Game Theory in String Diagrams

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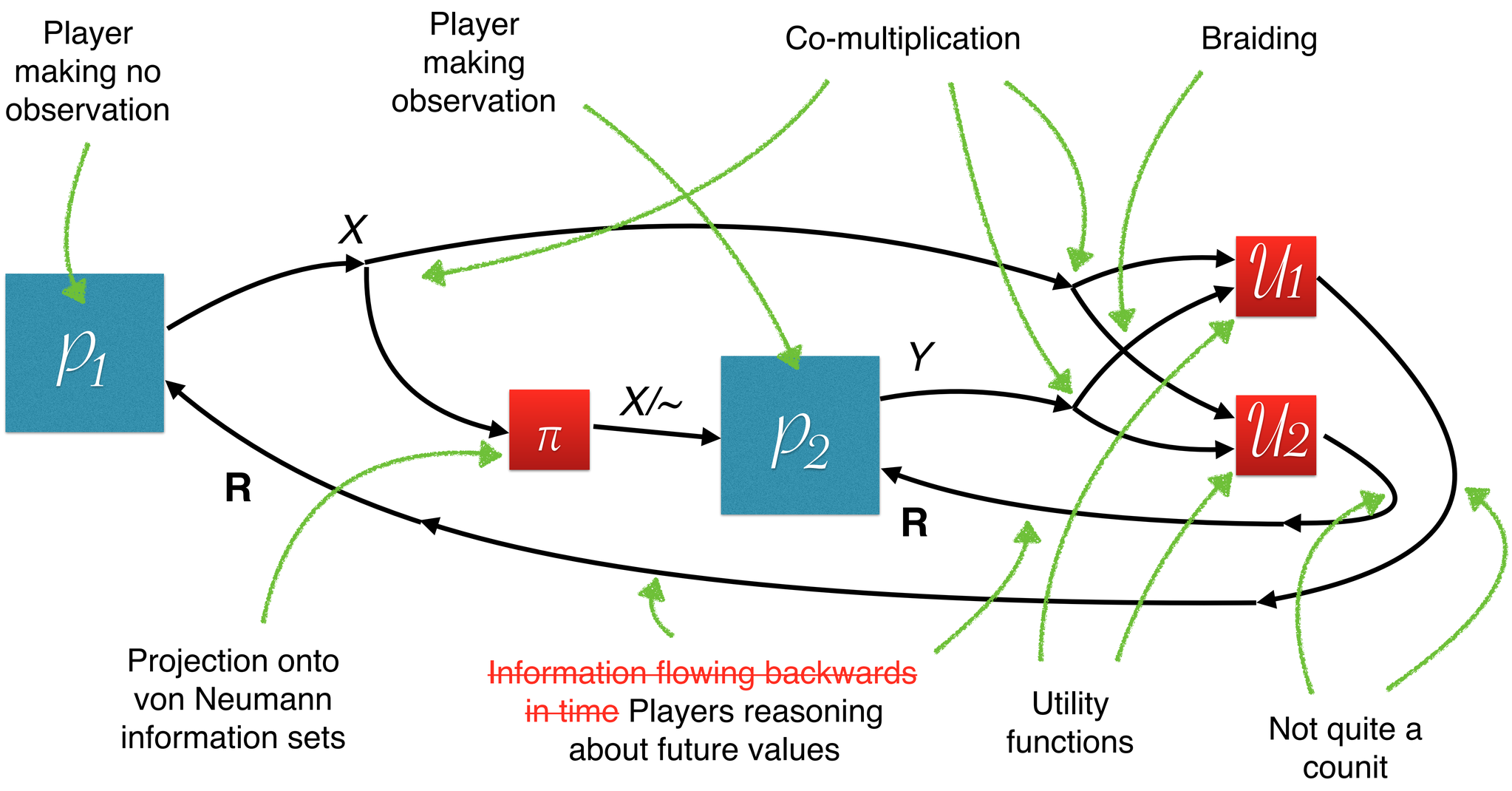
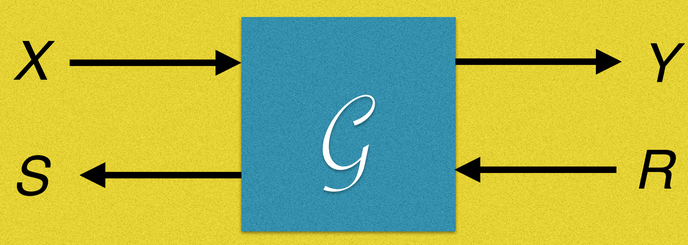


Morphism ≈ String diagram ≈ (Component of) game	Categorical composition ≈ End-to-end composition ≈ Sequential play	Monoidal product ≈ Side-by-side composition ≈ Simultaneous play

Semantics

Objects are pairs of sets (cf. Int/GoI construction, Chu spaces, dialectica categories, …)



$\mathcal{G} : (X, S) \longrightarrow (Y, R)$

X : observations Y : choices
S : "co-utilities" R : utilities

Formally $\mathcal{G} = (\sum, \mathbf{P}, \mathbf{C}, \mathbf{B})$ where

- $\sum$ set of strategy profiles
- $\mathbf{P} : \sum \longrightarrow \hom_{\mathcal{C}}(X, Y)$ plays a strategy
- $\mathbf{C} : \sum \longrightarrow \hom_{\mathcal{C}}(X \times R, S)$ "dual" of playing
- $\mathbf{B} : \hom_{\mathcal{C}}(I, X) \times \hom_{\mathcal{C}}(Y, R) \longrightarrow (\mathcal{P}\sum)^{\Sigma}$

Delimited continuation (secret ingredient)

Best response function (apply fixpoint theorem to this)

Motivation

- Classical game theory is not at all compositional
- Intuitive, graphical, formal language for specifying and reasoning about games

Foundations

Can be built over several starting categories, giving different types of strategies:

- **Set**: pure strategies
- $\mathbf{Set}_{\mathcal{D}}$ ($\mathcal{D}$ distribution monad): mixed strategies
- **Rel**: nondeterministic strategies
- More generally, Kleisli categories of commutative strong monads
- Can get correlated & Bayesian equilibria for free by suitable choices of monad