SMT-Style Program Analysis with Value-based Refinements

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NSV-3
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Outline

Imprecision in Abstract Interpretation

- Abstract interpretation sound but not complete.
- Incompleteness manifests in imprecision during the analysis.

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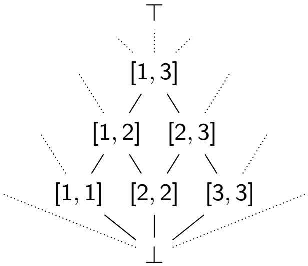

Example: Domain of Intervals

Imprecisions in the Domain

Imprecision in join

```
x:=*;
if(x > 5)
  y := -1;
else
  y := 1;

assert(y != 0);
```

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$\longrightarrow y \in [-1, -1], x \in [6, \infty]$

Imprecisions in the Domain

Imprecision in join

```
x:=*;
if(x > 5)
  y := -1;
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  y := 1;

assert(y != 0);
```

$\texttt{y := -1;} \longrightarrow y \in [-1, -1], x \in [6, \infty]$

$\texttt{y := 1;} \longrightarrow y \in [1, 1], x \in [-\infty, 5]$

Imprecisions in the Domain

Imprecision in join

```
x:=*;
if(x > 5)
  y := -1;
else
  y := 1;

assert(y != 0);
```

$\longrightarrow y \in [-1,-1], x \in [6,\infty]$ (after `y := -1;`)

$\longrightarrow y \in [1,1], x \in [-\infty,5]$ (after `y := 1;`)

$\longrightarrow y \in [-1,1]$ (at `assert(y != 0);`)

The disjunction $y = 1 \vee y = -1$ cannot be expressed as an interval.

Imprecisions in the Domain

Imprecision in transformer

```
x:=y;
if(x > 5)
 assert(y > 5);
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Imprecisions in the Domain

Imprecision in transformer

```
x:=y;
if(x > 5)
  assert(y > 5);
```

$\longrightarrow \top$ (beside `x:=y;`)

$\longrightarrow x \in [6, \infty]$ (beside `assert(y > 5);`)

Intervals cannot express relational information.

Imprecisions in the Analysis

Imprecision in widening

```
while(x < 50000)
{
  x++;

