Folding Domain-Specific Languages:
Deep and Shallow Embeddings

Jeremy Gibbons
www.cs.ox.ac.uk/jeremy.gibbons
University of Oxford

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Abstract

A domain-specific language can be implemented by embedding within a
general-purpose host language. This embedding may be deep or shallow, de-
pending on whether terms in the language construct syntactic or semantic
representations. The deep and shallow styles are closely related, and inti-
mately connected to folds; in this paper, we explore that connection.

1 Introduction

General-purpose programming languages (GPLs) are great for generality. But this
very generality can count against them: it may take a lot of programming to estab-
lish a suitable context for a particular domain; and the programmer may end up
being spoilt for choice with the options available to her—especially if she is a domain
specialist rather than primarily a software engineer. This tension motivates many
years of work on techniques to support domain-specific languages (DSLs) such as
VHDL, SQL and PostScript: languages specialized for a particular domain, incor-
porating the contextual assumptions of that domain and guiding the programmer
specifically towards programs suitable for that domain.

There are two main approaches to DSLs. Standalone DSLs provide their own
custom syntax and semantics, and standard compilation techniques are used to
translate or interpret programs written in the DSL for execution. Standalone DSLs
can be designed for maximal convenience to their intended users. But the exercise
can be a significant undertaking for the implementer, involving an entirely separate
ecosystem—compiler, editor, debugger, and so on—and typically also much reinven-
tion of standard language features such as variables, definitions, and conditionals.

The alternative approach is to embed the DSL within a host GPL, essentially
as a collection of definitions written in the host language. All the existing facilities
and infrastructure of the host environment can be appropriated for the DSL, and
familiarity with the syntactic conventions and tools for the host language can be
carried over to the DSL. Whereas the standalone approach is the most common one within object-oriented circles [5], the embedded approach is typically favoured by functional programmers [11]. It seems that core FP features such as algebraic datatypes and higher-order functions are extremely helpful in defining embedded DSLs; conversely, it has been said that language-oriented tasks such as DSLs are the killer application for FP.

Amongst embedded DSLs, there are two further refinements. With a deep embedding, terms in the DSL are implemented simply to construct an abstract syntax tree (AST); this tree is subsequently transformed for optimization and traversed for evaluation. With a shallow embedding, terms in the DSL are implemented directly as the values to which they evaluate, bypassing the intermediate AST and its traversal. The deep and shallow embeddings are closely related, and intimately connected to folds; the purpose of this paper is to explore that connection.

2 Expressions

Consider a very simple language of arithmetic expressions, involving integer constants and addition. As a deeply embedded DSL, this can be captured by the following algebraic datatype:

```haskell
data Expr :: * where
  Val :: Integer → Expr
  Add :: Expr → Expr → Expr
```

(We have used Haskell’s ‘generalized algebraic datatype’ notation, in order to make the types of the constructors Val and Add explicit; but we are not using the generality of GADTs here, and the old-fashioned way would have worked too.) The expression $3 + 4$ is represented by the term $Add (Val 3) (Val 4)$ in the DSL. Observations of terms in the DSL are defined as functions over the algebraic datatype; for example, here is how to evaluate an expression:

```haskell
eval :: Expr → Integer
eval (Val n) = n
eval (Add x y) = eval x + eval y
```

A shallow embedding eschews the algebraic datatype, which records the abstract syntax of the language; instead, the language is defined directly in terms of its semantics. For example, if the semantics is to be evaluation, then we could define:

```haskell
type Expr = Integer
val :: Integer → Expr
val n   = n
add :: Expr → Expr → Expr
add x y = x + y
```
One might see the deep and shallow embeddings as duals, in a variety of senses. For one sense, the language constructs Val and Add in the deep embedding do none of the work, leaving this entirely to the observation function eval; in contrast, in the shallow embedding, the language constructs val and add do all the work, and the observer (of type Expr → Integer) is simply the identity function and so is omitted.

For a second sense, it is trivial to add a second observer to the deep embedding—just define another function alongside eval—but awkward to add new constructs: doing so entails revisiting the definitions of all existing observers to add an additional clause. In contrast, adding a construct to the shallow embedding is trivial—alongside val and add—but introducing an additional observer entails completely revising the semantics by changing the definitions of all existing constructs. This tension is precisely the conflict of forces addressed by the Visitor design pattern in object-oriented programming [6].

