Programming Concurrent Garbage Collectors With Pushouts and Monads

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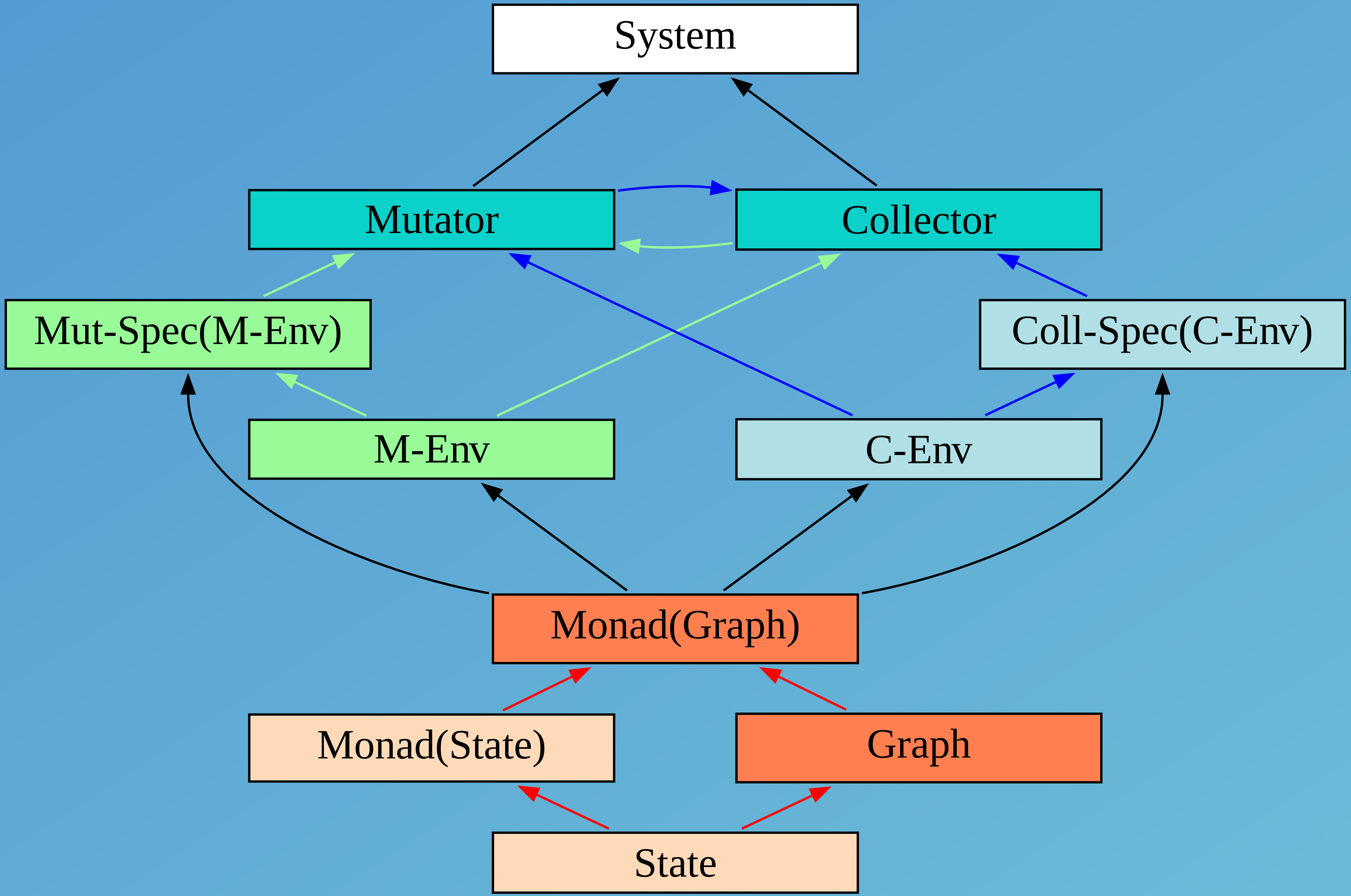
1 The Overall System

☞ The system is a parallel combination of

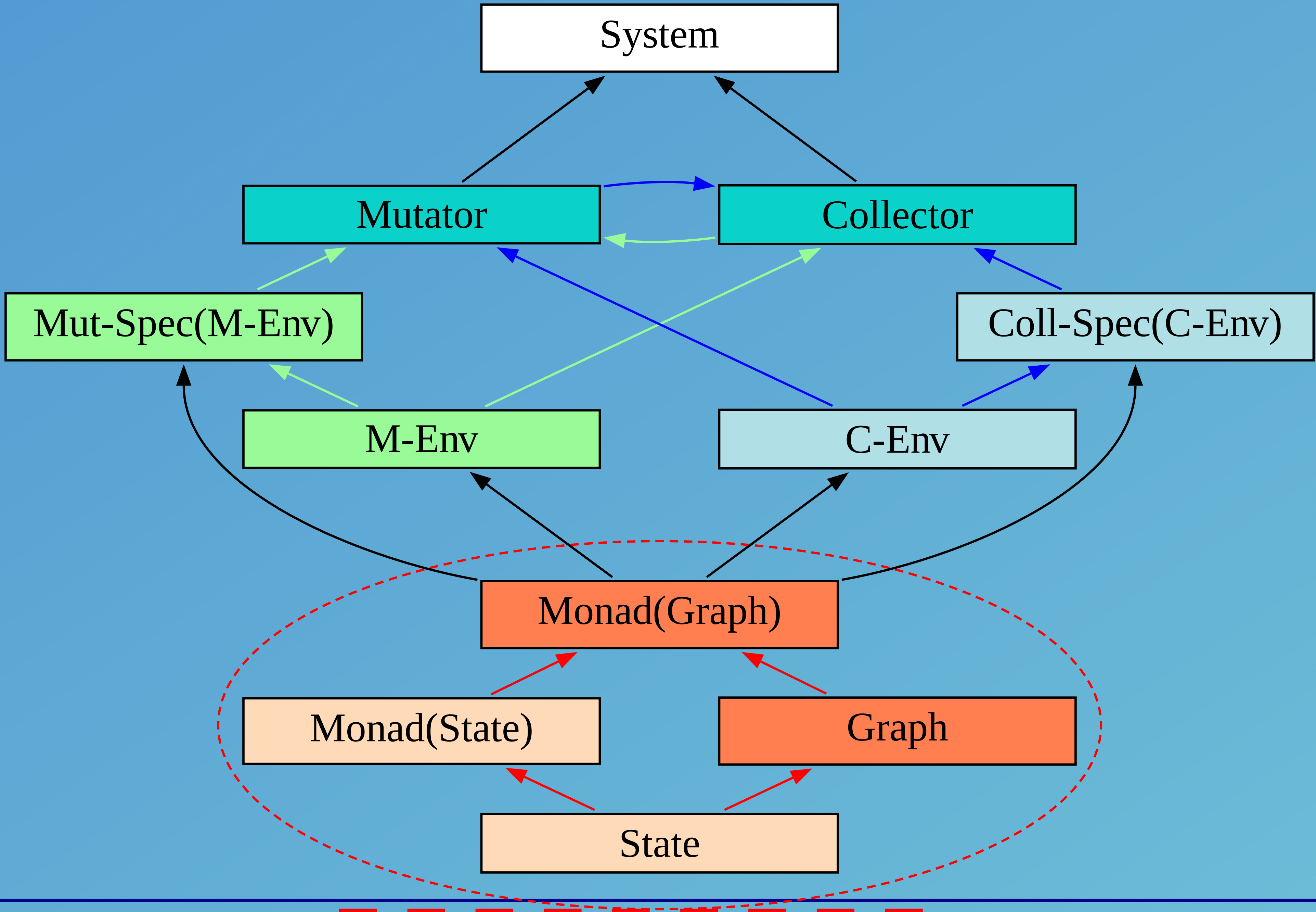
➢ the mutator (the "program")