  if(y < x)
    y++;
}
```

$x \in [0, 0], y \in [0, 0]$

$x \in [0, 1], y \in [0, 1]$

widening

$x \in [0, \infty], y \in [0, \infty]$

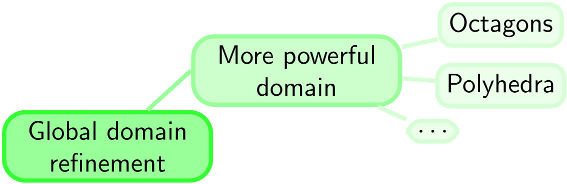
$x \in [50000, 50000]$, $y \in [0, \infty]$

Precision can be lost in the the analysis
Refinement of widening studied by, e.g., Gulavani et. al (TACAS 2008), Wang et al. (CAV 2007)

Refining Abstract Domains

Global domain refinement

Refining Abstract Domains



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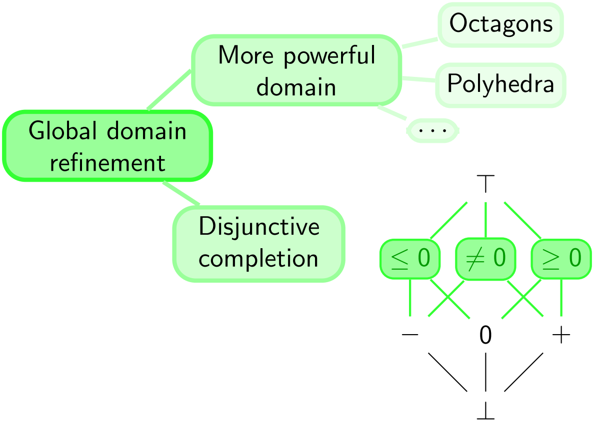

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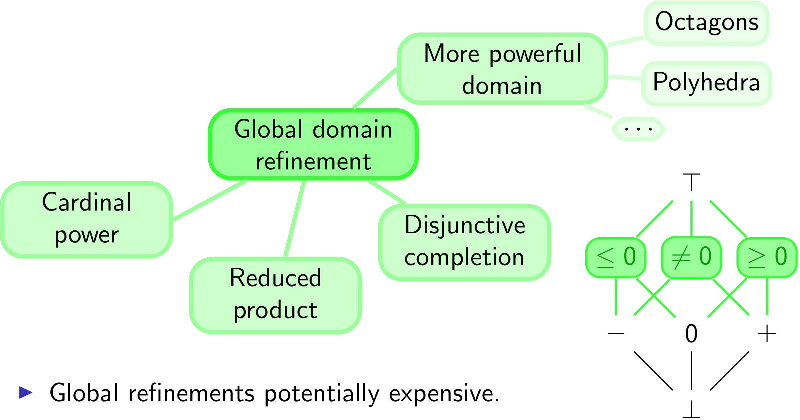

- Global refinements potentially expensive.
- How can we locally refine an abstract domain?

Trace Partitioning

- Trace partitioning allows for flexible and local refinement

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  - Consider separately different sets of traces through a program
  - Similar to case splits in a mathematical proof.

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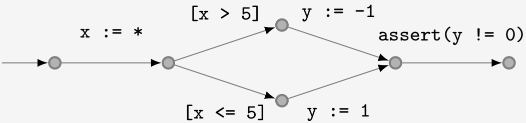
Control-flow based trace partitioning



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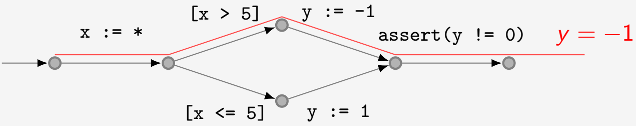

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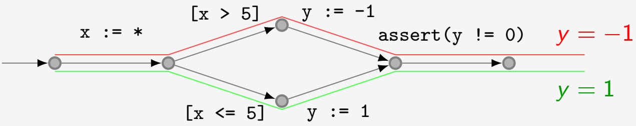

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- Wide range of partitionings possible
  - control flow,
  - values of variables,
  - number of iterations through a loop, etc.

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Value-based partitioning

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x:=y;
if(x > 5)
  assert(y > 5);
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Trace Partitioning

- Wide range of partitionings possible
  - control flow,
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Value-based partitioning

```
assume(y > 5);
         x:=y;
         if(x > 5)
           assert(y > 5);
```

$y > 5$

Trace Partitioning

- Wide range of partitionings possible
  - control flow,
  - values of variables,
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Value-based partitioning

