CHAPTER 12: MAKING GROUP DECISIONS

An Introduction to Multiagent Systems

http://www.csc.liv.ac.uk/~mjw/pubs/imas/
Social Choice Theory

Social choice theory is concerned with group decision making. Formally, the issue is combining preferences to derive a social outcome. A classic example of social choice theory is voting.

Social Choice Theory

An Introduction to Multiagent Systems 2e
Chapter 12
An Introduction to Multiagent Systems

ComponentsofaSocialChoiceModel

• Assume a set \( A \) of voters.

• Voters make group decisions wrt a set of outcomes \( \Omega = \{ \omega_1, \omega_2, \ldots, \omega_m \} \).

• If \( |\Omega| = 2 \), we have a pairwise election. Think of these as the candidates.

• These are the entities who will be expressing preferences.

• Assume a set \( A \) of voters.
Preferences

• Each voter has preferences over $\Omega$: an ordering over the set of possible outcomes.

Example. Suppose the set of possible outcomes $\Omega$, Each voter has preferences over $\Omega$: an ordering over preferences.

brandy $\succ_m w$ rum $\succ_m w$ gin $\succ_m w$ whisky

meaning

$(\text{brandy, rum, gin, whisky}) = \omega_{mw}$

then we might have agent $mw$ with preference order:

$\{\text{gin, rum, brandy, whisky}\} = \Omega$

Preferences
The fundamental problem of social choice theory: given a collection of preference orders, one for each voter, how do we combine these to derive a group decision, that reflects as closely as possible the preferences of voters?

Two variants of preference aggregation:

- Social welfare functions
- Social choice functions

The fundamental problem of social choice theory: Preference Aggregation
Social Welfare Functions

- Let $\Pi(\Omega)$ be the set of preference orderings over $\Omega$.

- A social welfare function takes the voter preferences and produces a social preference order:

$$\left(\bigotimes \Pi \times \cdots \times \bigotimes \Pi \right) \cup f$$

- We let $\succ^*$ denote the outcome of a social welfare function.

- Example: beauty contest.
Sometimes, we just want to select one of the possible candidates, rather than a social order.

Example: presidential election.

\[ \text{This gives social choice functions:} \]

\[ U \leftarrow (U \times \cdots \times U) : f \]

Social Choice Functions
Voting Procedures: Plurality

If we have only two candidates, then plurality is a simple majority election.

Example: Political elections in UK.

Winner is the one with largest number of points.

Each candidate gets one point for every preference order that ranks them first.

Each voter submits preferences.

Social choice function: selects a single outcome.

Social choice function: Plurality.
Chapter 12
An Introduction to Multiagent Systems

Anomalies with Plurality

With plurality, \( \omega_1 \) gets elected even though a clear majority (60%) prefers another candidate!

• 40% of voters voting for \( \omega_1 \)
• 30% of voters voting for \( \omega_2 \)
• 30% of voters voting for \( \omega_3 \)

Suppose

\[ |A \delta| = 100 \text{ and } n = 100 \text{ and } U = \{ \omega_1, \omega_2, \omega_3 \} \]

Anomalies with Plurality

http://www.csc.liv.ac.uk/~mjw/pubs/imas/
Suppose your preferences are \( \omega_1 \succ_i \omega_2 \succ_i \omega_3 \) while you believe 49% of voters have preferences \( \omega_2 \succ_i \omega_1 \succ_i \omega_3 \) and you believe 49% have preferences \( \omega_3 \succ_i \omega_1 \succ_i \omega_2 \). While you believe 49% of voters have preferences \( \omega_1 \succ_i \omega_2 \succ_i \omega_3 \), you may do better voting for \( \omega_2 \), even though this is not your true preference profile.

This is tactical voting: an example of strategic manipulation of the vote.

Strategic Manipulation by Tactical Voting
Condorcet’s Paradox

• Suppose $A = \{1, 2, 3\}$ and $\{1, 2\} \cup \{1, 3\} = \emptyset$.

• An Introduction to Multiagent Systems
Sequential Majority Elections

A variant of plurality, in which players play in a series of rounds: either a linear sequence or a tree (knockout tournament).

