Game Semantics and Block-Structured State

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What this talk is about

Evaluation strategies vs scoping.

- Call-by-value and mobility: RML [AM98], Reduced ML [MT09], etc.
- Call-by-name and blocks: Idealized Algol [AM97].

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Call-by-value and (base-type) blocks?

- There is a gap.
- Name-mobility has been be described as a semantical intricacy of ML-like languages (e.g. [PS98]).

Simply-typed λ-calculus with integers and references.

- Types:

$$\theta ::= \ \mathsf{unit} \mid \mathsf{int} \mid \mathsf{int\,ref} \mid \theta \to \theta$$

- Terms:

$$\begin{aligned} M ::= \ & x \mid \lambda x.M \mid M\,M \mid () \mid \Omega \\ & \mid n \mid M \odot M \mid \mathsf{if}\, M\, \mathsf{then}\, M\, \mathsf{else}\, M \\ & \mid a \mid \mathsf{ref}\, M \mid\ !\,M \mid M := M \end{aligned}$$

$$V ::= \ n \mid () \mid a \mid x \mid \lambda x.M$$

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- Moves carry carefully selective stores: to be included in the store a name has to be available to both participants.

- Names adhere to freshness conditions: an X-name which is fresh in the view of the other participant must be fresh.

These plays give full abstraction: see paper for details!

Go simpler: RML

$$\begin{aligned} \text{RML} &= \text{Reduced ML} \\ &- \ a \\ &+ \ \text{mkvar} \end{aligned}$$

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- No names
- $\mathsf{new}\ x\ \mathsf{in}\ M \triangleq (\lambda x.\, M)(\mathsf{ref}\, 0)$

Games for RML [AM98]

- Games for PCF$_v$ [HY99], without Innocence
- $[\![\mathsf{int\,ref}]\!] = ([\![\mathsf{unit}]\!] \Rightarrow [\![\mathsf{int}]\!]) \times ([\![\mathsf{int}]\!] \Rightarrow [\![\mathsf{unit}]\!])$
- $\mathsf{cell} : [\![\mathsf{unit}]\!] \longrightarrow [\![\mathsf{int\,ref}]\!]$

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$$(\ * \quad , \quad * \ )_{PA}$$