```
assume(y > 5);        assume(y <= 5);
          x:=y;
          if(x > 5)
            assert(y > 5);
```

$y > 5$ $\quad\quad$ $\perp$

Finding Partitioning Functions

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The main question is:
How can we find a good partitioning?

- Precise enough to prove the property, and
- abstract enough to be efficient.

Finding Partitioning Functions

- Leino and Logozzo (APLAS 2005): Value-based trace partitionings based on counter examples
- Gulavani et al. (TACAS 2008): DAG-based Exploration of control-flow paths inside loops with splitting on demand.
- Gulwani et al. (PLDI 2009): Control-flow refinement for bounds analysis.
- Harris et al. (POPL 2010): Satisfiability Modulo Path Programs

Value-based Trace Partitionings

- If the abstract transformer $\hat{F}$ is too imprecise, find a set of transformers $\hat{F}_1, \ldots, \hat{F}_k$, such that

$$\bigcup_{1 \le i \le k} \gamma(\mu X.\ \hat{F}_i(X)) \ \supseteq \ \mu X.\, F(X)$$

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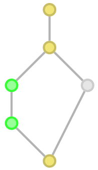

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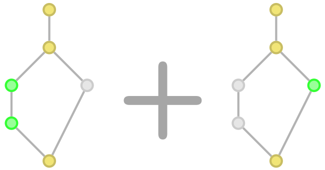

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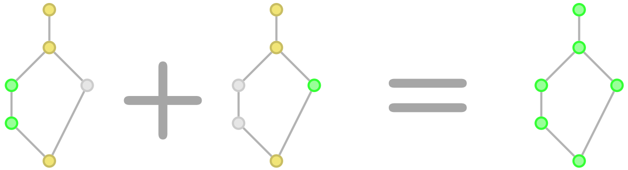

