Interpolation for guarded logics

Michael Vanden Boom

University of Oxford

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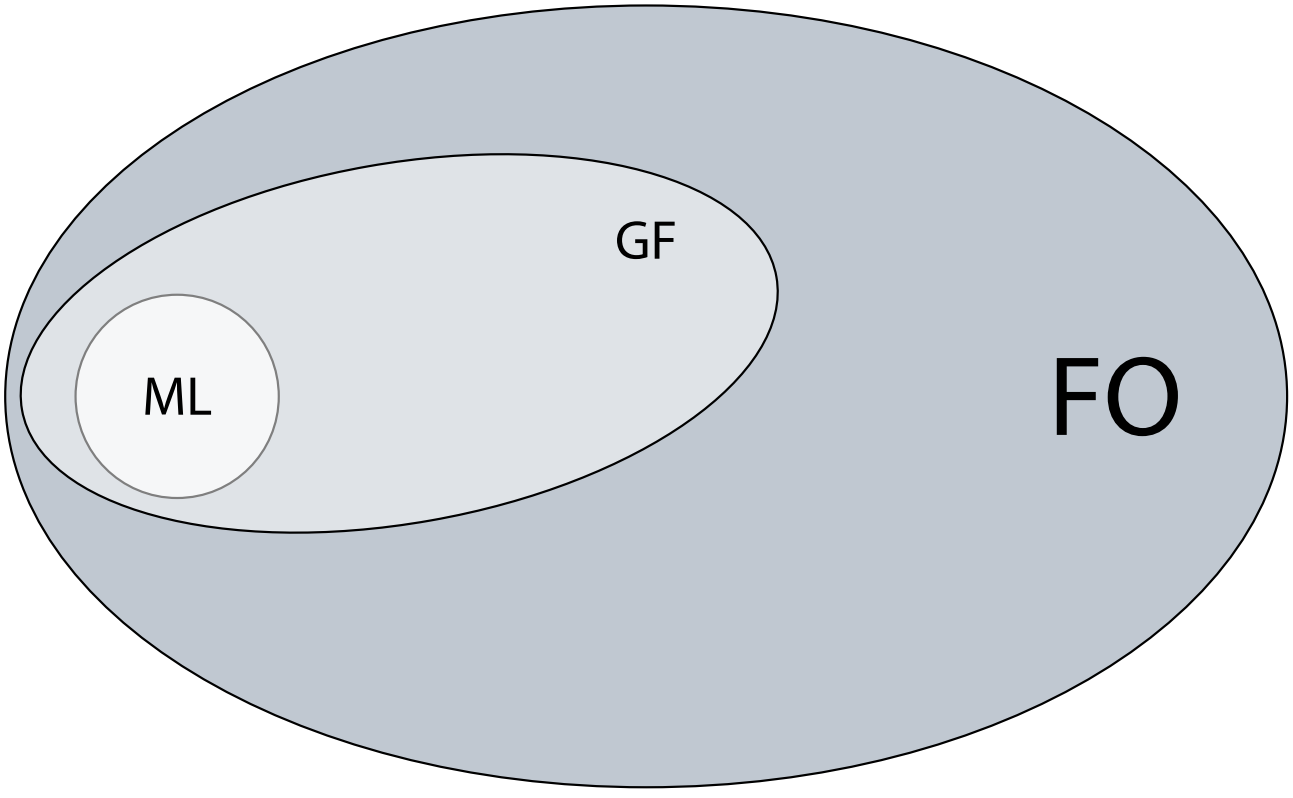
Joint work with
Michael Benedikt and Balder ten Cate

Some guarded logics



constrain
quantification

$\exists \boldsymbol{x}(G(\boldsymbol{xy}) \wedge \psi(\boldsymbol{xy}))$
$\forall \boldsymbol{x}(G(\boldsymbol{xy}) \rightarrow \psi(\boldsymbol{xy}))$

[Andréka, van Benthem, Németi '95-'98]