$$\mathsf{rd}_{OQ} \qquad \mathsf{w}(n)_{OQ}$$

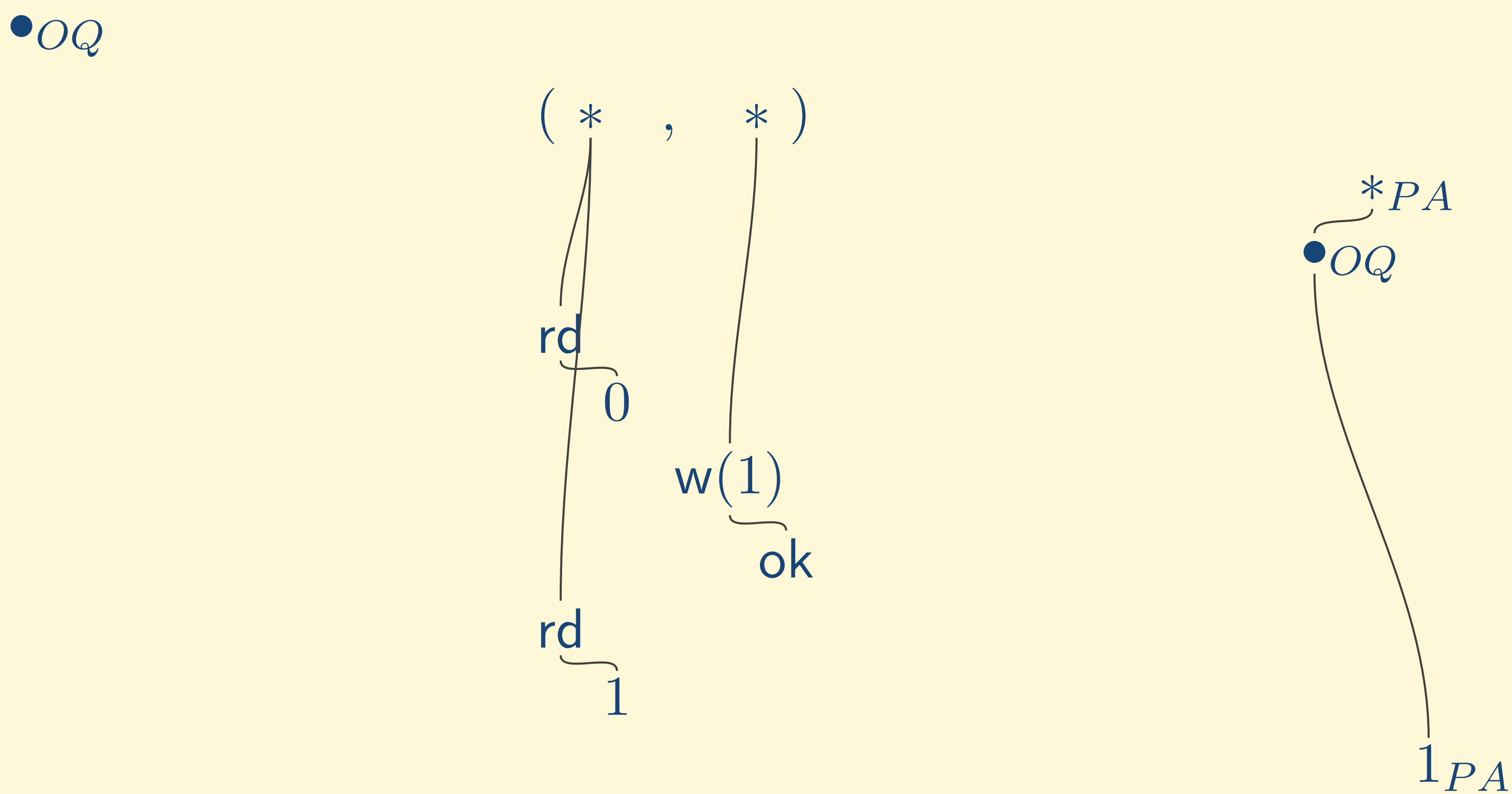
$$m_{PA} \qquad \mathsf{ok}_{PA}$$

- $\mathsf{cell} : [\![\mathsf{unit}]\!] \longrightarrow [\![\mathsf{int\,ref}]\!]$

Example

Taking $M \triangleq \lambda y.\,(x := {!}x{+}1\,;{!}x)$, $[\![\text{new}\ x\ \text{in}\ M : \text{unit} \to \text{int}]\!] =$

$$[\![\text{unit}]\!] \xrightarrow{\text{cell}} ([\![\text{unit}]\!] \Rightarrow [\![\text{int}]\!]) \times ([\![\text{int}]\!] \Rightarrow [\![\text{unit}]\!]) \xrightarrow{[\![M]\!]} [\![\text{unit}]\!] \Rightarrow [\![\text{int}]\!]$$



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$\bullet_{OQ}$

$(\ *\ ,\ *\ )$

$*_{PA}$

$\bullet_{OQ}$

$\vdots$

1_{PA}

$\bullet_{OQ}$

$\vdots$

2_{PA}

$\vdots \qquad \vdots$

- Non-innocence in this case crucially depends on $\mathsf{new}\,\_\,\mathsf{in}\,\_ : \mathsf{unit} \Rightarrow \mathsf{int}$

- What if we use blocks? Take $\beta = \{\mathsf{unit}, \mathsf{int}\}$ and:

$$\begin{aligned} \mathsf{bRML} = \mathsf{RML} & \\ & - \ \mathsf{ref} \\ & + \ \mathsf{new}\,\_\,\mathsf{in}\,\_ : \mathsf{int}\,\mathsf{ref} \to \beta \to \beta \end{aligned}$$

RML with blocks: bRML

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Explicitly, bRML terms are:

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Observations

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$$
\begin{tikzcd}
\mathrm{PCF}_v \arrow[d, mapsto] & \text{bRML} \arrow[d, mapsto] & \text{RML} \arrow[d, mapsto] \\
\text{Innocence} & ?? & \text{Knowledge}
\end{tikzcd}
$$

bRML strategies exhibit a particular kind of uniformity:

$$A \longrightarrow B_1 \to B_2 \to \cdots \to B_n \to \beta$$

$\bullet_{OQ}$

$\boxed{\vdots}$

$*_{PA}$

Binnocence

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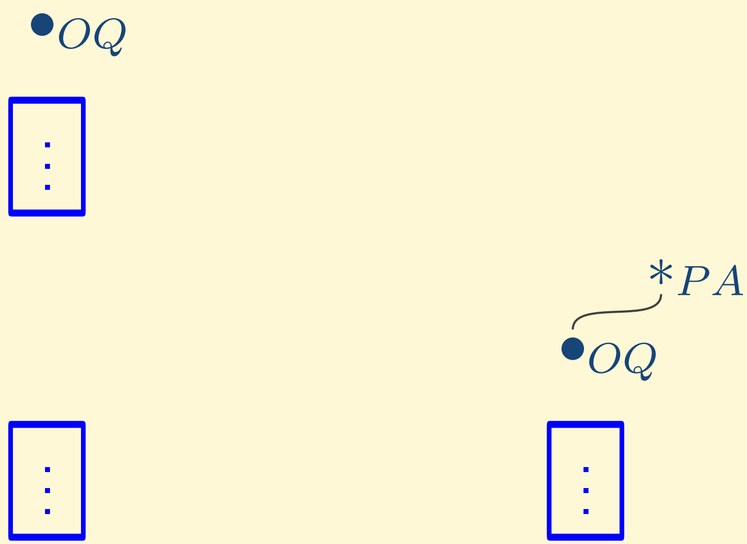

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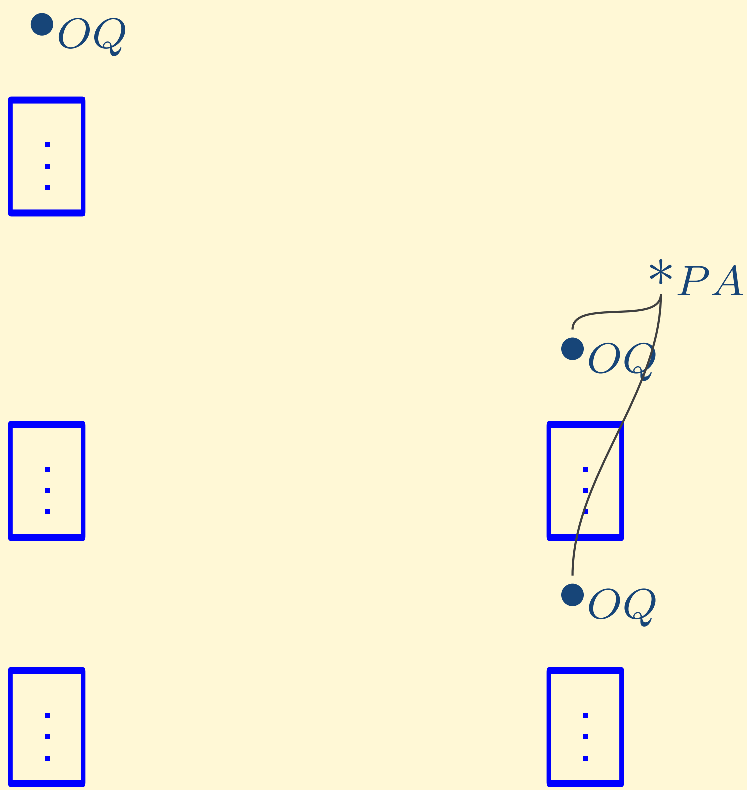

