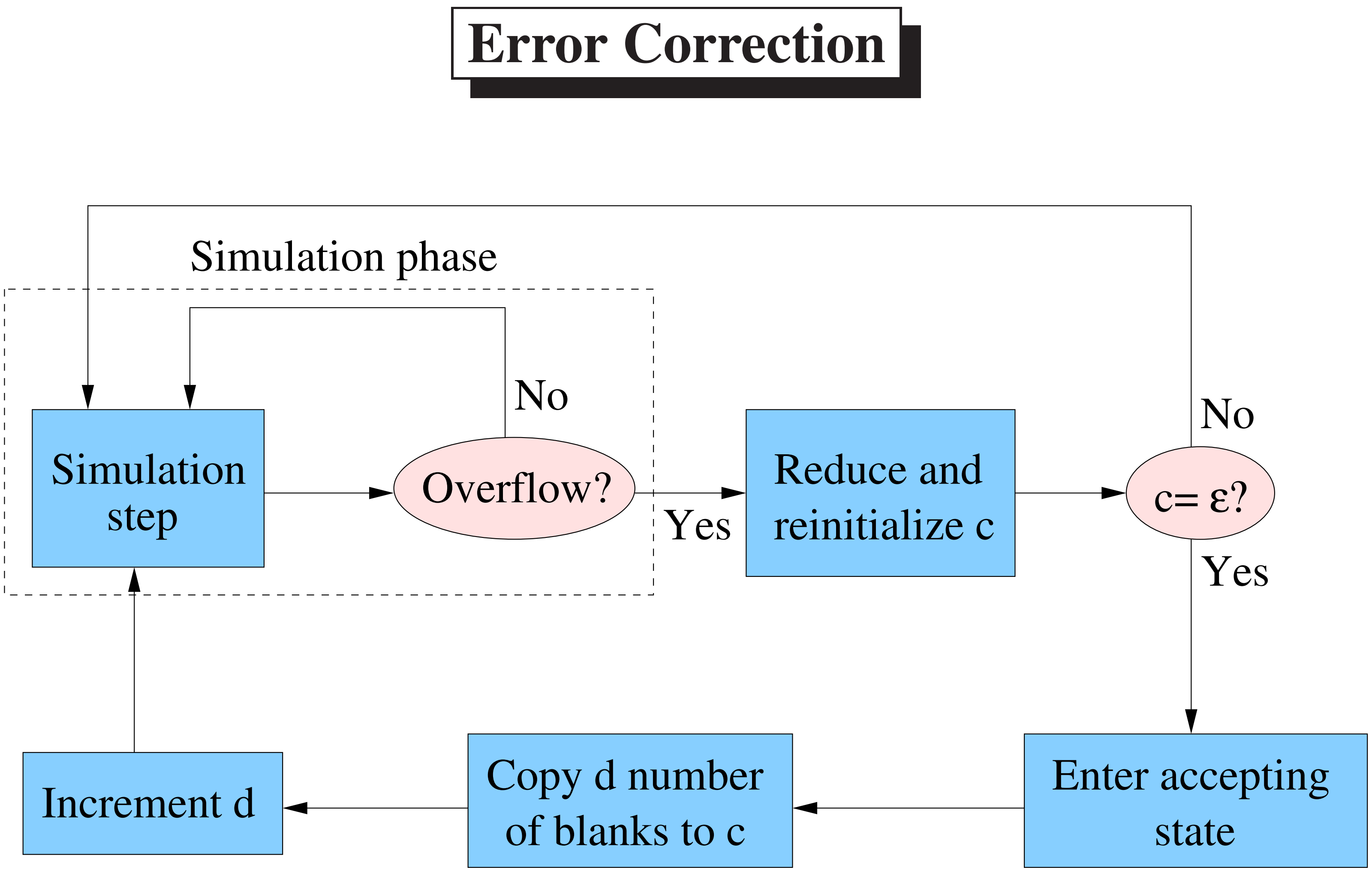
- Two problems:
  - Could run out of space during simulation phase.
  - Could also be computing gibberish!

Error Correction



Error Correction




Error Correction
Simulation phase
Simulation step
Overflow?
No
Yes
Reduce and reinitialize c
c= ε?
No
Yes
Enter accepting state
Copy d number of blanks to c
Increment d

Second Main Result

Lemma. The recurrent-state problem for channel machines with insertion errors and zero testing is undecidable.

Theorem. Both the satisfiability and model checking problems for MTL over infinite words are undecidable.

Our Contribution

	Satisfiability	Model Checking
MTL (Inf.)	Undec.	Undec.
MTL (Fin.)	Dec. / NPR	Dec. / NPR
Safety MTL (Inf.)	??	Dec. / ??

- Safety MTL: bounded timing constraints on all eventuality operators.

One Last Thing

	Satisfiability	Model Checking
MTL (Inf.)	Undec.	Undec.
MTL (Fin.)	Dec. / NPR	Dec. / NPR
Safety MTL (Inf.)	Dec. / NPR	Dec. / ??

- Safety MTL: bounded timing constraints on all eventuality operators.

Summary and Bibliography

- Decidability and complexity results for MTL:
  - Safety properties (e.g., invariance and time-bounded response) are decidable.
  - Liveness properties are undecidable.
- Bibliography:

  - J. Ouaknine and J. Worrell. On the Decidability of Metric Temporal Logic. *Proceedings of LICS'05, IEEE Computer Society Press*, 2005.
  - J. Ouaknine and J. Worrell. Metric Temporal Logic and Faulty Turing Machines. *In preparation.*