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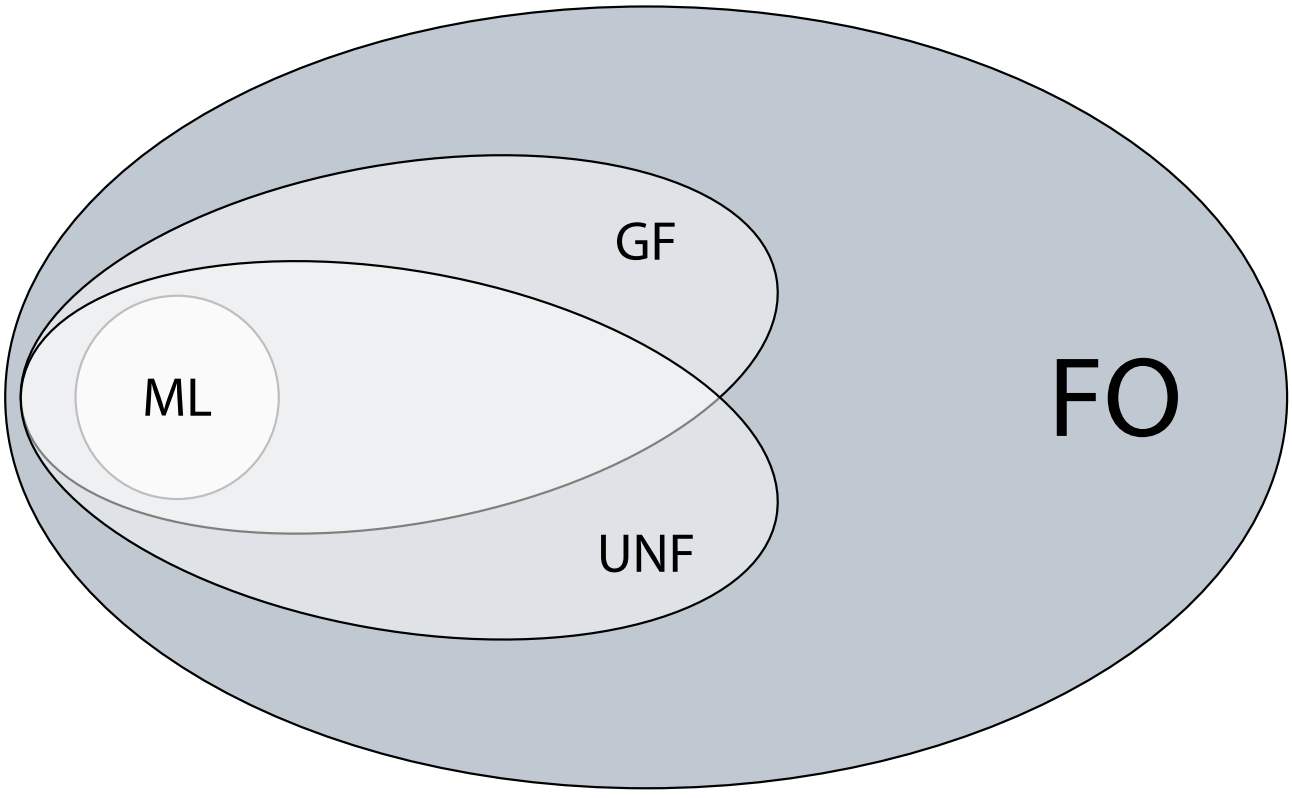

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constrain negation

$\exists \boldsymbol{x}(\psi(\boldsymbol{xy}))$
$\neg\psi(x)$

[ten Cate, Segoufin '11]