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How can we find such a set of elements $a_1, \ldots, a_k$*?*

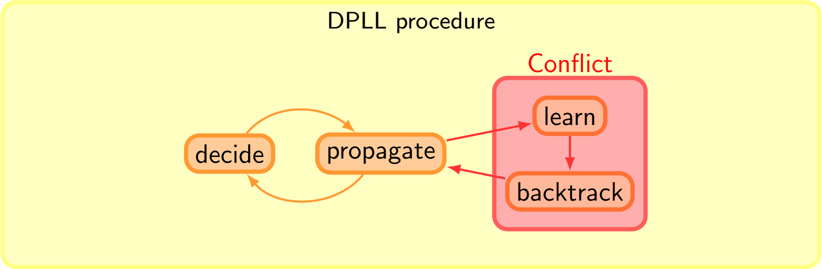
Value-based Trace Partitionings

New question:

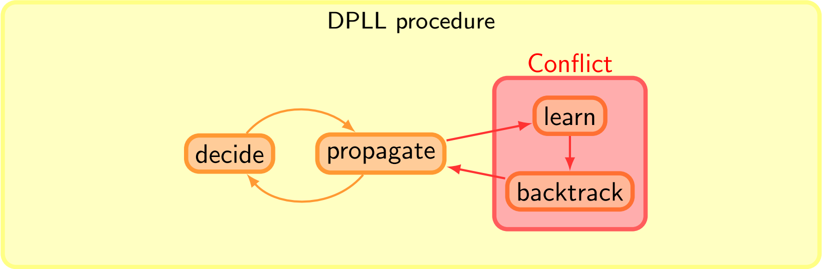
How can we find such a set of elements $a_1, \ldots, a_k$?

Use the search architecture of a SAT solver!

DPLL framework

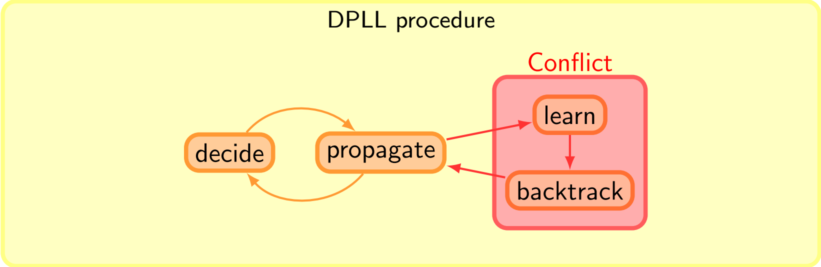


DPLL framework



- Main phases of the DPLL procedure:

DPLL framework



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  Decision Assume a value for an undetermined variable

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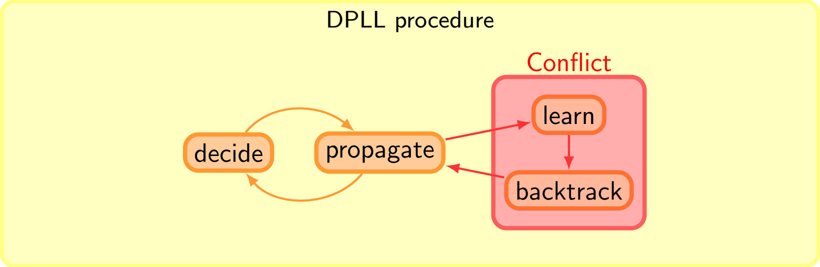

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Decision	Assume a value for an undetermined variable
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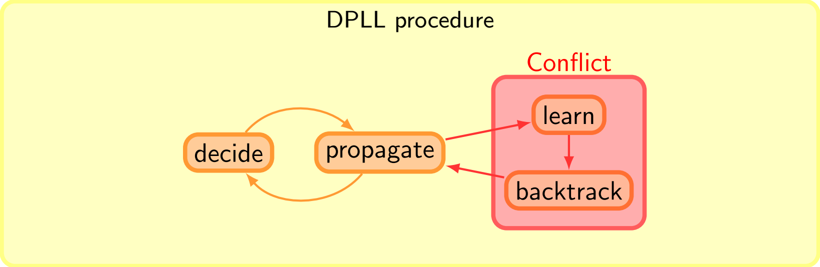

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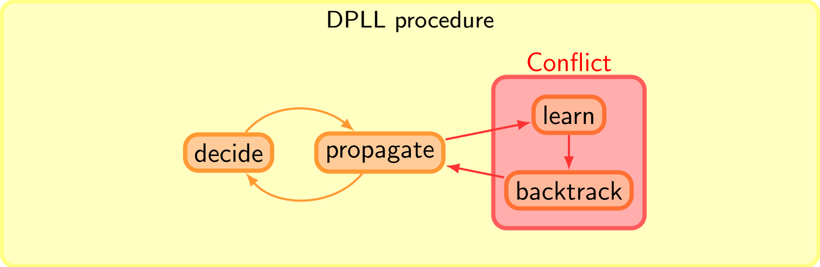

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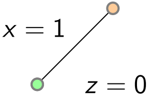

Propagation

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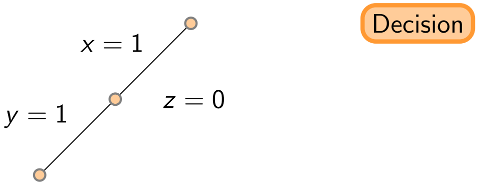

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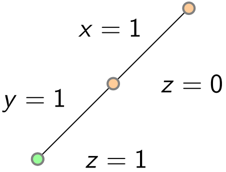

Propagation