➢ the collector (the "garbage collector")

The Overall System



The Overall System



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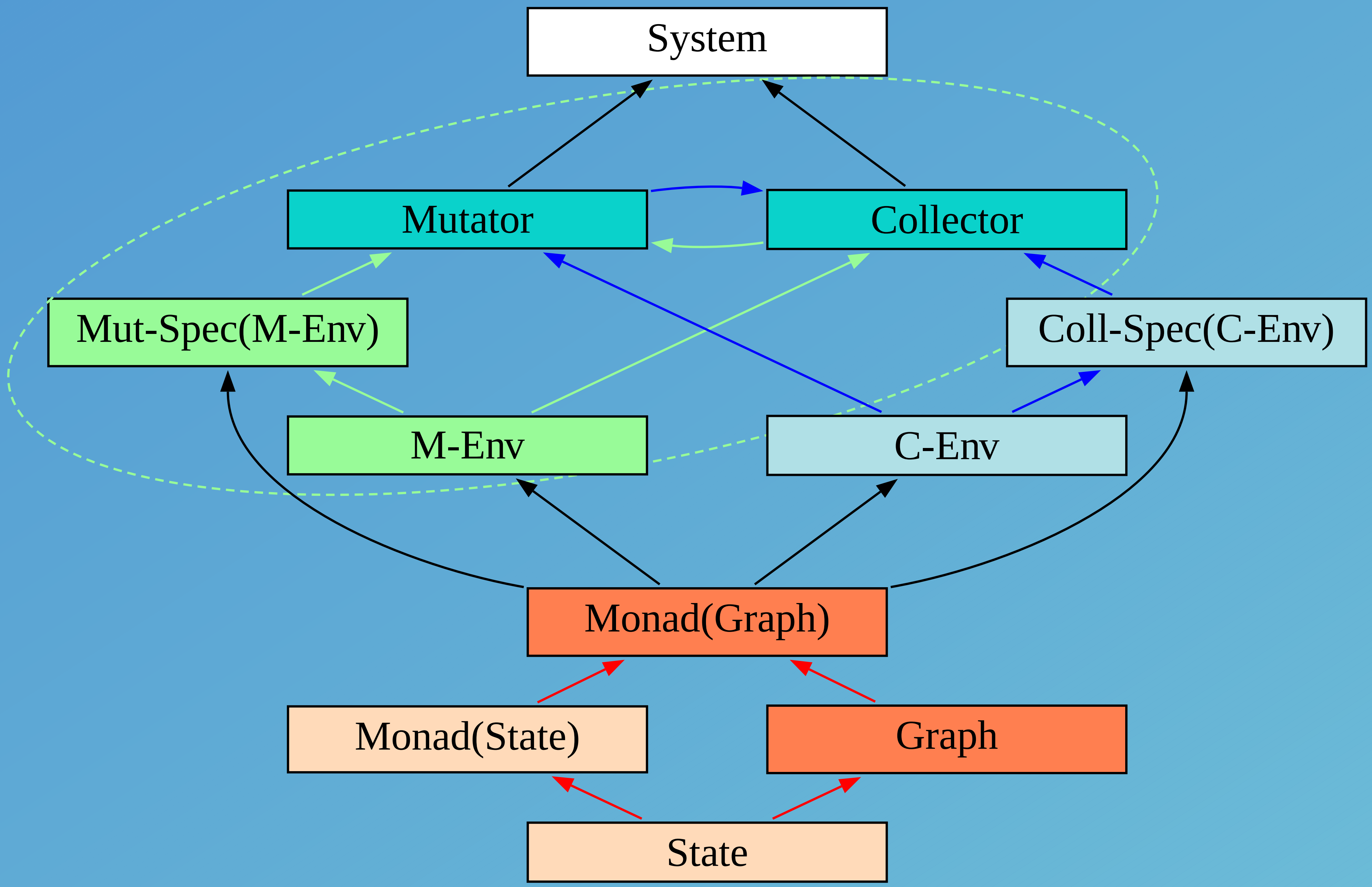
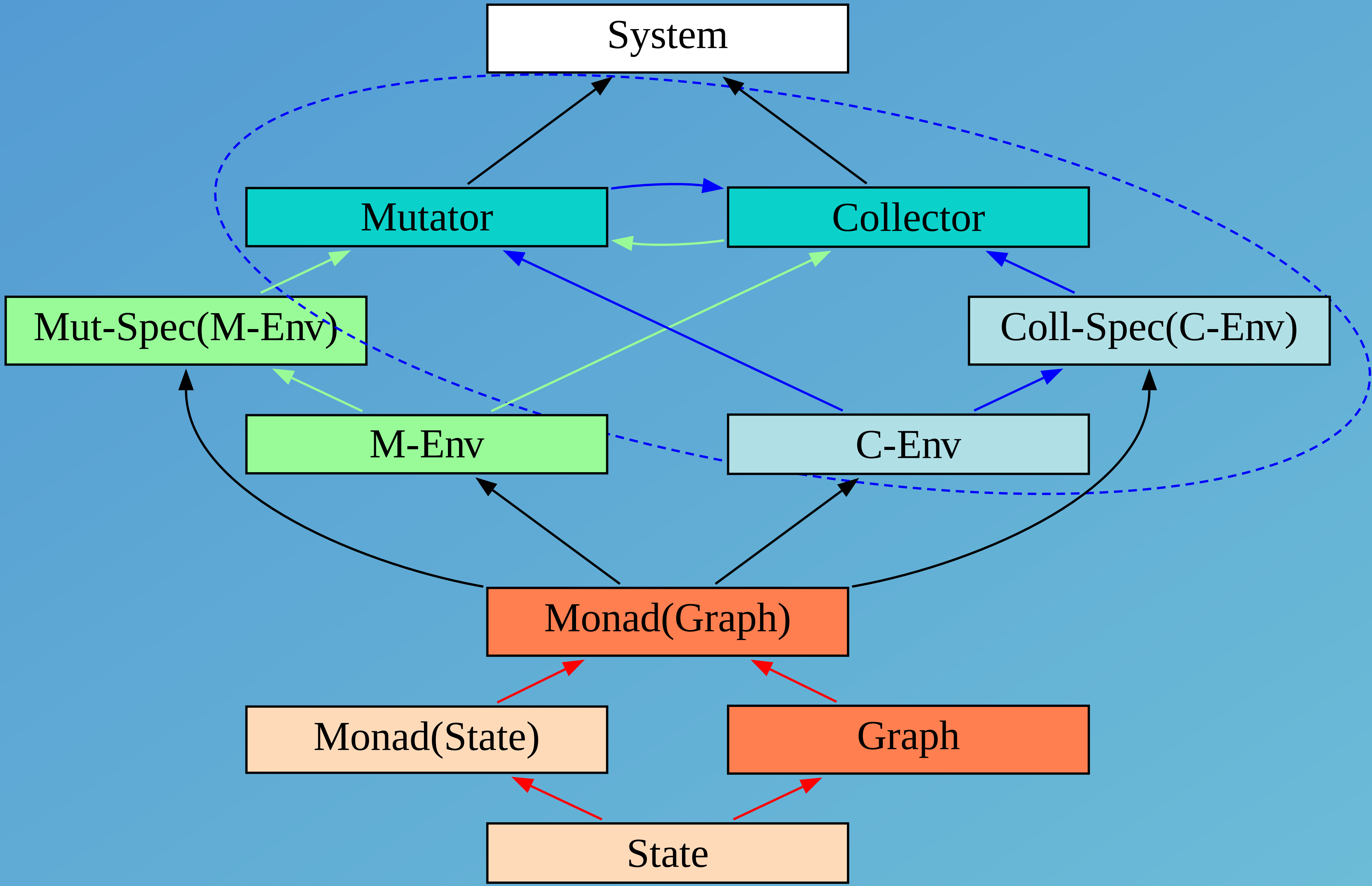