The types of val and add in the shallow embedding coincide with those of Val and Add in the deep embedding; moreover, the definitions of val and add in the shallow embedding correspond to the ‘actions’ in each clause of the definition of the observer in the deep embedding. The shallow embedding presents a compositional semantics for the language, since the semantics of a composite term is explicitly composed from the semantics of its components. Indeed, it is only such compositional semantics that can be captured in a shallow embedding; it is possible to define a more sophisticated non-compositional semantics as an interpretation of a deep embedding, but not possible to represent that semantics directly via a shallow embedding. In other words, shallow embeddings correspond to folds over the abstract syntax captured by a deep embedding.

Note that we do not claim duality in the categorical sense of reversing arrows. Similarly, deep and shallow embeddings have been called the ‘initial’ and ‘final’ approaches [3], but only in an informal sense; in fact, the two approaches both correspond to initial algebras, and neither to final coalgebras.

### 3 Folds

Folds are the natural pattern of computation induced by algebraic datatypes. We consider here just polynomial algebraic datatypes, namely those with one or more constructors, each constructor taking zero or more arguments to the datatype being defined, and each argument either having a fixed type independent of the datatype, or being a recursive occurrence of the datatype itself. For example, the polynomial algebraic datatype Expr above has two constructors; Val takes one argument, of the fixed type Integer; Add takes two arguments, both recursive occurrences. Thus, we rule out contravariant recursion, polymorphic datatypes, higher kinds, and other such esoterica.

The general case is captured by a shape, an instance of the Functor type class:

```haskell
class Functor f where
    fmap :: (a → b) → (f a → f b)
```

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For \( \text{Expr} \), the shape is as follows:

\[
\text{data } \text{ExprF} :: \ast \to \ast \text{ where} \\
\quad \text{ValF} :: \text{Integer} \to \text{ExprF } a \\
\quad \text{AddF} :: a \to a \to \text{ExprF } a
\]

\[
\text{instance } \text{Functor } \text{ExprF} \text{ where} \\
\quad \text{fmap } f \ (\text{ValF } n) = \text{ValF } n \\
\quad \text{fmap } f \ (\text{AddF } x \ y) = \text{AddF } (f \ x) (f \ y)
\]

For a given functor such as \( \text{ExprF} \) expressing a language shape, the deeply embedded DSL of that shape is the so-called initial algebra of the functor:

\[
\text{data } \text{Deep } :: (\ast \to \ast) \to \ast \text{ where} \\
\quad \text{In} :: \text{Functor } f \Rightarrow f \ (\text{Deep } f) \to \text{Deep } f
\]

\[
\text{type } \text{Expr} = \text{Deep } \text{ExprF}
\]

Compositional interpretations are precisely the folds for these initial algebras, morphisms to other algebras for the functor \( f \):

\[
\text{type } \text{Algebra } f \ a = f \ a \to a \\
\text{fold } :: \text{Functor } f \Rightarrow \text{Algebra } f \ a \to \text{Deep } f \to a \\
\text{fold } \phi \ (\text{In } x) = \phi \ (\text{fmap } (\text{fold } \phi) \ x)
\]

For example, \( \text{eval} \) is a fold for the deeply embedded DSL of shape \( \text{ExprF} \):

\[
\text{evalAlg} :: \text{Algebra } \text{ExprF } \text{Integer} \\
\text{evalAlg} \ (\text{ValF } n) = n \\
\text{evalAlg} \ (\text{AddF } x \ y) = x + y \\
\text{eval} :: \text{Expr} \to \text{Integer} \\
\text{eval} = \text{fold } \text{evalAlg}
\]

The shallow embedding is simply the algebra, such as \( \text{evalAlg} \). So an observation function for the deep embedding, such as \( \text{eval} \), is precisely a fold using the shallow embedding as the algebra. This insight is very revealing: we know a lot about folds, and this tells us a lot about embedded DSLs. We discuss these consequences next.