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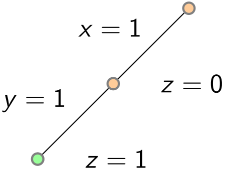

Propagation

Conflict

DPLL Procedure

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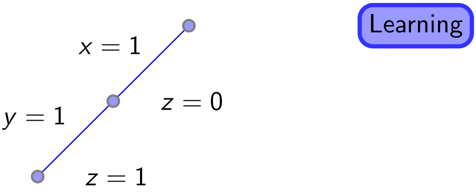

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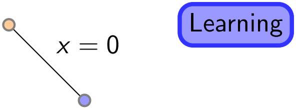
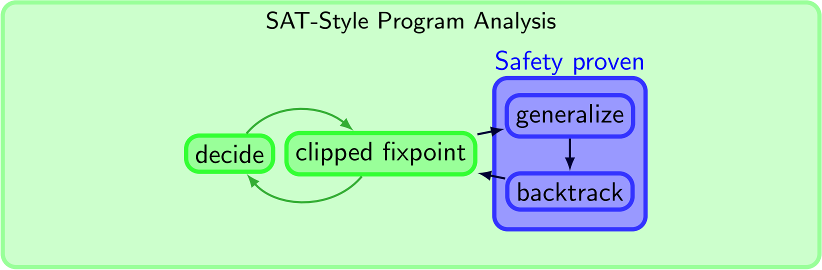


SAT-Style Program Analysis



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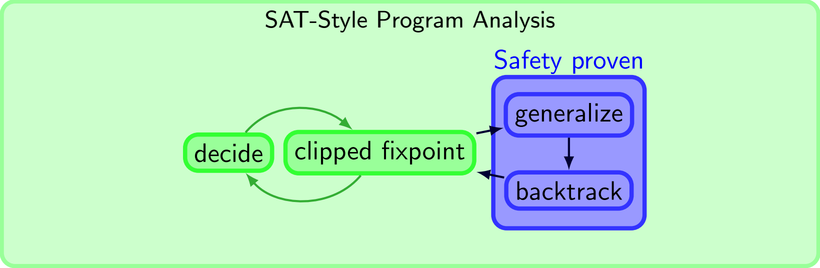

Decision Refine current element a by $a' \sqsubset a$

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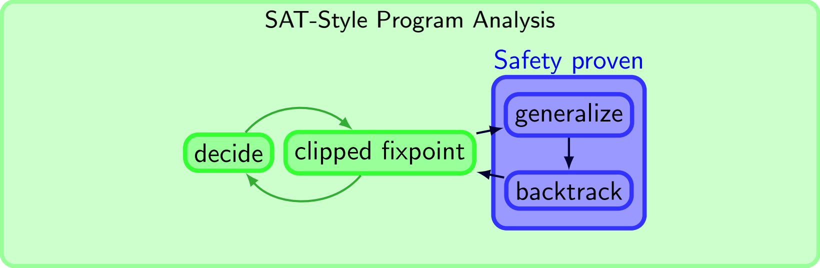

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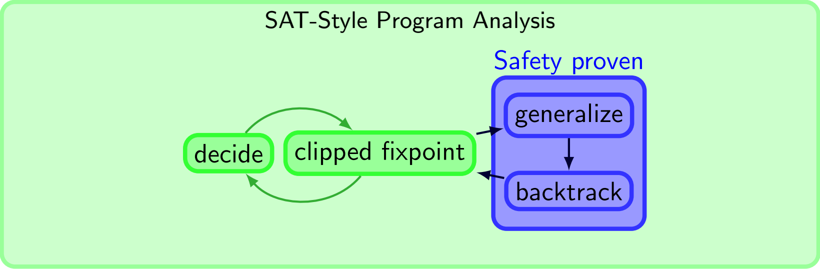

Decision	Refine current element a by $a' \sqsubset a$
Propagation	Compute clipped fixpoint $\mu X.\hat{T}(X) \sqcap a'$
Learning	Find $a'' \sqsupseteq a'$, such that $\mu X.\hat{F}(X) \sqcap a''$ is safe.

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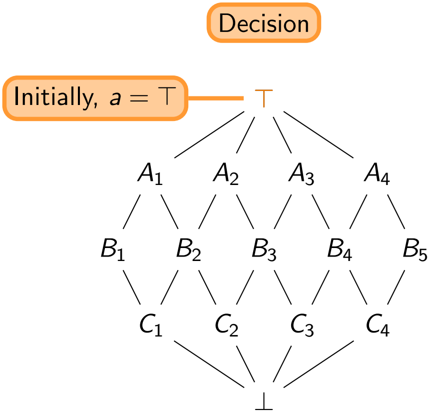

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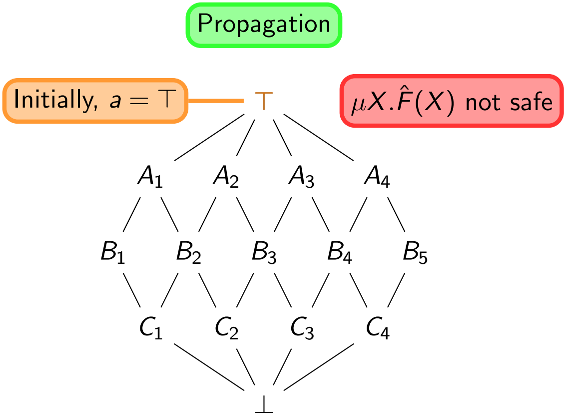
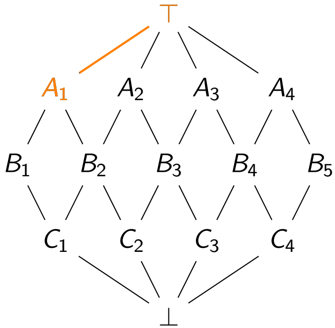


SAT-Style Program Analysis

Decision

Decision: refine a



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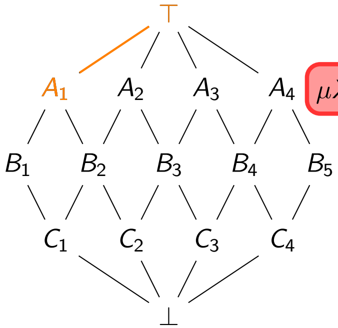