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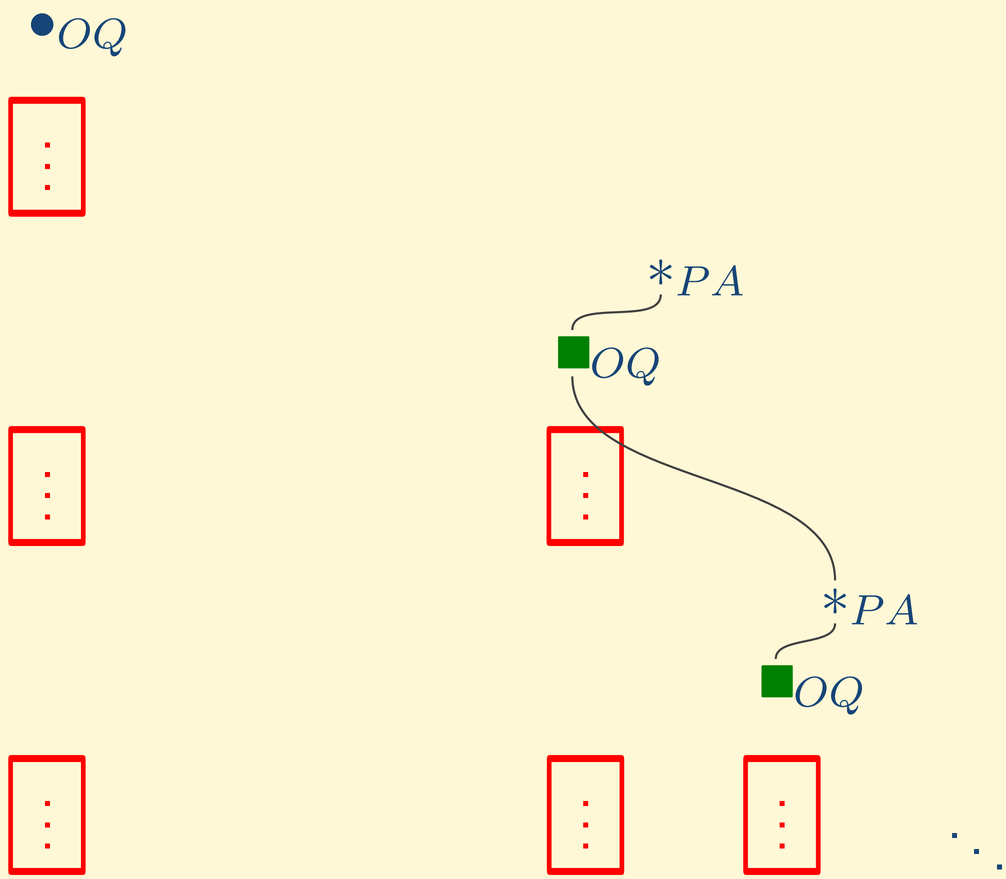