The Overall System



The Overall System Specification

SPEC System =
  IMPORT Mutator, Collector
  FUN run : M[Void] = (mutate ‖ collect)   -- *parallel*

SPEC Mutator = Mutator-Spec(Collector)

SPEC Mutator-Spec(Env : Mutator-Environment) = . . .

The Overall System Specification

$$\text{SPEC } \mathtt{System} =$$
$$\quad \text{IMPORT } \mathtt{Mutator}, \mathtt{Collector}$$
$$\quad \text{FUN } \mathtt{run} : \mathtt{M}[\mathtt{Void}] = (\ \mathtt{mutate}\ \|\ \mathtt{collect}\ ) \qquad \text{-- } \textit{parallel}$$

$$\text{SPEC } \mathtt{Mutator} = \mathtt{Mutator\text{-}Spec}(\mathtt{Collector})$$

$$\text{SPEC } \mathtt{Collector} = \mathtt{Collector\text{-}Spec}(\mathtt{Mutator})$$

$$\text{SPEC } \mathtt{Mutator\text{-}Spec}(\mathtt{Env} : \mathtt{Mutator\text{-}Environment}) = \ldots$$

$$\text{SPEC } \mathtt{Collector\text{-}Spec}(\mathtt{Env} : \mathtt{Collector\text{-}Environment}) = \ldots$$

2 Graph Specification

Graphs are modelled here by

- ☞ a fixed set of nodes
- ☞ a set of entry points ("roots")
- ☞ a successor function (representing the arcs)
- ☞ a freelist

```
SPEC Graph =
    ...
```

```
SORT Node
  -- the (coalgebraic ) type for graphs
SORT Graph OBSERVED BY roots, sucs, free
FUN roots : Graph → Set Node
FUN sucs : Node → Graph → Set Node
FUN free : Graph → Set Node
```

SORT Node

-- *the (coalgebraic) type for graphs*

SORT Graph OBSERVED BY roots, sucs, free

FUN roots : Graph $\longrightarrow$ Set Node

FUN sucs : Node $\longrightarrow$ Graph $\longrightarrow$ Set Node

FUN free : Graph $\longrightarrow$ Set Node

-- *reachability*

OBS reachable : $(R : \text{Set Node}) \longrightarrow (G : \text{Graph}) \longrightarrow (S' : \text{Set Node})$

POST $S' = \text{LEAST } S.\ (R \subseteq S) \wedge (\cup/(G.\text{sucs}) * S \subseteq S)$

-- *reachable from root*

OBS black : $(G : \text{Graph} \longrightarrow \text{Set Node}) = G.\text{reachable}(G.\text{roots})$

-- *alternative name for freelist*

OBS gray : $(G : \text{Graph} \longrightarrow \text{Set Node}) = G.\text{free}$

-- *totally unreachable nodes (garbage)*

OBS white : $(G : \text{Graph} \longrightarrow \text{Set Node}) = \{ n : \text{Node} \} \setminus (G.\text{black} \cup G.\text{gray})$

AXM 1 : $G.\text{black} \cap G.\text{gray} = \emptyset$

-- *add /delete arc (coinductive definition)*

FUN $\mathtt{add} : (\mathtt{x} : \mathtt{Node}, \mathtt{y} : \mathtt{Node}) \longrightarrow (\mathtt{G} : \mathtt{Graph}) \longrightarrow (\mathtt{G}' : \mathtt{Graph})$

POST $\mathtt{G}'.\mathtt{sucs}(\mathtt{x}) = \mathtt{G}.\mathtt{sucs}(\mathtt{x}) \oplus \mathtt{y}$

FUN $\mathtt{cut} : (\mathtt{x} : \mathtt{Node}, \mathtt{y} : \mathtt{Node}) \longrightarrow (\mathtt{G} : \mathtt{Graph}) \longrightarrow (\mathtt{G}' : \mathtt{Graph})$

POST $\mathtt{G}'.\mathtt{sucs}(\mathtt{x}) = \mathtt{G}.\mathtt{sucs}(\mathtt{x}) \ominus \mathtt{y}$

-- *add / delete arc (coinductive definition)*

$\texttt{FUN}\ \texttt{add} : (\texttt{x} : \texttt{Node}, \texttt{y} : \texttt{Node}) \longrightarrow (\texttt{G} : \texttt{Graph}) \longrightarrow (\texttt{G}' : \texttt{Graph})$

$\quad \texttt{POST}\ \texttt{G}'.\texttt{sucs}(\texttt{x}) = \texttt{G}.\texttt{sucs}(\texttt{x}) \oplus \texttt{y}$

$\texttt{FUN}\ \texttt{cut} : (\texttt{x} : \texttt{Node}, \texttt{y} : \texttt{Node}) \longrightarrow (\texttt{G} : \texttt{Graph}) \longrightarrow (\texttt{G}' : \texttt{Graph})$

$\quad \texttt{POST}\ \texttt{G}'.\texttt{sucs}(\texttt{x}) = \texttt{G}.\texttt{sucs}(\texttt{x}) \ominus \texttt{y}$

-- *obtaining a free node*

$\texttt{FUN}\ \texttt{new} : (\texttt{G} : \texttt{Graph}) \longrightarrow (\texttt{G}' : \texttt{Graph},\ \texttt{n} : \texttt{Node})$

$\quad \texttt{PRE}\ \ \texttt{G}.\texttt{free} \neq \emptyset$

$\quad \texttt{POST}\ \texttt{n} \in \texttt{G}.\texttt{free}$

$\quad\quad \texttt{G}'.\texttt{free} = \texttt{G}.\texttt{free} \ominus \texttt{n}$

$\quad\quad \texttt{G}'.\texttt{sucs}(\texttt{n}) = \emptyset$

-- *recycling a garbage node*

$\texttt{FUN}\ \texttt{recycle} : (\texttt{G} : \texttt{Graph}) \longrightarrow (\texttt{G}' : \texttt{Graph})$

$\quad \texttt{PRE}\ \ \texttt{G}.\texttt{white} \neq \emptyset$

$\quad \texttt{POST}\ \texttt{G}'.\texttt{free} = \texttt{G}.\texttt{free} \oplus \texttt{n}\ \texttt{WHERE}\ \texttt{n} \in \texttt{G}.\texttt{white}$

3 Monads

Modelling of imperative aspects by monads

- ☞ The graph becomes the (hidden) state
- ☞ Parallel execution of mutator/collector is interleaving of monadic operations.

