CHAPTER 14: ALLOCATING SCARCE RESOURCES

Multiagent Systems

http://www.csc.liv.ac.uk/~mjw/pubs/tmas/
Overview

• Allocation of *scarce* resources amongst a *number of agents* is central to multiagent systems.

• Resource might be:
  – a physical object
  – the right to use land
  – computational resources (processor, memory, …)
If the resource isn’t scarce, there is no trouble allocating it.

If there is no competition for the resource, then there is no trouble allocating it.

If the resource isn’t scarce, there is no trouble allocating it.
In practice, this means we will be talking about auctions.

- Adword auctions
- eBay
- Profitable auction things that weren’t previously feasible

However, auctions have grown massively with the Frictionless commerce Web/Internet.

Theses used to be rare (and not so long ago).

Now feasible auction things that weren’t previously profitable.

- eBay
- Adword auctions
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An Introduction to Multiagent Systems

What is an auction?

Exchange is the free alteration of allocations of goods and money between traders.

Assume some initial allocation.

- Money, which is divisible.
- Units of an indivisible good, and

Concerned with traders and their allocations of:

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What is an auction?
Each trader has a value or limit price that they place on the good. A seller who exchanges a good for less than their limit price makes a loss. A buyer who exchanges more than their limit price for a good also makes a loss.

Limit Price
Limit prices clearly have an effect on the behavior of traders. There are several models, embodying different assumptions about the nature of the good. These are the models you will find most often adopted in the literature.

- Correlated Value
- Common Value
- Private Value

Three commonly used models:
Private value

- Good has an value to me that is independent of what it is worth to you.
- Textbook gives the example of John Lennon’s last dollar bill.
Common Value

- Winner’s curse
- Differring estimates of what it is:
  - The good has the same value to all of us, but we have
Our values are related.

The more you are prepared to pay, the more I should be prepared to pay.

Correlated Value
A market institution defines how the exchange takes place.

- Each trader has a non-zero component: each trader in the market.
- Difference between allocations is net trade: The change of allocation is market clearing.
- Messages: Defines how the final allocation depends on the exchange.
- Defines what messages can be exchanged: A market institution defines how the exchange takes place.
Traders with positive good component are buyers.

Traders with negative good component are sellers.

One-way traders are either buyers or sellers but not both.

The good component is the money component divided by the absolute value of the money component.
Yes, but what is an auction?

An *auction* is a market institution in which messages from traders include some price information—this information may be an offer to buy at a given price, in the case of a *bid*, or an offer to sell at a given price, in the case of an *ask*—and which gives priority to higher bids and lower asks.

This definition, as with all this terminology, comes from Dan Friedman.

http://www.csc.liv.ac.uk/~mjw/pubs/imas/
A good place to start. This particular classification is a bit zoological, but it is:

- It can help us to think about implementation.
- It suggests parameterization.
- It gives us a handle on all the kinds there might be.

This gives us a handle on all the kinds there might be.

A taxonomy.

Being good computer scientists, we draw up a categories.

We can split auctions into a number of different categories.

The zoology of auctions.
Single versus multi-dimensional auctions

- Single dimensiona auctions
  - The only content of an offer are the price and quantity of some specific type of good.
  - "I'll bid $200 for those two red chairs.

- Multi dimensional auctions
  - Offers can relate to many different aspects of many different goods.
  - "I'm prepared to pay $200 for those two red chairs, but $300 if you can deliver them tomorrow."

Multi dimensional auctions

Single versus multi-dimensional auctions
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Single versus double-sided markets.

- Single-sided markets with one seller and many buyers.
  - Either one buyer and many sellers, or one seller and many buyers.

- Two-sided markets:
  - Many buyers and many sellers.
  - The latter is the thing we normally think of as an auction.

Single-sided markets with one seller and many buyers are "sell-side" markets.

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Single-sided markets with one buyer and many sellers are “buy-side.”
Open-cry versus sealed-bid

- Open-cry
  - Traders announce their offers to all traders
  - Clearly as a bidder in an open-cry auction you have more information.

