Control Systems vs. Reactive Systems

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Control Systems vs. Reactive Systems

Plan

- Motivations
- Dynamics
- Games
- Optimal and winning strategies
- Challenges

Style

- Reading notes
- Personal surprises and views
- No formulae, no pictures

(I) Motivations

Control systems

- Around -200: Ktesibios' water-controller in clocks
- Around 1750: Watt's flyball governor in steam engines
- 20th century: systems for transportation, production, health care

Computing systems

- From the fourties: sequential, batch, finite-time systems
- From the sixties: interactive, cooperative, concurrent, infinite-time, reactive, distributed systems

Same challenges: safety and optimality

Growing, deep convergence

(II) Dynamics

Dynamics

- Ditto: behaviours, histories, plays, processes, traces, trajectories
- Kinds: discrete time or not, deterministic or not, finite or not
- Discrete time: automata, graphs, programs, difference equations
- Continous time: differential equations or inclusions

Commonalities

- Controlling systems, implemented on computers, akin to

  Reactive computing systems, interacting with physical components
- Computer-based dynamics: Simulation;

  Simulation programming: Object-oriented programming
- Dynamics: the basis for control systems and for reactive ones

Dynamics: Analysis

Invariance

- Dynamics: invariant sets, since Poincaré, 19th century.
- Programs: invariant predicates, since the sixties.

Progress

- Stability, attraction, absorption in continuous dynamics:
  real-valued Lyapunov function with negative derivative.
- Termination in programs: decreasing integer-valued Floyd function
- Floyd functions: discrete abstractions of Lyapunov ones. Surprise!

Induction

- Discrete: iteration
- Continuous: integration

(III) Games

Interaction between dynamics

Kinds of games

- Time: discrete or continuous
- State space: finite or infinite
- Plays: one move, or finitely or infinitely many moves
- Moves: atomic or durative
- Goals: qualitative or quantitative
- Information on states: perfect or imperfect
- Winning strategies: forgetful or not, with or without equilibria

Games

Optimal or Winning strategies: policies ensuring goals

Duality

- Proponent: Or, Exists. Eve, Eloïse, Pro
- Opponent: And, All. Adam, Abélard, Opp
- And/Or trees. And/Or programs. Max-Min principles

Control systems as games

- Pro: controlling sub-system, designed by engineers
- Opp: controlled sub-system, e.g. plant, physical environment
- Problem: to synthesize optimal or winning strategies for Pro

Highlights in Game Theory

- 1910-30, Principles: Zermelo, Kalmar, Borel, von Neumann
- 1940-50, Max-Min strategies: von Neumann, Morgenstern
- 1950-70, Differential games: Isaacs
- 1960-80, Infinite automata-games: Church, Büchi, Landweber

Surprise! Zermelo's Work

- Game of Chess.
- Determinacy theorem
- Game of the Swimmer: Pro is a tired swimmer,
  Opp is a wild river in a narrow canyon with a safe island downstream.
- Methods: backward inductive reasoning (informal),
  set-based derivation of well-orderings, fixed points in graphs

Impulse games

Differential or dynamical games

- Continuous interactions between Pro and Opp
- Examples: Homicidal Chauffeur, Lady in the Lake

Impulse games

- Discontinuous interactions between Pro and Opp
- Pro and Opp may use discrete or continuous time
- ”Hybrid” systems: atomic Pro-moves, continuous Opp-moves

Observations

- The difference between discrete and dense time is a red herring
- Reactive systems are instances of dynamical systems and games

(IV) Optimal and Winning Strategies: Dynamics

Optimality: quantitative goals (e.g. maximization)

Winning: qualitative goals (e.g. a binary set of quantities)

Optimal discrete-time dynamics

- Discrete Bellman-principle of dynamic programming
- Solutions: algorithms using dynamic programming

Optimal dense-time dynamics

- Continuous Bellman-principle of dynamic programming, Hamilton-Jacobi(-Bellman) differential equations
- Solutions: Bellman principle, or Pontryagin principle of optimality

Surprise!? Same principle for discrete/continuous shortest paths.

Optimal Strategies: Games

Max-Min versions of strategies for dynamics

Optimal continuous-time games

- Max-Min continuous Bellman-principle of dynamic programming
  Hamilton-Jacobi(-Bellman)-Isaacs equations
- Solutions: max-min variants of those for dynamics, integration

Optimal discrete-time games

- Max-Min discrete Bellman-principle of dynamic programming
  Max-Min difference equations
- Solutions: max-min variants of those for dynamics, iterations

Surprise! Structurally similar approaches for continuous and discrete time

Winning strategies: Games

Borel hierarchy of dynamics

- Level 0: Invariance (safety), Inevitability (reachability)
- Level 1: Liveness (recurrence), Persistence
- Levels 2,...

Finite-state, infinite-execution automata. Impulse games

- Properties: Levels 0, 1
- Winning regions: least and greatest fixed-points, nested iterations
- Winning strategies: well-orderings

Differential, dynamic games, with continuous Pro/Opp interactions:

open for Levels 1,...

Surprise! Similar structure of optimal and winning strategies.

(V) Challenges

- To scale up: many players, teams, goals, rôles, levels
- Hierarchical decomposition and refinement
- Clear, shared theoretical bases
- Iterative synthesis of winning strategies
- Further properties: security, probability
- Further solutions: equilibria, negociation, incomplete information
- Development, adaptation or reuse of design-support systems, also from related fields.
- Cooperation between software engineers and other system-engineers, between computing scientists and applied mathematicians

Paradigmatic Challenge

A non-trivial undertaking

- To develop a distributed computer-based system
  assisting drivers and
  managing road traffic
- Goals: to maximize
  the survival of persons,
  the satisfaction of transportation needs, and
  the preservation of the environment.
- Possible approach: a land-based variant of
  control systems for air traffic and airplanes

See you then?