- Monads are parameterized by an internal `State`
- Monads provide a polymorphic type $\mathtt{M}[\alpha]$

```
SPEC Monad (TYPE State) =
  TYPE M[α]
```

- Monads are parameterized by an internal `State`
- Monads provide a polymorphic type $\mathtt{M}[\alpha]$

$$\mathrm{SPEC}\ \mathtt{Monad}\ (\mathrm{TYPE}\ \mathtt{State}) =$$
$$\quad \mathrm{TYPE}\ \mathtt{M}[\alpha]$$

The monad type $\mathtt{M}[\alpha]$ consists of two functions

- the evolution of the internal (hidden) state
- the observation of the (visible) associated value

$$\quad \texttt{--}\ \textit{coalgebraic view of } M\,[\alpha]$$
$$\mathrm{OBS}\ \mathtt{evolution} : \mathtt{M}[\alpha] \longrightarrow (\mathtt{State} \rightarrow \mathtt{State})$$
$$\mathrm{OBS}\ \mathtt{observer} : \mathtt{M}[\alpha] \longrightarrow (\mathtt{State} \rightarrow \alpha)$$

We can lift a value to a monad (with no evolution)

$$\mathrm{FUN}\ \mathtt{yield}[\alpha] : \alpha \longrightarrow \mathtt{M}[\alpha]$$
$$\mathrm{DEF}\ (\mathtt{yield}\ \mathtt{a}).\mathtt{evolution} = \mathtt{id}$$
$$\mathrm{DEF}\ (\mathtt{yield}\ \mathtt{a}).\mathtt{observer} = \mathtt{K}\ \mathtt{a}$$

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We can compose two monads

$$\text{FUN } (\_\ ;\ \_)[\alpha, \beta] : \texttt{M}[\alpha] \times \texttt{M}[\beta] \longrightarrow \texttt{M}[\beta]$$
$$\text{DEF } \big(\texttt{m}_1\ ;\ \texttt{m}_2\big).\texttt{evolution} = \texttt{m}_2.\texttt{evolution} \circ \texttt{m}_1.\texttt{evolution}$$
$$\text{DEF } \big(\texttt{m}_1\ ;\ \texttt{m}_2\big).\texttt{observer} = \texttt{m}_2.\texttt{observer} \circ \texttt{m}_1.\texttt{evolution}$$

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$$\texttt{DEF}\ (\texttt{m}_1 ; \texttt{m}_2).\texttt{observer} = \texttt{m}_2.\texttt{observer} \circ \texttt{m}_1.\texttt{evolution}$$

We can compose a monad with a continuation

$$\texttt{FUN}\ (\_ ; \_)[\alpha, \beta] : \texttt{M}[\alpha] \times (\alpha \rightarrow \texttt{M}[\beta]) \rightarrow \texttt{M}[\beta]$$
$$\texttt{DEF}\ \texttt{m}_1 ; \texttt{f} = (\texttt{f} \circ \texttt{m}_1.\texttt{observer})\ \texttt{S}\ (\texttt{m}_1.\texttt{evolution})$$

Note: $(\mathrm{h}\ \mathrm{S}\ \mathrm{g})(\mathrm{x}) = (\mathrm{h}\ \mathrm{x})(\mathrm{g}\ \mathrm{x})$

Automatic casting to monadic operations

Let $\mathtt{M}[\alpha]$ be defined by $\mathtt{Monad}(\mathtt{Graph})$:

$\boxed{\text{FUN } \mathtt{f} : \mathtt{Graph} \to \alpha}$ is lifted to

$$\boxed{\begin{array}{l} \text{FUN } \mathtt{f} : \mathtt{M}[\alpha] \\ \text{DEF } \mathtt{f}.\mathtt{observer} = \mathtt{f} \\ \text{DEF } \mathtt{f}.\mathtt{evolution} = \mathtt{id} \end{array}}$$

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$$\text{FUN } \mathtt{f} : \mathtt{Graph} \rightarrow \mathtt{Graph}$$

is lifted to

$$\begin{array}{l} \text{FUN } \mathtt{f} : \mathtt{M}[\mathtt{Void}] \\ \text{DEF } \mathtt{f}.\mathtt{observer} = \mathtt{K}\ \mathtt{nil} \\ \text{DEF } \mathtt{f}.\mathtt{evolution} = \mathtt{f} \end{array}$$

The monadic graph `Monad(Graph)`:

Automatic lifting yields

FUN **roots** : M[Set Node]
FUN **sucs** : Node ⟶ M[Set Arc]
FUN **free** : M[Set Node]
FUN **add** : Node × Node ⟶ M[Void]
FUN **cut** : Node × Node ⟶ M[Void]
FUN **new** : M[Node]
FUN **recycle** : M[Void]
OBS **reachable** : Set Node ⟶ M[Set Node]
OBS **black** : M[Set Node]
OBS **gray** : M[Set Node]
OBS **white** : M[Set Node]

Special Predicates on Monads

$\mathrm{SPEC}\ \mathtt{Monad}\ (\mathrm{TYPE}\ \mathtt{State}) =$

$\quad \ldots$

-- *invariance*

$\mathrm{OBS}\ \mathtt{invariant} : \mathtt{M}[\alpha] \longrightarrow \mathtt{Bool}$

$\mathrm{DEF}\ \mathtt{invariant}(\mathtt{f}) \ = \ \mathtt{preserve}(=)(\mathtt{f})$

-- *monotonicity*

$\mathrm{OBS}\ \mathtt{monotone} : \mathtt{M}[\alpha] \longrightarrow \mathtt{Bool}$

$\mathrm{DEF}\ \mathtt{monotone}(\mathtt{f}) \ = \ \mathtt{preserve}(\preceq)(\mathtt{f})$

-- *preservation of a relation*

$\mathrm{OBS}\ \mathtt{preserve} : (\preceq : \alpha \times \alpha \longrightarrow \mathtt{Bool}) \longrightarrow \mathtt{f} : \mathtt{M}[\alpha] \longrightarrow \mathtt{Bool}$

$\quad \mathrm{PRE}\ \ \mathtt{f.evolution} = \mathtt{id}$

$\quad \mathrm{POST}\ \forall \mathtt{m} : \mathtt{M}[\alpha] : \ \mathtt{f.observer} \preceq \mathtt{f.observer} \circ \mathtt{m.evolution}$

Iterators on Monads

```
SPEC Monad (TYPE State) =
      ...
   -- infinite repetition
  FUN forever : M[Void] → M[Void]
  DEF forever m  =  m ; forever(m)

   -- repeat as often as possible
  FUN iterate : M[Void] → M[Void]
  DEF iterate m  =  IF APPLICABLE (m) THEN m ; iterate(m)
                                       ELSE nop FI
```

4 The Mutator

Any program that uses only the operations
`roots`, `sucs`, `add`, `cut`, `new`
is an acceptable instance of the mutator.

```
SPEC Mutator-Spec ( Env : Mutator-Environment ) =
  IMPORT Monad(Graph) ONLY roots, sucs, add, cut, new
  FUN mutate : M[Void]
  THM monotone white
```

The mutator guarantees that the (unreachable) white nodes remain white.