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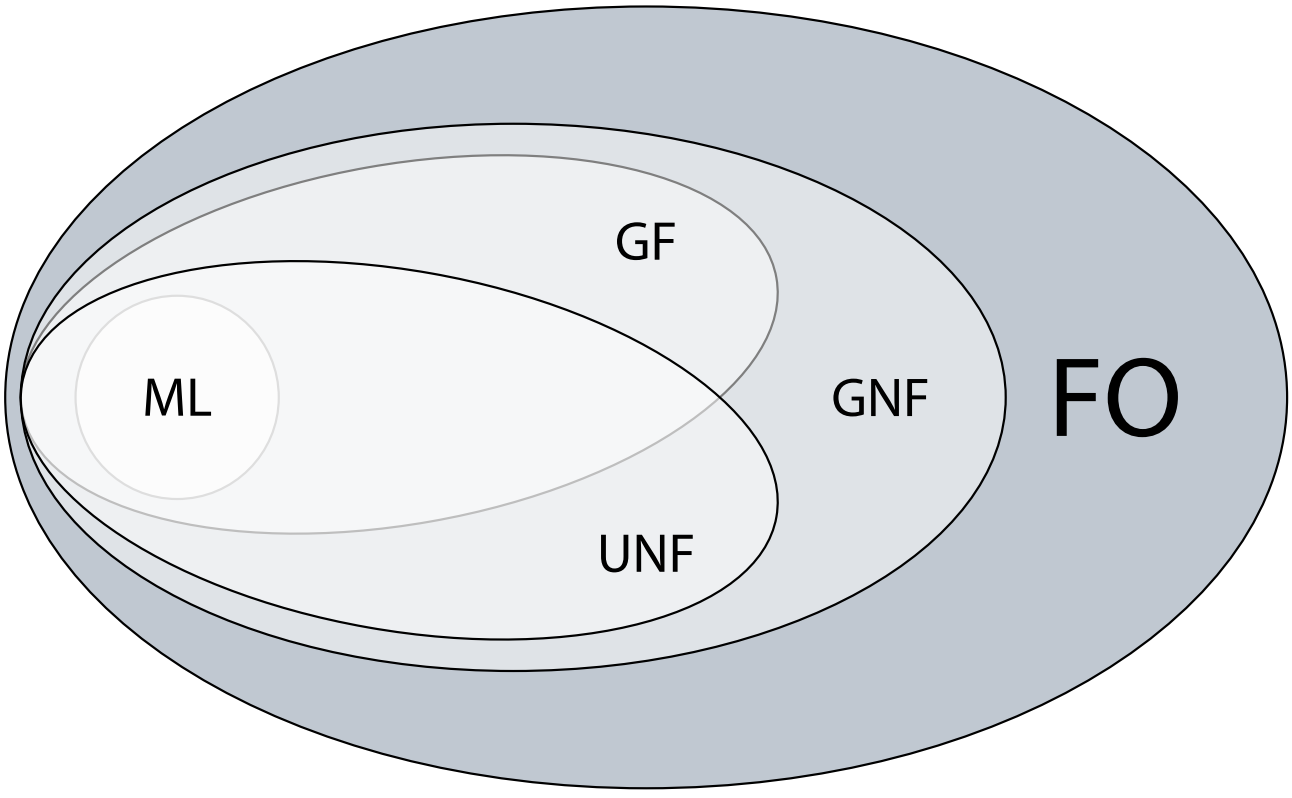