$\mu X.(\hat{F}(X) \sqcap A_1)$ not safe

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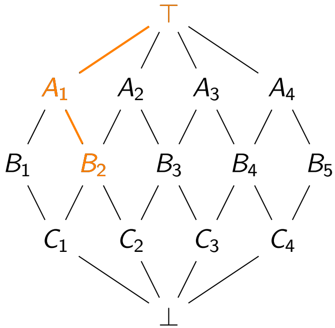

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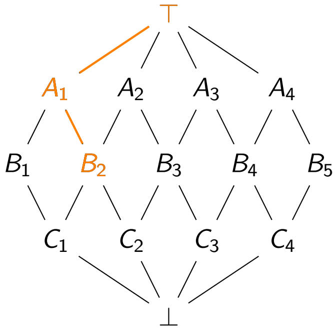
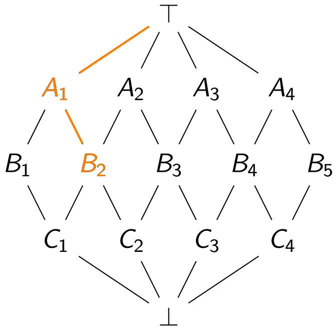


$\mu X.(\hat{F}(X) \sqcap B_2)$ safe

SAT-Style Program Analysis

Generalization



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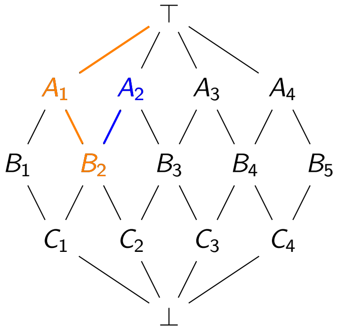

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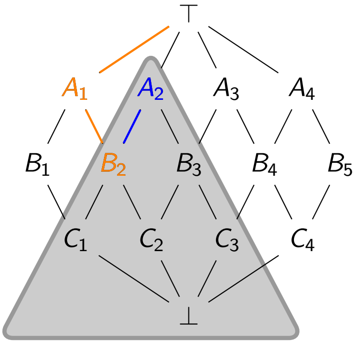

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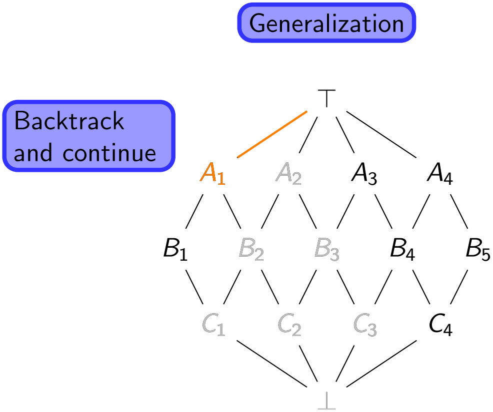

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- When can we efficiently prove safety with this?
  - When there is a small and finite number of elements $a_1, \ldots, a_k$ such that the fixpoints $\mu X.(\hat{F}(X) \sqcap a_i)$ can be put together to form a concrete postfixpoint.
- Specific implementation issues:
  - Generalization step
  - Decision heuristic

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We have created a preliminary instantiation of this framework for the domain of intervals.

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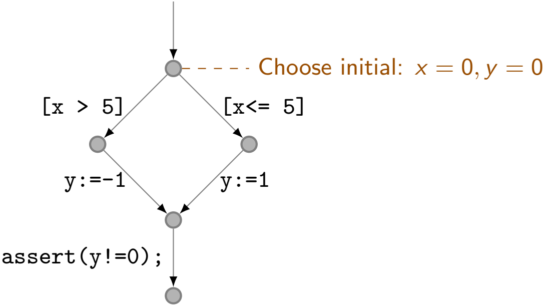