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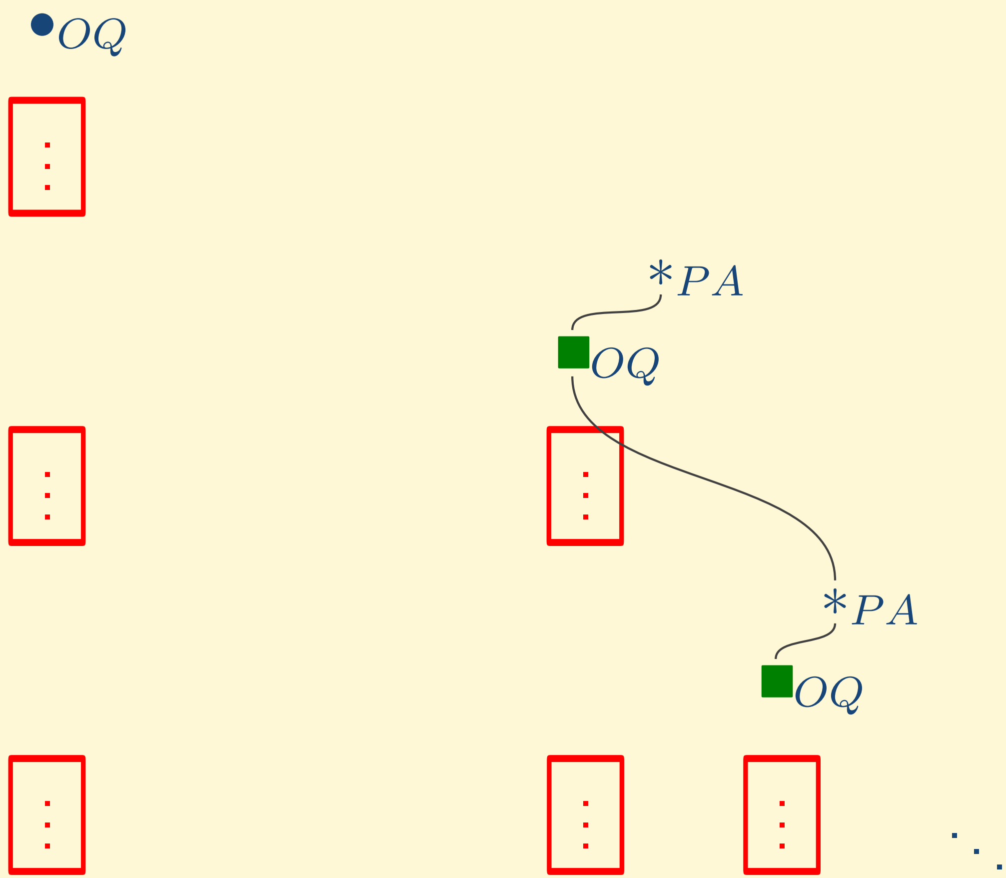
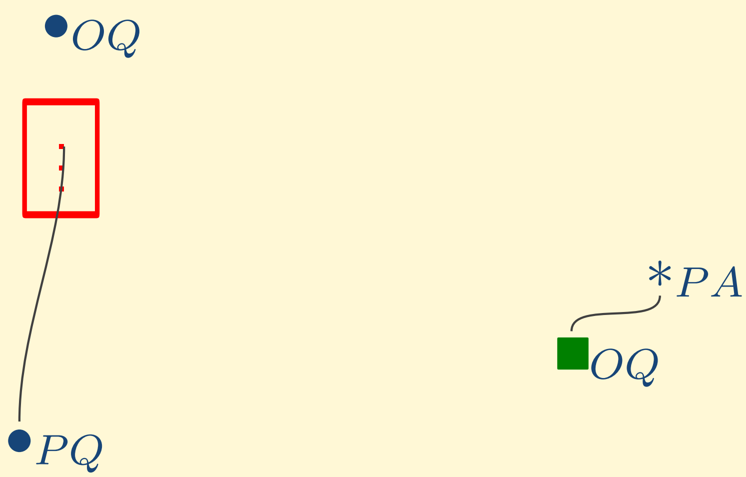


Binnocence can be described as a "recursive version" of 3rd-move-binnocence (*thread-independence*).

A problem

$$A \longrightarrow B_1 \to B_2 \to \cdots \to B_n \to \beta$$



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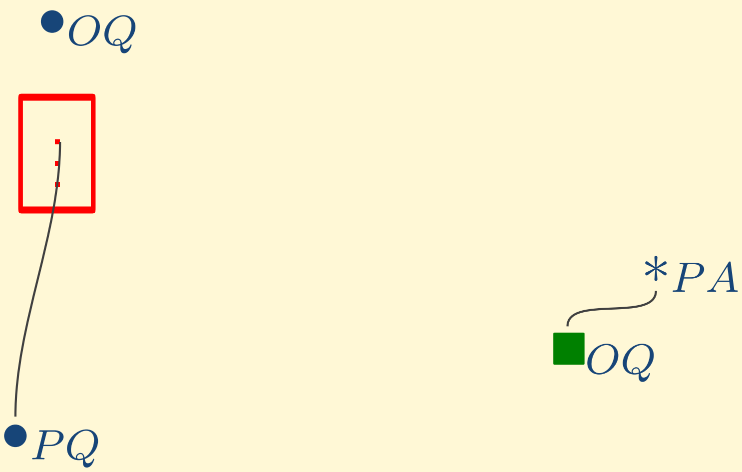

- If the play on the left is non-uniform then P cannot play $\bullet_{PQ}$

A problem

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$\bullet_{OQ}$

$\boxed{\vdots}$

$*_{PA}$

$\blacksquare_{OQ}$

$\bullet_{PQ}$

- If the play on the left is non-uniform then P cannot play $\bullet_{PQ}$
- In fact, P cannot play $\bullet_{PQ}$ if it is justified by a move in an open block

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- Annotations give us a means to express blocks, but binnocence has become very complicated.
- Use stores as annotations, and go innocent.