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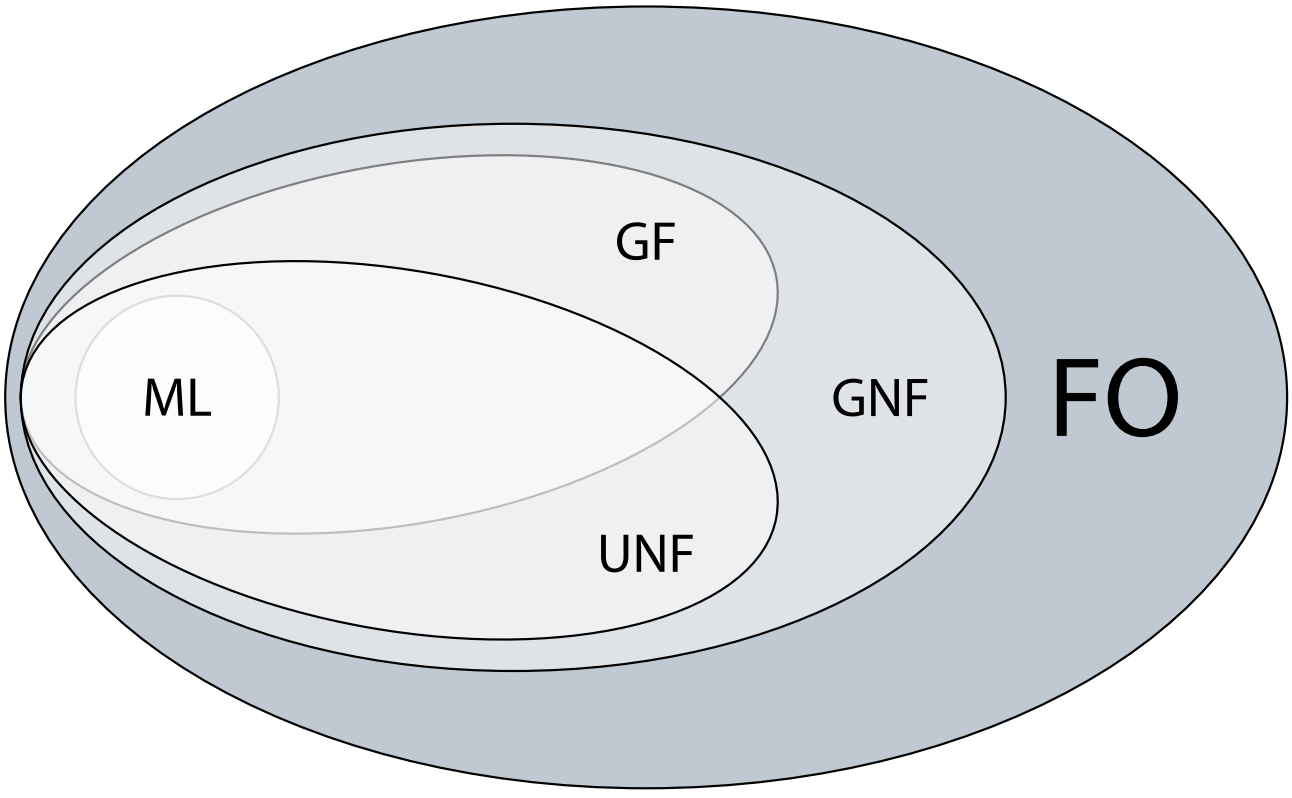

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These guarded logics are decidable, and expressive enough to capture many query languages and integrity constraints of interest in databases and knowledge representation.

Interpolation

$$\varphi \quad \vDash \quad \psi$$

Interpolation

$$\varphi \models \underset{\text{interpolant}}{\chi} \models \psi$$

χ only uses relations in both φ and ψ

Interpolation example

$$\exists xyz(Txyz \wedge Rxy \wedge Ryz \wedge Rzx) \quad \models \quad \exists xy(Rxy \wedge ((Sx \wedge Sy) \vee (\neg Sx \wedge \neg Sy)))$$

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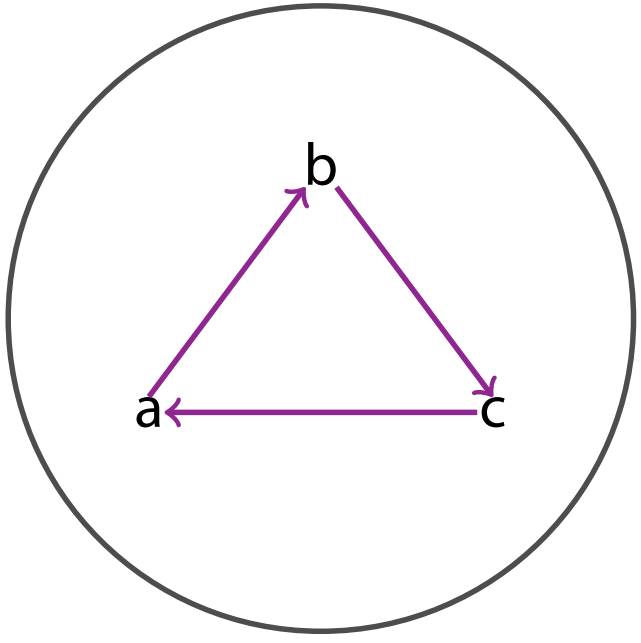