The mutator relies on the proper behaviour of its environment

```
SPEC Mutator-Environment =
  EXTEND Monad(Graph) BY
  AXM 1: invariant black
  AXM 2: ∀ n ∈ black: invariant sucs(n)
```

The environment must not change the black part of the graph, that is,

- not change the black nodes
- not change the arcs between black nodes

5 The Collector (naive view)

The collector
continuously recycles
white (unreachable) nodes.

```
SPEC Collector-Spec ( Env : Collector-Environment ) =
  IMPORT Monad(Graph) ONLY recycle, white
  FUN collect : M[Void] = forever(recycle)
  THM invariant black
  THM ∀n ∈ black : invariant sucs(n)
```

The collector guarantees that the
black part of the graph remains untouched

The collector relies on the proper behaviour of its environment

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SPEC Collector-Environment =
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The environment must not make white nodes reachable

Correctness:

Collector and mutator meet each others rely conditions

$$\begin{array}{lcl}\texttt{Collector-Spec} & \vdash & \texttt{Mutator-Environment} \\ \texttt{Mutator-Spec} & \vdash & \texttt{Collector-Environment}\end{array}$$

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Moreover:

Each unreachable node will eventually be in the freelist

$$\texttt{n} \in \texttt{white} \;\Rightarrow\; \Diamond\, \texttt{n} \in \texttt{gray}$$

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Moreover:

Each unreachable node will eventually be in the freelist

$$\texttt{n} \in \texttt{white} \Rightarrow \Diamond\, \texttt{n} \in \texttt{gray}$$

However:

The collector depends on the non-implementable function white.

6 Implementation

Idea:

Mark (a subset of) the unreachable nodes

GOAL $\texttt{marked} \subseteq \texttt{white}$

Algorithm:

- Start with all nodes marked
- Clean the reachable nodes
- Scavenge the marked nodes

We introduce the workset of currently considered nodes

$$\text{FUN } \mathtt{red} : \mathtt{Graph} \to \mathtt{Set\ Node} \quad \texttt{--}\ \textit{a coalgebraic observer}$$
$$\text{OBS } \mathtt{pink} : (\mathtt{G} : \mathtt{Graph} \to \mathtt{Set\ Node}) = \mathtt{G.reachable(G.red)}$$

- red is the wavefront of currently considered nodes
- pink are all nodes that still need to be visited.

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- ☞ red is the wavefront of currently considered nodes
- ☞ pink are all nodes that still need to be visited.

Central invariant:

$$\texttt{marked} \cap (\texttt{black} \cup \texttt{gray}) \subseteq \texttt{pink}$$

(The reachable nodes that are still marked will still be visited)

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Central invariant:

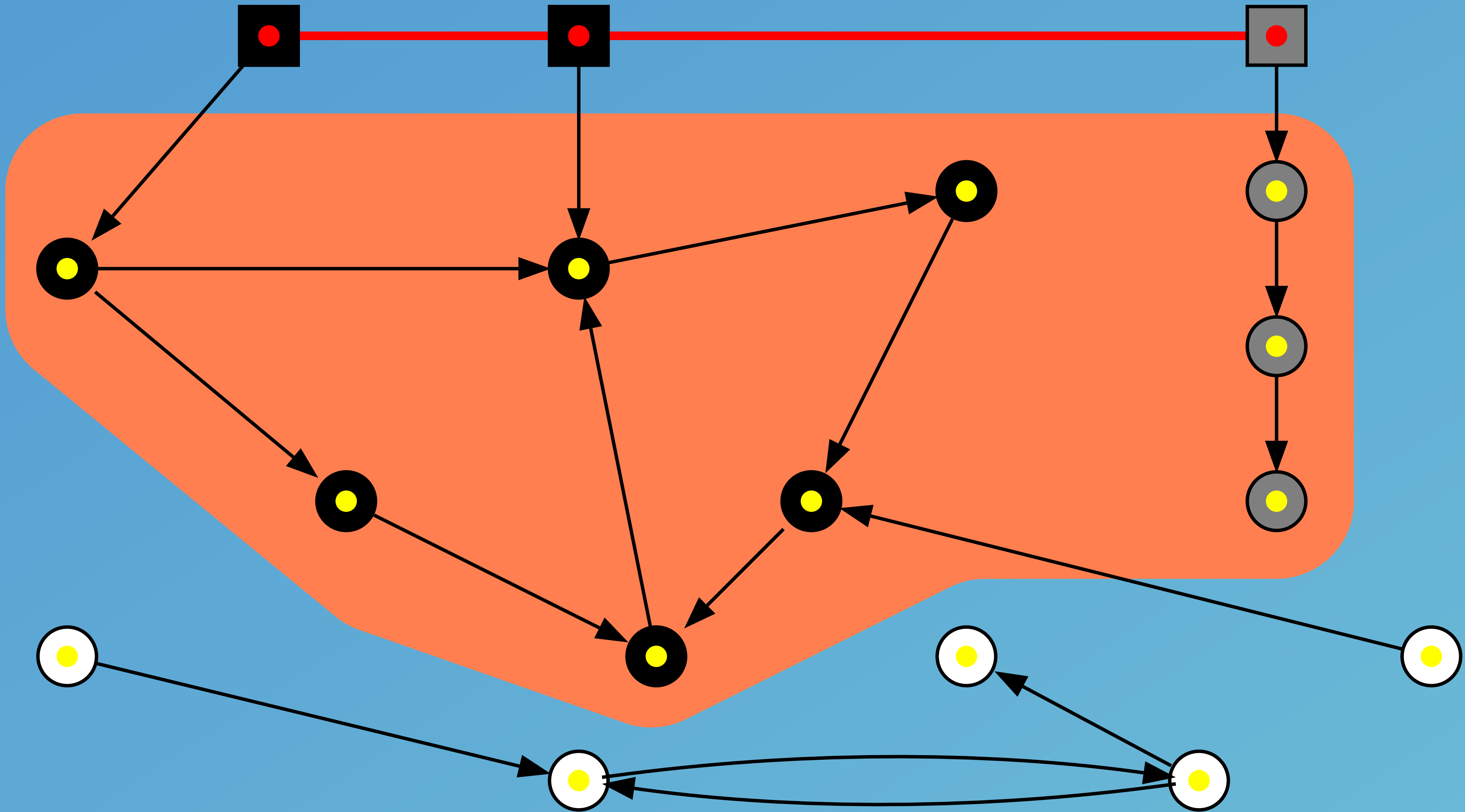
$$\texttt{marked} \cap (\texttt{black} \cup \texttt{gray}) \subseteq \texttt{pink}$$

(The reachable nodes that are still marked will still be visited)