### 3.1 Multiple interpretations

As mentioned above, the deep embedding smoothly supports additional observations. For example, suppose that we also wanted to print expressions as strings; no problem—we can just define another observation function \( \text{print} \).

\[
\text{print} :: \text{Expr} \to \text{String} \\
\text{print} \ (\text{Val } n) = \text{show } n \\
\text{print} \ (\text{Add } x \ y) = \text{paren } (\text{print } x + "\ + " + \text{print } y)
\]
where we define for later reuse

\[
\text{paren } s = "(" + s + ")"
\]

But what about a shallow embedding? With this approach, expressions can only have a single semantics, so how do we accommodate both evaluation and printing? It’s not difficult; we simply make the semantics a pair, providing both interpretations simultaneously, so that the observation functions \textit{eval} and \textit{print} become projections rather than just the identity function.

\[
\text{type } \text{Expr} = (\text{Integer, String})
\]

\[
\text{val} :: \text{Integer} \rightarrow \text{Expr}
\]

\[
\text{val } n = (n, \text{show } n)
\]

\[
\text{add} :: \text{Expr} \rightarrow \text{Expr} \rightarrow \text{Expr}
\]

\[
\text{add } x \text{ y} = (\text{eval } x + \text{eval } y, \text{paren } (\text{print } x + " + " + \text{print } y))
\]

\[
\text{eval} :: \text{Expr} \rightarrow \text{Integer}
\]

\[
\text{eval} = \text{fst}
\]

\[
\text{print} :: \text{Expr} \rightarrow \text{String}
\]

\[
\text{print} = \text{snd}
\]

Of course, this works best under lazy evaluation: if only one of the two interpretations on an expression is needed, only that one is evaluated.

Seen from the fold perspective, this step is no surprise: the ‘banana split law’ \cite{4} tells us that tupling two independent folds gives another fold, so multiple interpretations can be provided in the shallow embedding nearly as easily as in the deep embedding.

### 3.2 Strengthening the invariant

A shallow embedding supports only compositional interpretations, whereas a deep embedding provides full access to the AST and hence also non-compositional manipulations. Here, ‘compositionality’ of an interpretation means that the interpretation of a whole may be determined from the interpretations of its parts; it is both a valuable property for reasoning and a significant limitation to expressivity.

For example, recall the \textit{print} interpretation above, which produces a fully parenthesized string; suppose one wanted a slightly more sophisticated rendering instead, using parentheses only where necessary to capture the structure of the expression. (Of course, addition is associative, so the parenthesization of an expression makes no difference to its value; but value isn’t everything.) The function \textit{mprint} ‘minimally prints’ an expression, only parenthesizing subexpressions, and then only if they are \textit{Adds} rather than \textit{Vals}:

\[
\text{mprint } (\text{Add } (\text{Val } 3) (\text{Add } (\text{Val } 4) (\text{Val } 5))) = "3+(4+5)"
\]

We have:

5
mprint :: Expr → String
mprint (Val n) = show n
mprint (Add x y) = pprint x ++ "+" ++ pprint y
where
  pprint e = (if isAdd e then paren else id) (mprint e)
isAdd (Val _) = False
isAdd (Add _) = True

This is a non-compositional interpretation of the abstract syntax, because mprint
depends on isAdd of subexpressions as well as their recursive image under mprint.
(True, you can reconstruct isAdd e from mprint e, if you try hard enough. But in
general, one interpretation might depend on a second that is not derivable from the
first, as for example the average of a list depends on both its sum and its length,
neither of which is derivable from the other.)

What can we do about such non-compositional interpretations in the shallow
embedding? Again, fold theory comes to the rescue: mprint and isVal together
form a mutomorphism [4]—that is, two mutually dependent folds—and the tuple of
these two functions again forms a fold. (In fact, this is a special case, a zygomorphism
[4], since the dependency is only one-way; and a particularly simple example of a
zygomorphism at that, because isVal is a trivial non-recursive fold. Simpler still, in
the banana split above, neither of the two folds depends on the other.)

type Expr = (String, Bool)
val :: Integer → Expr
val n = (show n, False)
add :: Expr → Expr → Expr
add x y = (pprint x ++ "+" ++ pprint y, True)
where
  pprint e = (if isAdd e then paren else id) (mprint e)
mprint :: Expr → String
mprint = fst
isAdd :: Expr → Bool
isAdd = snd

Tupling functions in this way is analogous to strengthening the invariant of an
imperative loop to record additional information [12], and is a standard trick in
program calculation [10]. For example, when solving the ‘maximum segment sum’
problem [1] as a loop, one strengthens the invariant that s is the maximum segment
sum seen so far:

\[ s = (\max j, k : 0 \leq j \leq k < n : (\sum i : j \leq i < k : a[i])) \]  \hspace{1cm} (*)

by adding the conjunct that t is the maximum suffix sum seen so far:

\[ t = (\max j : 0 \leq j < n : (\sum i : j \leq i < n : a[i])) \]  \hspace{1cm} (**)
Conjoining the main invariant (∗) with the auxilliary invariant (∗∗) is analogous to tupling the ‘maximum segment sum’ function with the ‘maximum suffix sum’ function.