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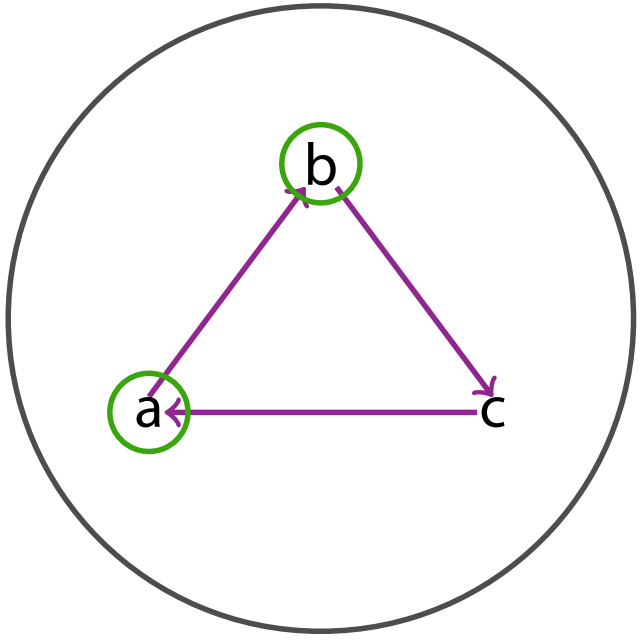

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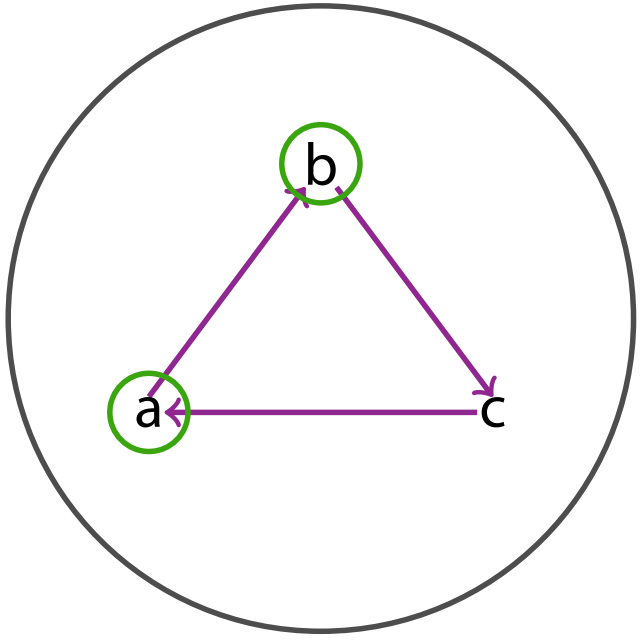

interpolant $\chi := \exists xyz(Rxy \wedge Ryz \wedge Rzx)$

"there is a 3-cycle using R"

Why do we care?

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Why might someone care?

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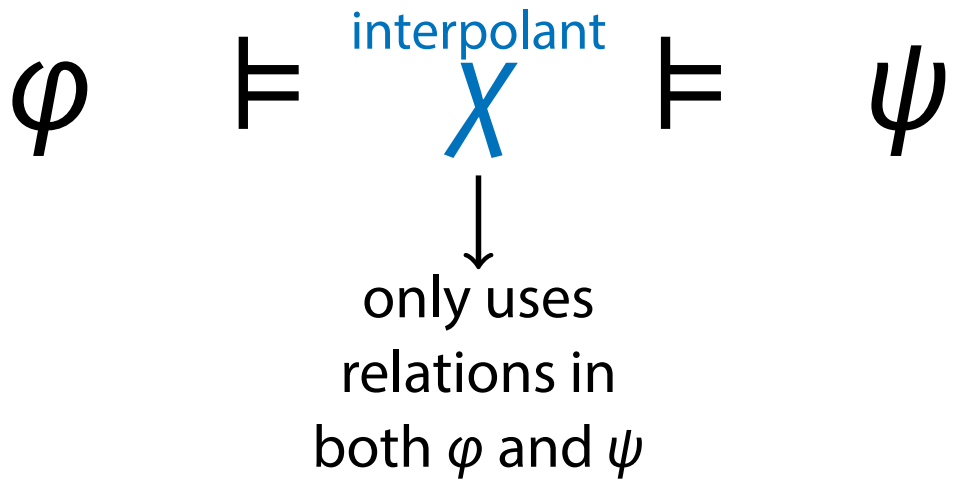
- interpolation is a benchmark property of modal logic
- interpolation implies the Beth definability property
  (implicit definability = explicit definability)
  which indicates a good balance between syntax and semantics
- for these guarded logics with connections to databases,
  interpolation is related to query rewriting over views

Interpolation



Theorem (ten Cate, Segoufin '11; Barany, Benedikt, ten Cate '13)

Given GNF (respectively, UNF) formulas φ and ψ such that $\varphi \models \psi$, there is a GNF (respectively, UNF) interpolant χ.