Generalization and Learning:
Generalize the result by locally generalizing intervals. Remove generalized initial values from selection pool

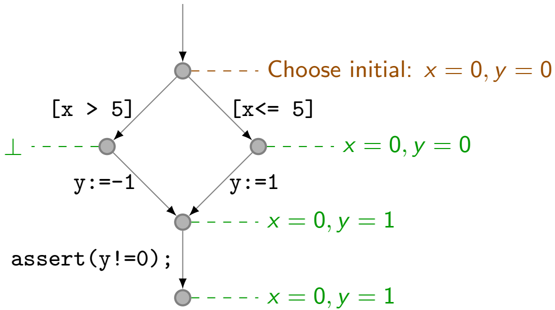
Example 1

Decision



Example 1

Propagation



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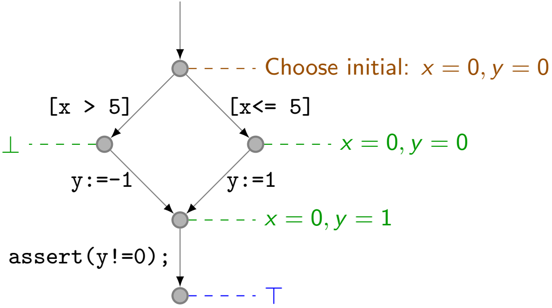

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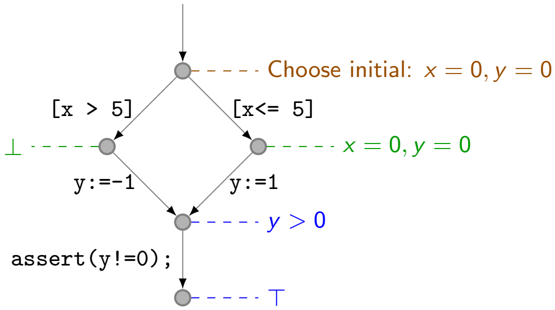

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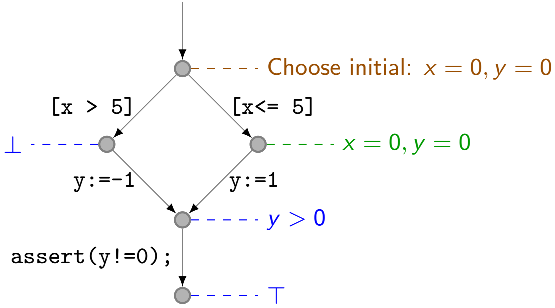

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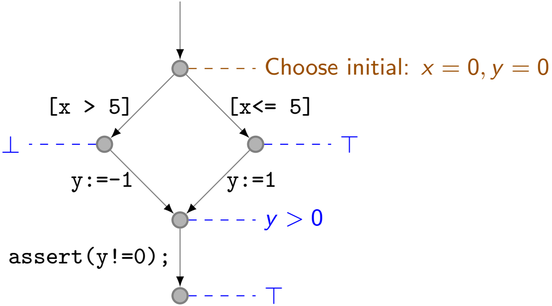
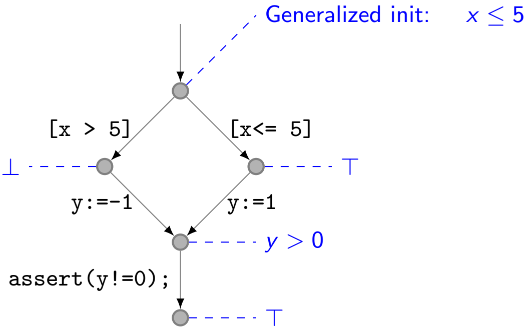


Example 1

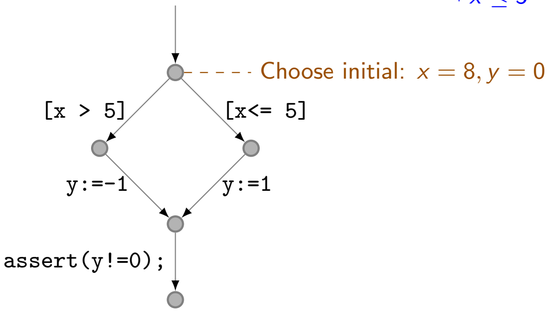
Generalization



Example 1

Decision

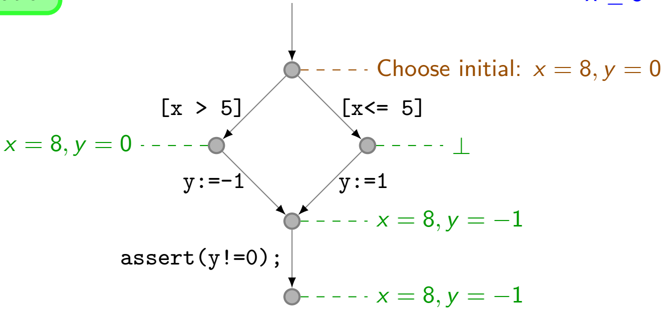
$\neg x \leq 5$



Example 1

Propagation

$\neg\ x \leq 5$



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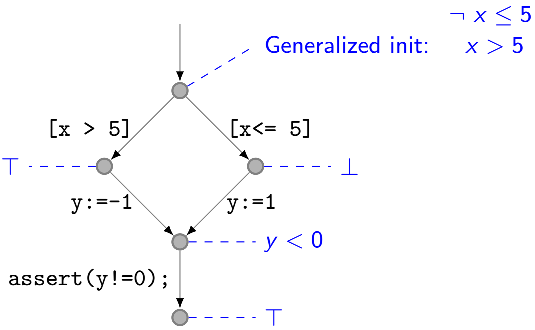

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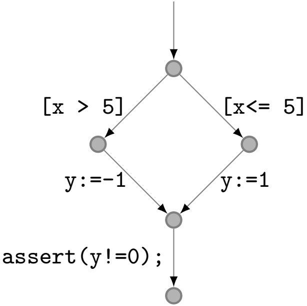
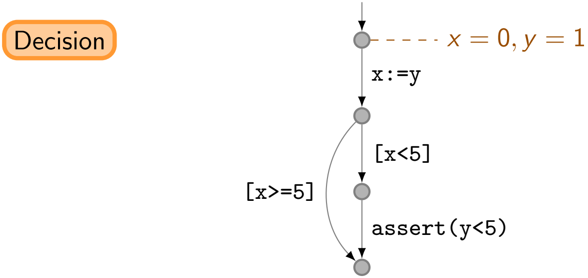


$\neg\ x \leq 5$

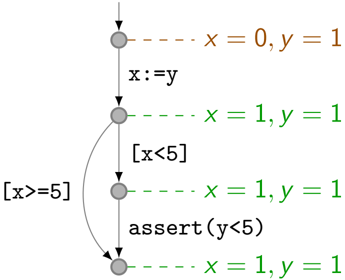
$\neg\ x > 5$

Example 2



Example 2

Propagation



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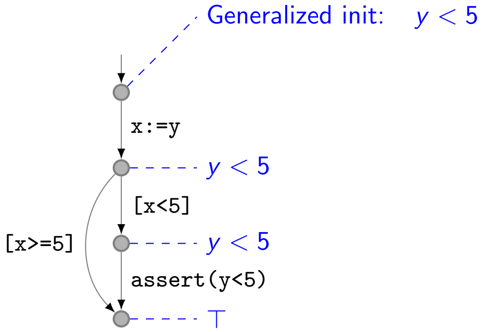