- Sealed-bid
  - Only the auctioneer sees the offers.

In some auction forms you pay for preferential access to information.
Single-unit versus multi-unit

• How many units of the same good are we allowed to bid for?
  
  • Single-unit
    – One at a time.
    – Might repeat if many units to be sold.
  
  • Multi-unit
    – Bid both price and quantity.
    – Might repeat if many units to be sold.
    – One at a time.

“Unit” refers to the indivisible unit that we are selling.

Single-fish versus box of fish.
Does the winner pay the highest price, the second highest price, the kth highest price?

First price versus kth price
Single item versus multi-item

- Not so much quantity as heterogeneity.
- Multi-item
  - Just the one indivisible thing that is being auctioned.
  - Bid for a bundle of goods.
  - "Two red chairs and an orange couch, or a purple beanbag."
- Single item
  - Just the one indivisible thing that is being auctioned.

Valuations for bundles are not linear combinations of the values of the constituents.
Also the so-called Japanese auction.

Also Vickrey auction

Also First-price sealed bid auction

Also Dutch auction

Also English auction

We will look at the four „standard“ auctions:

[Standard auction types]
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English auction

This is the kind of auction everyone knows.

- When there is no more bidding activity.
- At a fixed time (internet auctions); or

Auction ends:

- Acceptable price.

The seller may set a reserve price, the lowest

In some instances the auctioneer may call out prices.

Buyers call out bids, bids increase in price.

Typical example is sell-side.

This is the kind of auction everyone knows.
The "last man standing" pays their bid.
Around 95% of internet auctions are of this kind. A classic use is sale of antiques and artwork.

- Single item
- First-price
- Single unit
- Open-cry
- Single-sided
- Single-dimensional

Classified in the terms we used above:
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Unlikely tales

The former president of Parke-Bent reports that a dealer attending a
sale of eighteenth-century French furniture had arranged to unbutton
his overcoat whenever he wished to bid; buttoning the overcoat again
would signal that he had ceased bidding. The dealer, coat unbuttoned,
was in the midst of bidding for a Louis XVI sofa when he saw someone
outside to whom he wished to speak and suddenly left the room. The
auctioneer continued to bid for the dealer who, when he returned to the
room, found he had become the owner of the sofa at an unexpectedly
high price. An argument then followed as to whether an unbuttoned
coat not in the auction room is the same as an unbuttoned coat in the
auction room.
**Dutch auction**

- Also called a “descending clock” auction
  - Some auctions use a clock to display the prices.
- Starts at a high price, and the auctioneer calls out descending prices.
- One bidder claims the good by indicating the current price is acceptable.

Ties are broken by restarting the descent from a slightly higher price than the one occurred at. The winner pays the price at which they "stop the clock".

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Classified in the terms we used above:

- Tobacco in Canada.
- Fish in Spain and Israel.
- Flowers in the Netherlands (eg. Aalsmeer).

Often used to sell perishable goods:

- High volume (since auction proceeds swiftly).
- Single-unit; First-price; Single item.
- Single-dimensional; Single-sided; Open-cry; Single.

http://www.csc.liv.ac.uk/~mjw/pubs/imas/
An Introduction to Multiagent Systems
April 23rd 2008 (page 18–19)

The Guardian states that the Aalsmeer auction trades 1.9 million flowers and 2 million plants... every day.

The Guardian
First-price sealed bid auction

• In an English auction, you get information about how much a good is worth.
• In a sealed bid auction, none of that happens.
• Other people's bids tell you things about the market.
• As its name suggests, the winner pays that highest price (which is what they bid).
• In the FPSB auction the highest bid wins as always.
• At most you know the winning price after the auction.

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• Property can also be sold this way (as in Scotland):
  - US recently changed to SPSB.
  - UK still does.
  - Treasury bonds.