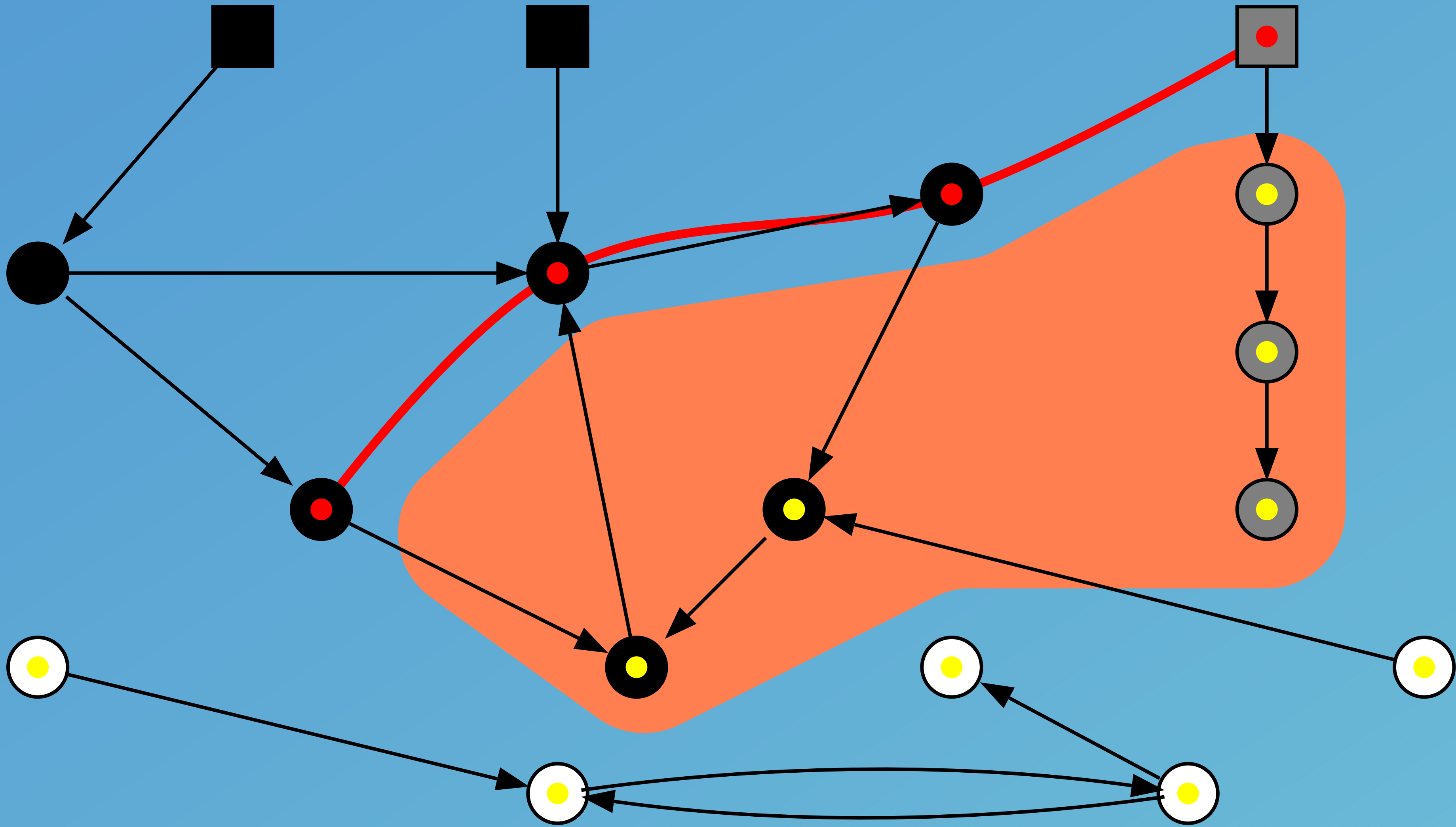
Conclusion

$$\texttt{red} = \emptyset \quad \vdash \quad \texttt{pink} = \emptyset \quad \vdash \quad \texttt{marked} \subseteq \texttt{white}$$

Initial situation



Intermediate snapshot



7 The Extended Collector

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➢ cleaning
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➢ cleaning
(removing the marks of the reachable nodes)

➢ scavenging
(adding the remaining marked nodes to the freelist)

```
SPEC XCollector-Spec ( Env : XCollector-Environment ) =
  IMPORT Monad(XGraph)

  IMPORT Cleaner ONLY clean
  IMPORT Scavenger ONLY scavenge

  FUN collect : M[Void] = forever(clean ; scavenge)

  THM invariant black
  THM ∀n ∈ black : invariant sucs(n)
```

☞ The black subgraph has to remain untouched!

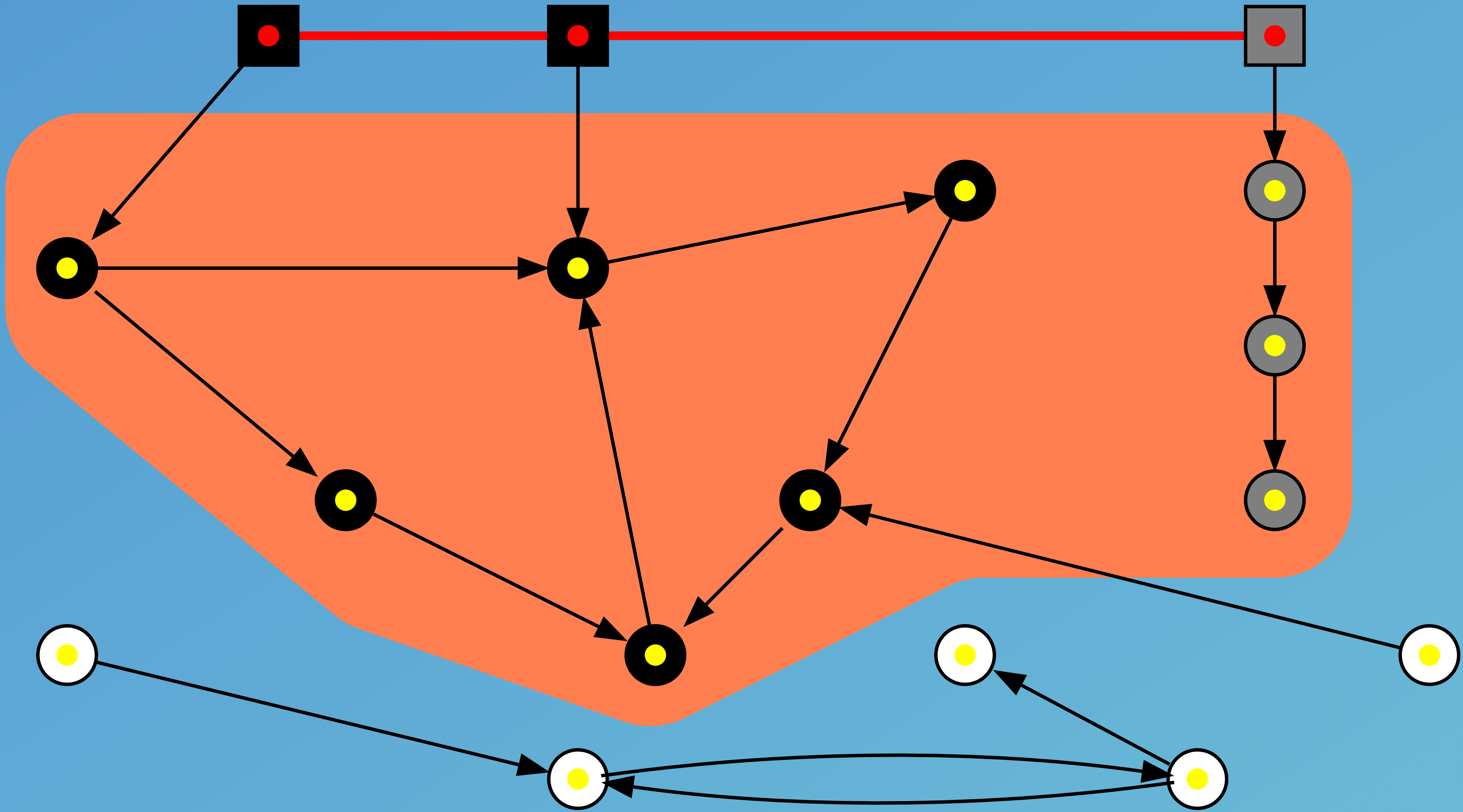
7.1 The Cleaning Phase ("unmark")