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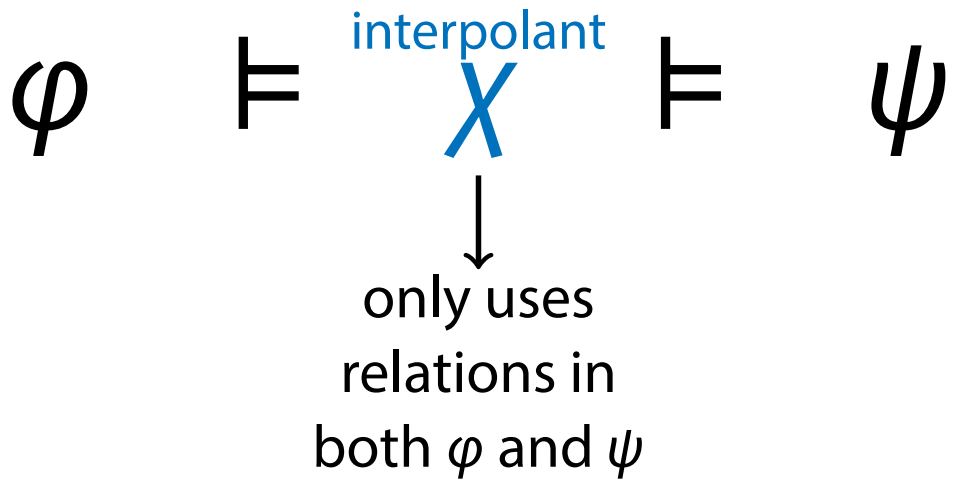

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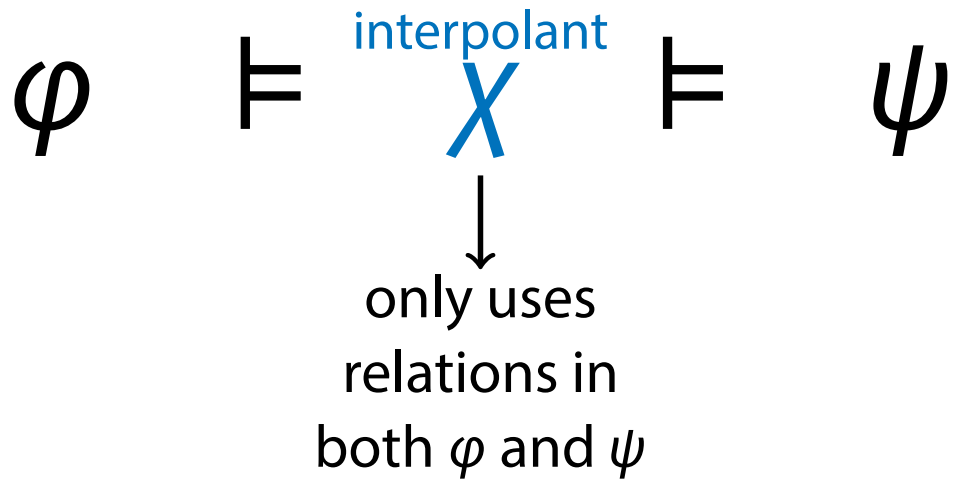
No idea how to **compute** interpolants
(or other rewritings related to interpolation).

Interpolation



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Theorem (Benedikt, ten Cate, VB. '14)

Given GNF (respectively, UNF) formulas φ and ψ s.t. $\varphi \models \psi$, we can construct a GNF (respectively, UNF) interpolant χ of doubly exponential DAG-size.

Conclusion

	ML	GF	UNF	GNF
Interpolation?	✓	✗	✓	✓

adapted
mosaic method
from ML

[Benedikt,ten Cate,VB.'14]

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Interpolation?	✓	✗	✓	✓	✓	✗	✓	✗

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used **automata** for L_μ [Benedikt,ten Cate,VB. unpublished]

Open question

Is there a decidable extension of GNFP that has interpolation?