Governments often use this mechanism to sell:
  - First-price
  - Single unit
  - Single-bid
  - Single-sided
  - Single-dimensional

Classified in the terms we used above:
English auction.
- Dutch auction starts from twice the final price of the English auction.
- When down to the final two bidders, start a Dutch auction.
- Start with an English auction.

Since medieval time, property in the low countries has traditionally been sold using the "Amsterdam" auction.

The Amsterdam auction
The Vickrey auction is a sealed bid auction.

- It is in the bidders’ interest to bid their true value.
- This sounds odd, but it is actually a very smart design.
- The winner pays the amount of the second highest bid.
- The winning bid is the highest bid, but the winning bidder pays the second highest bid.
- However, it is not a panacea, as the New Zealand government found out.

- Incentive compatible in the usual terminology.

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• Again, classified as above, it is:

- Second-price
- Single-unit
- Sealed-bid
- Single-sided
- Single-dimensional
Why does the Vickrey auction work?

- Suppose you bid more than your valuation.
- You may win the good.
- If you do, you may end up paying more than you think the good is worth.
- Not so smart.
Suppose you bid less than your valuation.

- Not so smart.
- Paying the same.
- However, even if you do win it, you will end up
  paying less.
- You stand less chance of winning the good.

- You stand less chance of winning the good.
there is no point in bidding above or below your valuation.

So: there is no point in bidding above or below your valuation.

Of course, this really assumes there are a large number of bidders (see the New Zealand case).
The auction form used to sell fish in Tokyo is different:

Japanese fish auction

This is thus simultaneous bidding and rather like an FPSB auction.
Ties are "not uncommon[ly] broken by playing Jan Ken Pon (or 'paper, rock, scissors').
Combinatorial Auctions

Auctions for bundles of goods.

For the 1.7 to 1.72 GHz band for Brooklyn to be useful, you need a license for Manhattan, Queens, Staten Island.

Most valuable are the licenses for the same bandwidth.

But a different bandwidth license is more valuable than no license.

A good example of bundles of goods are spectrum licenses.
The FCC spectrum auctions, however, did not use a combinatorial auction mechanism.
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• Let \( Z = \{z_1, \ldots, z_m\} \) be a set of items to be auctioned.

• We gave the usual set of agents \( \mathcal{A} \) and we capture preferences of agent \( i \) with the valuation function:

\[ v_i : 2^Z \rightarrow \mathbb{R} \]

meaning that for every possible bundle of goods \( Z \subseteq Z \),

\[ (Z)_i, \] says how much \( Z \) is worth to \( i \).

• If \( v_i(\emptyset) = 0 \) then we say that the valuation function for \( i \) is normalised.

• Let \( \mathcal{Z} \) be a set of items to be auctioned.
Another useful idea is free disposal:

\[ Z \subseteq Z \]

In other words, an agent is never worse off having more stuff.
• We already mentioned the idea of allocation.

• Formally, allocation is a list of sets, \( Z_1, \ldots, Z_n \), one for each agent \( A^8 \). With the stipulation that:

\[
\forall i \in \mathbb{A}, \forall Z \subseteq \bigcup_{i \neq i} Z_i \quad \text{and for all } i, j \in A^8 \text{ such that } i \neq j, \text{ we have } Z_i \cap Z_j = \emptyset.
\]

• Thus, no good is allocated to more than one agent.

\( \mathcal{A}_{\mathcal{Z}, A^8} \)

• The set of all allocations of \( Z \) to agents \( A^8 \) is:

\[
\mathcal{L}_{\mathcal{Z}, A^8} := \{ \mathcal{Z} \mid \mathcal{Z} \subseteq \bigcup_{i \neq i} Z_i \}
\]

• We already mentioned the idea of allocation.
If we design the auction, we get to say how the allocation is determined.

One natural way is to maximize social welfare:

Define a social welfare function:

- Sum of the utilities of all the agents.