- Cleaning unmarks reachable nodes as long as possible
- It bases on an extended spec of `Graph`

$$
\begin{array}{l}
\mathrm{SPEC}\ \mathtt{Cleaner}\ =\\
\quad \mathrm{IMPORT}\ \mathtt{Monad}(\mathtt{Cleaning\text{-}View\text{-}of\text{-}XGraph})\\
\\
\quad \mathrm{FUN}\ \mathtt{clean} : \mathtt{M}[\mathtt{Void}]\\
\qquad\qquad \mathrm{PRE}\quad \mathtt{marked} = \{\mathtt{n} : \mathtt{Node}\}\\
\qquad\qquad \mathrm{POST}\ \mathtt{marked} \subseteq \mathtt{white}\\
\quad \mathrm{DEF}\ \mathtt{clean} = (\ \mathtt{red} \leftarrow \mathtt{roots} \cup \mathtt{free}\ ;\ \mathtt{iterate}(\mathtt{unmark})\ )\\
\\
\quad \mathrm{THM}\ \mathtt{invariant\ black}\\
\quad \mathrm{THM}\ \forall \mathtt{n} \in \mathtt{black} : \mathtt{invariant\ sucs}(\mathtt{n})
\end{array}
$$

- The black subgraph remains untouched!

Initial situation



Unmarking a red node makes all its marked successors red (push red frontier forward)

SPEC $\texttt{Cleaning-View-of-XGraph}$ $=$

EXTEND $\texttt{XGraph}$ ONLY $\texttt{red}, \texttt{sucs}$ BY

-- *unmark some node in the workset*

FUN $\texttt{unmark} : (\texttt{G} : \texttt{Graph}) \rightarrow (\texttt{G}' : \texttt{Graph})$

PRE $\texttt{G.red} \neq \emptyset$

POST LET $\texttt{n} \in \texttt{G.red}$ IN

$\texttt{G}'\texttt{.red} = \texttt{G.red} \cup (\texttt{G.sucs}(\texttt{n}) \cap \texttt{G.marked}) \ominus \texttt{n}$

$\texttt{G}'\texttt{.marked} = \texttt{G.marked} \ominus \texttt{n}$

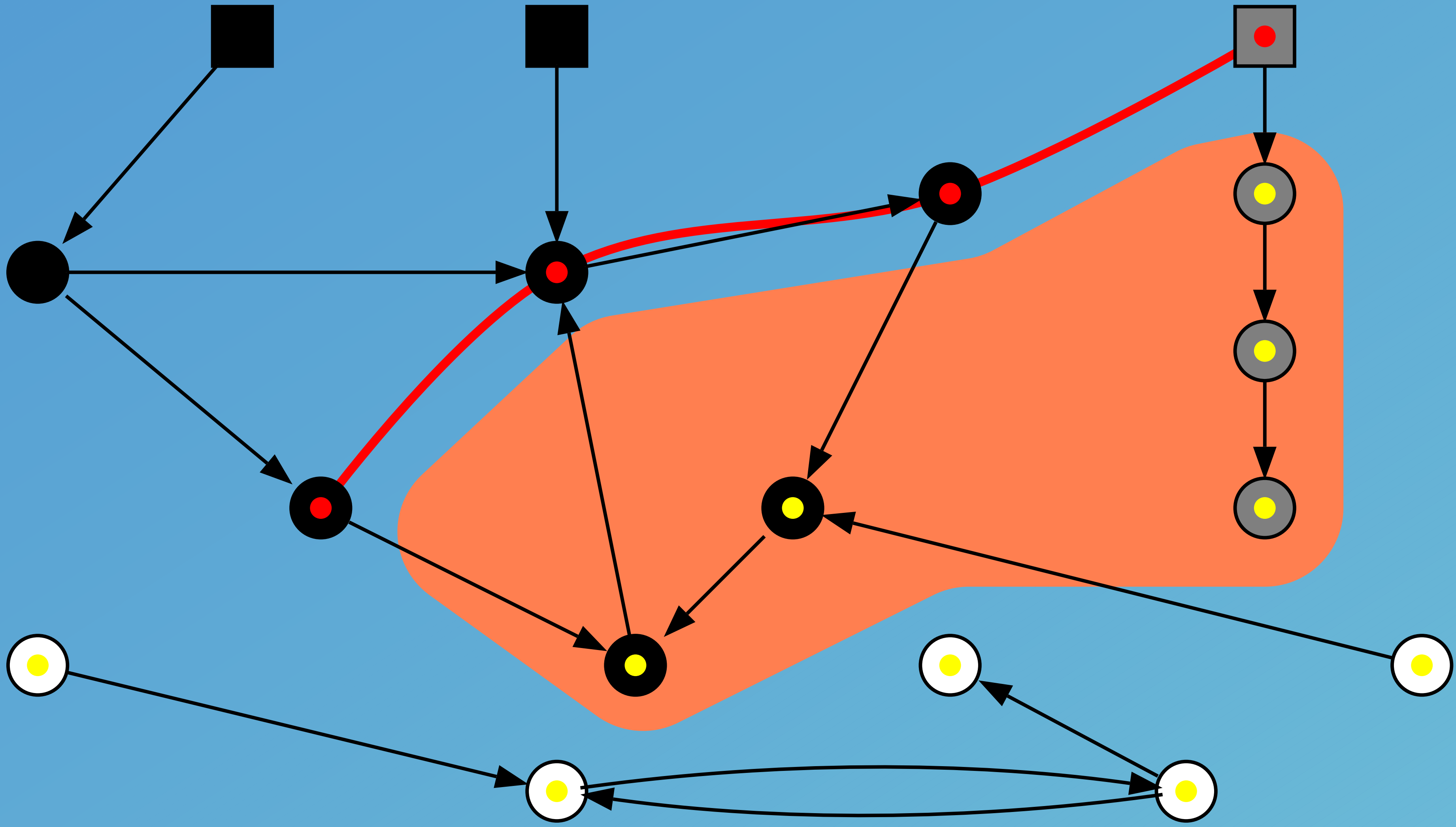
-- *the central invariant*

THM 1 : $\texttt{G.marked} \cap \texttt{G.dark} \subseteq \texttt{G.pink}$

-- *a helpful lemma*

THM 2 : $\texttt{G.red} = \emptyset \Rightarrow \texttt{G.marked} \subseteq \texttt{G.white}$

Intermediate snapshot



7.2 The Scavenging Phase ("Scan")

The scavenger recycles marked nodes as long as possible

```
SPEC Scavenger =
  IMPORT Monad(Scavenging-View-of-XGraph)

  FUN scavenge : M[Void]
      PRE marked ⊆ white
  DEF scavenge = iterate(recycle)

  THM invariant black
  THM ∀n ∈ black : invariant sucs(n)
```

☞ The black subgraph remains untouched!