### 3.3 Context-sensitivity

Another way of achieving minimal parenthesization is to print expressions differently depending on their context: Add subexpressions that are themselves arguments in enclosing expressions should be parenthesized, but outermost Add expressions and all Val expressions should not. More generally, expressions might be constructed from many different operators with different precedences, and an inner expression should be parenthesized iff it has lower precedence than the enclosing operator.

This too can be achieved using standard fold techniques. We use an *accumulating parameter* to carry the context into a subexpression:

```haskell
mprint :: Expr → String
mprint e = cprint False e

cprint :: Bool → Expr → String

-- print a VAL

-- print a ADD

cprint (Val n) = show n

cprint b (Add x y) = (if b then paren else id) (cprint True x ++ "+" ++ cprint True y)
```

Now, *cprint* is not a fold, because *cprint b* of an Add expression depends on a different function *cprint True* of its children; but flip *cprint* is a fold, yielding a result of type *Bool → String*, so we can use this as the semantics in a shallow embedding.

```haskell

-- type Expr = Bool → String

val :: Integer → Expr
val n _ = show n

add :: Expr → Expr → Expr
add x y b = (if b then paren else id) (x True ++ "+" ++ y True)

mprint :: Expr → String
mprint e = e False
```

A similar technique could be used for *eval*, if we wanted to introduce variable references and ‘let’ expressions into the language; the interpretation would be as functions from environments to values, extending the environment as ‘let’ bindings are encountered.

### 3.4 Generic interpretation

We have seen that it is not difficult to provide multiple interpretations with a shallow embedding, by constructing a tuple as the semantics of an expression and projecting the desired interpretation from the tuple. But this is still a bit clumsy: it entails
revising existing code each time a new interpretation is added, and ten-tuples are never pleasant to work with.

But as we have also seen, all compositional interpretations conform to a common pattern—they are folds. So we can provide a shallow embedding in terms of a single generic interpretation; that interpretation is a higher-order value, representing the fold.

\[
\text{type } Expr = \forall a . \text{Algebra } ExprF a \to a
\]

\[
\begin{align*}
\text{val} & : \text{Integer} \to Expr \\
\text{val } n \phi & = \phi (\text{ValF } n) \\
\text{add} & : Expr \to Expr \to Expr \\
\text{add } x \ y \phi & = \phi (\text{AddF } (x \phi) (y \phi))
\end{align*}
\]

That this encoding is generic is evidenced by the fact that it can be instantiated to yield evaluation and printing (and, of course, any other fold):

\[
\begin{align*}
\text{eval} & : Expr \to \text{Integer} \\
\text{eval } e & = e \text{ evalAlg} \\
\text{print} & : Expr \to \text{String} \\
\text{print } e & = e \text{ printAlg} \\
\text{where} \\
\text{printAlg} & : \text{Algebra } ExprF \text{ String} \\
\text{printAlg } (\text{ValF } n) & = \text{show } n \\
\text{printAlg } (\text{AddF } x \ y) & = \text{paren } (x + "+" + y)
\end{align*}
\]

In fact, the shallow embedding provides a universal generic interpretation as the Church encoding [2, 8] of the AST.

4 Discussion

The essential observation made here—that shallow embeddings correspond to folds over the abstract syntax captured by a deep embedding—is surely not new. For example, it was probably known to Reynolds [14], who contrasted deep embeddings (‘user defined types’) and shallow (‘procedural data structures’), and observed that the former were free algebras; but he didn’t explicitly discuss anything corresponding to folds. It is also implicit in the finally tagless approach [3], which uses a shallow embedding and observes that ‘this representation makes it trivial to implement a primitive recursive function over object terms’, providing an interface that such functions should implement; but this comment is made rather in passing, and their focus is mainly on taglessness. (The observation is more explicit in Kise-lyov’s lecture notes on the finally tagless approach [13], which go into more detail on compositionality.)

Nevertheless, the observation seems not to be widely appreciated. And it makes a nice application of folds: many results about folds evidently have interesting statements about shallow embeddings as corollaries. The three generalizations of folds
(banana split, mutumorphisms, and accumulating parameters) exploited in Section 3 are all special cases of adjoint fold [9]; perhaps other adjoint folds yield more interesting insights about shallow embeddings?

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References