An Introduction to Multiagent Systems
Here, we pick an ordering of the outcomes – the agenda – which determines who plays against who.

For example, if the agenda is:

• \( \omega_2, \omega_3, \omega_4, \omega_1 \)

of this election goes in an election with \( \omega_1 \), and the winner
winner goes on to an election with \( \omega_4 \), and the winner
then the first election is between \( \omega_2 \) and \( \omega_3 \), and the

Linear Sequential Pairwise Elections
Suppose:

• 33 voters have preferences $\omega_1 \succ_i \omega_2 \succ_i \omega_3$

• 33 voters have preferences $\omega_3 \succ_i \omega_1 \succ_i \omega_2$

• 33 voters have preferences $\omega_2 \succ_i \omega_3 \succ_i \omega_1$

Then for every candidate, we can fix an agenda for that candidate to win in a sequential pairwise election!

Anomalies with Sequential Pairwise Elections
Majority Graphs

- This idea is easiest to illustrate by using a majority election.

A directed graph with:

- Vertices = candidates
- An edge (i,j) if i would beat j in a simple majority election.

A compact representation of voter preferences.

This idea is easiest to illustrate by using a majority.

Majority Graphs
Majority Graph for the Previous Example

ω₁ wins with agenda \((ω₁, ω₃, ω₂)\),
ω₂ wins with agenda \((ω₁, ω₂, ω₃)\),
ω₃ wins with agenda \((ω₃, ω₁, ω₂)\),
Another Majority Graph

Give agendas for each candidate to win with the following majority graph.
A Condorcet winner is a candidate that would beat every other candidate in a pairwise election. Here, \( \omega_3 \) is a Condorcet winner.
One reason plurality has so many anomalies is that it ignores most of a voter’s preference orders. It only looks at the top ranked candidate. The Borda count takes whole preference order into account.

Voting Procedures: Borda Count

http://www.csc.liv.ac.uk/~mjw/pubs/tmas/
For each candidate, we have a variable, counting the strength of opinion in favour of this candidate.

If \( \omega_i \) appears first in a preference order, then we increment the count for \( \omega_i \) by \( k - 1 \); we then increment the count for the next outcome in the preference order by \( k - 2 \), \( \ldots \), until the final candidate in the preference order has its total incremented by 0.

After we have done this for all voters, then the totals give the ranking.
Can we classify the properties we want of a "good" voting procedure?

Desirable Properties of Voting Procedures

- Independence of Irrelevant Alternatives (IIA)
- The Pareto property

Two key properties:
The Pareto Property

If everybody prefers \( \omega_i \) over \( \omega_j \), then \( \omega_i \) should be ranked over \( \omega_j \) in the social outcome.

http://www.csc.liv.ac.uk/~mjw/pubs/imas/
Independence of Irrelevant Alternatives (IIA)

Whether \( \omega_i \) is ranked above \( \omega_j \) in the social outcome should depend only on the relative orderings of \( \omega_i \) and \( \omega_j \) in voters' profiles.
This is a negative result: there are fundamental limits to democratic decision making! Simply selected by one of the voters, a dictatorship, in which the social outcome is in fact a dictatorship, is a voting procedure satisfying the Pareto condition and IIA. For elections with more than 2 candidates, the only

Arrow’s Theorem
We already saw that sometimes, voters can benefit by strategically misrepresenting their preferences, i.e., lying — tactical voting.

Are there any voting methods which are non-manipulable, in the sense that voters can never benefit from misrepresenting preferences?

Strategic Manipulation
The answer is given by the Gibbard-Satterthwaite Theorem.
Gibbard-Satterthwaite only tells us that manipulation is possible in principle. It does not give any indication of how to misrepresent preferences.

Bartholdi, Tovey, and Trick showed that there are elections that are prone to manipulation in principle, but where manipulation was computationally complex.

"Single Transferable Vote" is NP-hard to manipulate!

Computationally Complexity to the Rescue!

http://www.csc.liv.ac.uk/~mjw/pubs/imas/