$\sum_{i=1}^{n} u(Z_i) = (u_1, \ldots, u_n, Z_1, \ldots, Z_n)$
Given this, we can define a combinatorial auction.

Given a set of goods $Z$ and a collection of valuation functions $v_1, \ldots, v_n$, the goal is to find an allocation $Z^*$ that maximizes $SW$, in other words:

$$u_{Z^*} = \max_{(Z_1, \ldots, Z_n) \in \text{alloc}(Z, Ag)} SW(Z_1, \ldots, Z_n).$$

Figuiring this out is winner determination.
How do we do this?

Well, we could get every agent to declare their valuation $\hat{v}_i$.

- The agent may lie!
- The hat denotes that this is what the agent says, not what it necessarily is.

Then we just look at all the possible allocations and figure out what the best one is.
One problem here is that valuations are exponential:

\[ v : \mathbb{Z}^+ \rightarrow \mathbb{R} \]

Searching through them is computationally intractable.

- A naive representation is impractical.
- In a bandwidth auction with 1122 licenses we would have to specify \( 2^{1122} \) values for each bidder.
Rather than exhaustive evaluations, allow bidders to construct valuations from the bits they want to mention.

A bundle \( Z' \) satisfies a bid \( (d, Z) \) if \( Z \subseteq Z' \).

Atomic bids \( (d, Z) \) where \( Z \subseteq Z' \).

In other words a bundle satisfies a bid if it contains at least the things in the bid.

Bidding Languages
Atomic bids definevaluations.

\[
\{ p | (Z', p) \text{ satisfies } (Z, p) \}
\]

Atomic bids alone don't allow us to construct very

interestingvaluations.

\[
(d, Z) = (Z')_{\forall}
\]

Atomic bids definevaluations.
To construct more complex valuations, atomic bids can be combined into more complex bids.

One approach is XOR bids: XOR because we will pay for at most one.

\((\{p,c\}, 3) \text{ XOR } (\{q,v\}, 5) = \{b\}\)

For a bundle that contains \(a, c\) and \(p\), but not \(a\) and \(q\), and I will pay 5 contains \(c\) and \(p\). I will pay 5 for a bundle that contains \(a\) and \(q\) but not \(c\) and \(p\), and I will pay 5 for a bundle that contains \(a\) and \(p\) and I would pay 3 for a bundle that contains \(a\) and \(q\) and \(c\).

From this we can construct a valuation, reading the bid to mean:

- XOR because we will pay for at most one.
- From this bid, we can construct a valuation.
Thus:

\[ \begin{align*}
\varphi &= (\{p', c, q, p\})^{Ig/\lambda} \\
\varphi &= (\{p', c\})^{Ig/\lambda} \\
\varphi &= (\{q, p\})^{Ig/\lambda} \\
0 &= (\{q\})^{Ig/\lambda} \\
0 &= (\{p\})^{Ig/\lambda}
\end{align*} \]
More formally, a bid like this defines a valuation $\vartheta$ like so:

$$(\vartheta^d, \vartheta^Z) \text{ XOR } \cdots \text{ XOR } (\vartheta^1, \vartheta^Z) = \vartheta$$

if $d$ doesn't satisfy any

$$\{ i \mid \vartheta^i \text{ satisfies } Z \} = \{ i \mid \vartheta^i \}$$

otherwise $\max \{ i \mid \vartheta^i \} = 0$$
In polynomial time.

However, the valuation of a bundle can be computed in polynomial time.

To do that, we may need an exponentially large number of atomic bids.

XOR bids are fully expressive, that is they can express any valuation function over a set of goods.
Winner Determination

• The basic problem is intractable.
  Common approach: code the problem as an integer linear program and use a standard solver — often works in practice.
  • Look for approximation algorithms.
  • Use heuristics, or
    • Can also forget optimality and either:
      • many cases.
      • But this is a worst case result, so it may be possible to develop approaches that are optimal and run well in

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The VCG Mechanism

- In general, we don’t know whether the $v_i$ are true.

- However, we would be easier if they were.