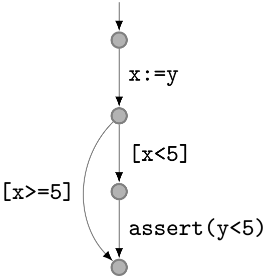
Generalization



Example 2

$\neg y < 5$

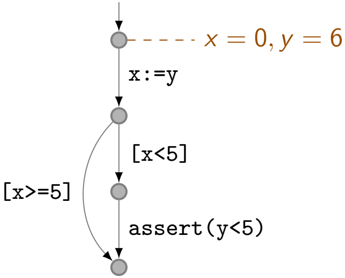
Generalization



Example 2

$\neg y < 5$

Decision



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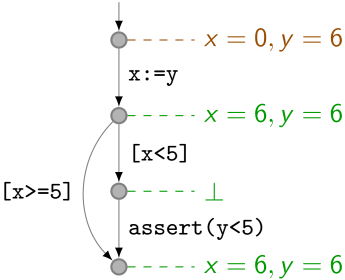
Propagation



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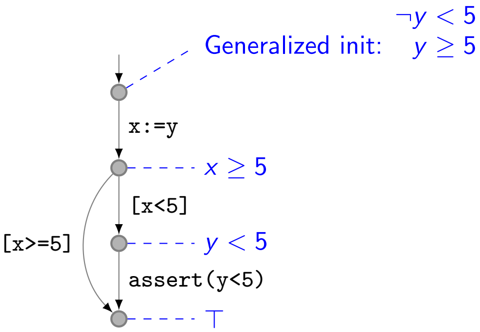
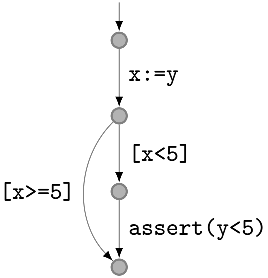
Generalization



Example 2

$\neg y < 5$
$\neg y \geq 5$



Notes on Implementation

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$0 \leq a \leq 5, b > 5, c < 10$ Repair using SAT solver

```
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$-10 \leq a \leq \infty, b > 5, c < 10$	Repair using SAT solver
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inf1.c	36	*	*
inf2.c	12	0.7	5
inf3.c	16	0.9	4
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inf5.c	28	2.1	19
inf6.c	32	0.9	4
inf7.c	27	1.7	7
inf8.c	40	3.3	9

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- Does not work if fully relational information is required (inf1.c,inf4.c)

```
assume(x > y);
assert(x > y);
```

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- Use trace partitioning and SMT/SAT-style analysis as “glue“ to combine a static analyzer with a bounded model checker.