The scavenger needs a primitive operation for recycling

SPEC $\mathtt{Scavanging\text{-}View\text{-}of\text{-}XGraph} =$

EXTEND $\mathtt{XGraph}$ ONLY $\mathtt{marked}$, $\mathtt{free}$ BY

-- *modify recycle*

FUN $\mathtt{recycle} : (\mathtt{G} : \mathtt{Graph}) \longrightarrow (\mathtt{G}' : \mathtt{Graph})$

PRE $\mathtt{G.marked} \neq \emptyset$

POST LET $\mathtt{n} \in \mathtt{G.marked}$ IN

$\mathtt{G}'.\mathtt{marked} = \mathtt{G.marked} \ominus \mathtt{n}$

$\mathtt{G}'.\mathtt{free} = \mathtt{G.free} \oplus \mathtt{n}$

-- *the central invariant*

AXM $1 : \mathtt{G.marked} \subseteq \mathtt{G.white}$

Adaptions for the Collector's Environment

☞ For retaining correctness the collector now relies on three properties of its environment

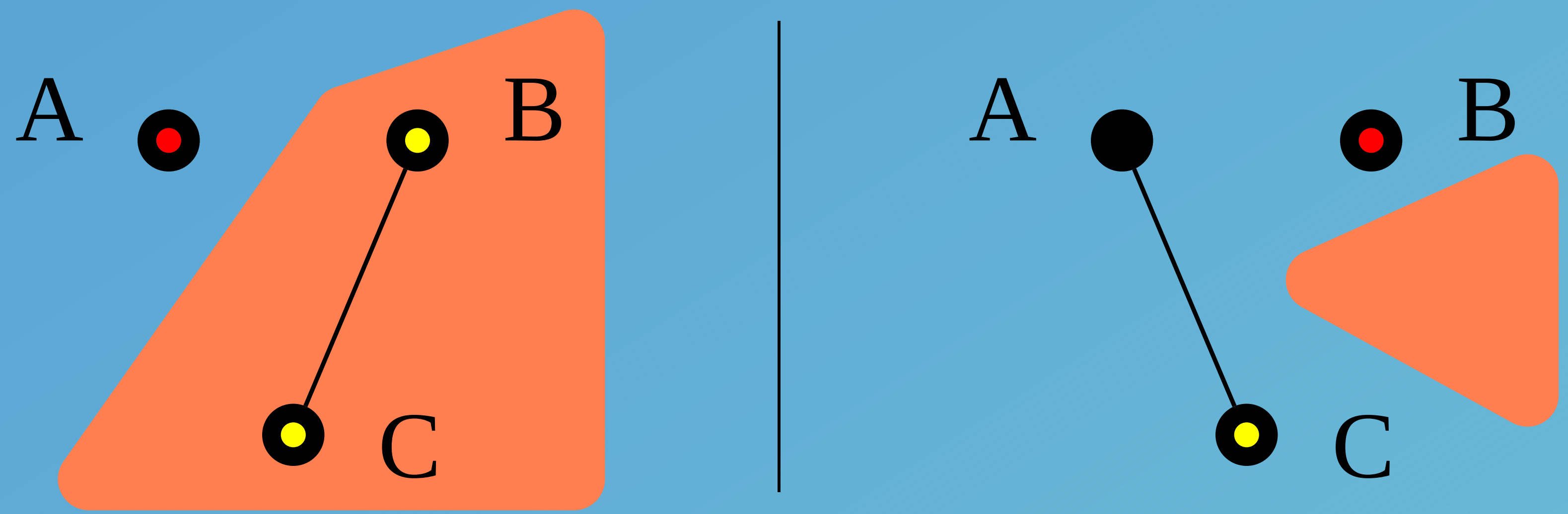
$$
\begin{array}{l}
\text{SPEC } \mathtt{XCollector\text{-}Environment} = \\
\quad \text{EXTEND } \mathtt{Monad}(\mathtt{XGraph}) \\
\quad \text{AXM } 1: \mathtt{G.marked} \cap \mathtt{G.dark} \subseteq \mathtt{G.pink} \\
\quad \text{AXM } 2: \mathtt{monotone\ white} \\
\quad \text{AXM } 3: \mathtt{invariant\ marked}
\end{array}
$$

(Note: $\mathtt{G.dark} = \mathtt{G.black} \cup \mathtt{G.gray}$)

8 The Extended Mutator

☞ The mutator now has to cooperate

Counterexample: A demonic mutator



```
SPEC XMutator-Spec ( Env : Mutator-Environment ) =
  IMPORT Monad(Mutator-View-of-XGraph)

  FUN mutate : M[Void]

  THM 1 : G.marked ∩ G.dark ⊆ G.pink
  THM 2 : monotone white
  THM 3 : invariant marked
```

The mutator guarantees:

1. all reachable marked nodes will still be visited
2. unreachable nodes remain unreachable
3. the marking remains untouched

The mutator needs a modified `add` operation

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- ☞ Whenever an arc to a (still) marked node is created, this node is coloured red (needs visiting)

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$$\text{SPEC } \mathtt{Mutator\text{-}View\text{-}of\text{-}XGraph} =$$

$$\quad \text{EXTEND } \mathtt{XGraph} \text{ ONLY } \mathtt{roots}, \mathtt{sucs}, \mathtt{cut}, \mathtt{new} \text{ BY}$$

-- *modify add*

$$\text{FUN } \mathtt{add} : (\mathtt{x} : \mathtt{Node}, \mathtt{y} : \mathtt{Node}) \to (\mathtt{G} : \mathtt{Graph}) \to (\mathtt{G}' : \mathtt{Graph})$$

$$\quad \text{POST } \mathtt{G}'.\mathtt{sucs}(\mathtt{x}) = \mathtt{G}.\mathtt{sucs}(\mathtt{x}) \oplus \mathtt{y}$$

$$\quad \mathtt{x} \notin \mathtt{G}.\mathtt{marked} \wedge \mathtt{y} \in \mathtt{G}.\mathtt{marked} \Rightarrow \mathtt{G}'.\mathtt{red} = \mathtt{G}.\mathtt{red} \oplus \mathtt{y}$$

9 Assessment of the Method

☞ Monads provide a sound and smooth way of integrating imperative concurrent programs with declarative specifications

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☞ Selective import plays a central role in the formulation and verification of properties:
It restricts quantification to selected operations.

(this avoids the clumsy $\Box\,(\mathtt{at}\ \pi \Rightarrow \ldots)$ known from temporal logic.)

Parameterized specifications and restricted imports in the Mutator specification