- Well, can we make them true valuations?

- Yes, in a generalization of the Vickrey auction.

- Mechanism is incentive compatible: telling the truth is a dominant strategy.

- Yes!

- Mechanism is incentive compatible: telling the truth is a dominant strategy.

- Vickrey/Clarke/Groves Mechanism.
And we can then define the VCG mechanism.

\[
\text{if } A \neq \emptyset \text{ for } i \neq j \\
\sum_{A \neq \emptyset \text{ for } i \neq j} (f(Z) - f(i) = (u_\Lambda, \ldots, u_{\Lambda_i}, u_{Z_i} - v_i) - sw_{-i}
\]

Is the social welfare function without i:

For all Z:

\[
0 = (Z) - f(A)
\]

Individual valuation function:

Need some more notation.
1. Every agent simultaneously declares a valuation $v_i$. 
The mechanism computes:

\[ Z^* = \arg\max_{(Z_1, \ldots, Z_n) \in \text{alloc}(Z, A)} \sum_{i=1}^{n} u_i(Z_i, \hat{v}_i) \]

and the allocation \( Z^* \) is chosen.
The mechanism also computes, for each agent $i$:
Every agent $i$ pays $p_i$, where:

$$\frac{(u_1^*, \ldots, u_n^*, z_1^*, \ldots, z_n^*) - MS}{(u_1^0, \ldots, u_n^0, z_1^0, \ldots, z_n^0) - MS} = d$$

4. Every agent $i$ pays $p_i$.
On other words, each agent pays out the cost, to other agents, of it having participated in the auction. On other words, each agent pays out the cost, to other agents, of it having participated in the auction.

Reducing your chance to win.

If you shade your bid, you reduce your chance to win.

As the Vickrey auction was before.

It is incentive compatible for exactly the same reason.

If you bid more than your valuation and win, well you end up paying back what the good is worth to everyone else, which is more then it is worth to you.

If you bid more than your valuation and win, well you end up paying back what the good is worth to everyone else, which is more then it is worth to you.

But even if you win you are still paying what everyone else thinks it is worth so you don’t save money by else thinks it is worth so you don’t save money by

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If you bid more than your valuation and win, well you end up paying back what the good is worth to everyone else, which is more then it is worth to you.

If you shade your bid, you reduce your chance to win.
So we get a dominant strategy for each agent that guarantees to maximise social welfare.
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Ebay

http://www.csc.liv.ac.uk/~mjw/pubs/imas/
- Activity rule
  not to use it.

- BTW, there is an easy fix to sniping, but eBay chose
  Many companies offer sniping services.

- Reduces the auction to a FPSB.
  To counter this, eBay offers an automated bidding agent.

- Vulnerable to sniping.
  eBay runs a variation of the English auction.

- eBay runs a variation of the English auction.
Adword auctions

http://www.csc.liv.ac.uk/~mjw/pubs/imas/
To decide which ads get shown in which position for which searches, an adword auction is run. This auction is a variation on the Vickrey auction. Though clearly bids are placed beforehand, this is run in real time.

- 85% of Google’s revenue ($4.1 billion) in 2005 came from these auctions.

- Auction is a variation on the Vickrey auction.

- Not clear what the best auction mechanism is for this application.

- Not clear what the best way to bid is.

- Very active area of research.

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Summary

- Allocating scarce resources comes down to auctions.
- We looked at a range of different simple auction mechanisms.
- The web looked at the popular field of combinatorial auctions.
  - Vickrey auction
  - First price sealed bid
  - Dutch auction
  - English auction
- Allocating scarce resources comes down to auctions.
We discussed some of the problems in implementing multiagent interaction. A rare ray of sunshine on the problems of combinatorial auctions: the Vickrey/Clarke/Groves mechanism, a rare ray of sunshine on the problems of multiagent interaction.

And we talked about the Vickrey/Clarke/Groves mechanism, a rare ray of sunshine on the problems of multiagent interaction.