Full abstraction for nominal exceptions and general references

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Summary

Semantics of nominal computation.

- What is it?
- The $\nu\varepsilon\rho$-calculus.
- Nominal games.

Nominal computation

Names in computation

Names: Identifiers used to distinguish different entities inside a computation.

Examples: references, objects, channels, exceptions, etc.

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We are interested in names which can be:

- created locally and dynamically,
- compared for equality,
- passed around.

\+ Other specifications (e.g. *raised* and *handled*).

Names are constants

Describe a functional language with names for general references.

Names are variables	*Names are constants*
$\Gamma, x : \texttt{ref}(A) \vdash x : \texttt{ref}(A)$ • $!\,x, x := V$ • $\nu x, [x = y], x$	
• should names vary? • what is $[\![\texttt{ref}(A)]\!]$? – *bad variables*	

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(ν-calculus [PS93])

Names are atoms (in Nominal Sets [GP99])

– *But what kind of constants?*

Names are atoms (in Nominal Sets [GP99])

Consider a collection of distinct sets of *atoms* and its group of *finite componentwise permutations*:

$$(\mathbb{A}_i)_{i\in I}\,, \quad \mathbb{A} \triangleq \bigcup_{i\in I} \mathbb{A}_i\,, \quad \text{and} \quad \mathsf{PERM}(\mathbb{A})\,,$$

$$\pi \in \mathsf{PERM}(\mathbb{A}) \iff \exists \pi_i \in \mathsf{PERM}(\mathbb{A}_{j_i}).\, \pi = \pi_1 \circ \cdots \circ \pi_n$$

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A ***nominal set*** is a set X with an *action* from $\mathsf{PERM}(\mathbb{A})$:

$$\pi\circ(\pi'\circ x)=(\pi\circ\pi')\circ x\,,\quad \mathtt{id}\circ x=x,$$

s.t. each $x\in X$ has *finite support* : $\mathsf{S}(x)\subset_{\text{fin}}\mathbb{A}$,

$$\forall\pi.\,(\forall a\in\mathsf{S}(x).\,\pi(a)=a)\implies\pi\circ x=x\,.$$

- $\mathbb{A}$ is a nominal set, with $\pi\circ a=\pi(a)$,
- $\mathbb{A}^{\#}$ is a nominal set, with $\pi\circ(a_1...a_n)=\pi(a_1)...\pi(a_n)$.

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$$\forall \pi.\, (\forall a \in \mathsf{S}(x).\, \pi(a) = a) \implies \pi \circ x = x\,.$$

We found our constructions on nominal sets,
and take atoms to stand for names.

The $\nu\varepsilon\rho$-calculus

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A λ-calculus with *nominal exceptions* and *nominal references*.

Types: $A, B ::= \mathbb{1} \mid \mathbb{N} \mid A \times B \mid A \to B \mid \mathbb{E} \mid [A]\,.$

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Terms are *nominal*:

Names for exceptions: $\dot{a}, \dot{b}, \dots \in \mathbb{A}_{\mathbb{E}} \in (\mathbb{A}_i)_{i \in \omega}$

Names for references: $\ddot{a}, \ddot{b}, \dots \in \mathbb{A}_{[A]} \in (\mathbb{A}_i)_{i \in \omega}$

Names generally: $a, b, \dots$

Typing:

$\vec{a} \mid \Gamma \vdash M : A$

$\rightsquigarrow$ free vars in Γ

$\rightsquigarrow$ free names in $\vec{a}$

Typing rules

λ-calculus $+$ products $+$ `if0 _ then _ else _` $+$

$$\frac{}{\vec{a} \mid \Gamma \vdash \dot{a} : \mathbb{E}}\ {\scriptstyle \dot{a} \in \vec{a} \wedge \dot{a} \in \mathbb{A}_{\mathbb{E}}} \qquad \frac{}{\vec{a} \mid \Gamma \vdash \ddot{a} : [A]}\ {\scriptstyle \ddot{a} \in \vec{a} \wedge \ddot{a} \in \mathbb{A}_{[A]}}$$

$$\frac{\vec{a}a \mid \Gamma \vdash M : B}{\vec{a} \mid \Gamma \vdash \nu a.M : B} \qquad \frac{\vec{a} \mid \Gamma \vdash M : A_\nu \quad \vec{a} \mid \Gamma \vdash N : A_\nu}{\vec{a} \mid \Gamma \vdash [M = N] : \mathbb{N}}\ A_\nu \in \{\mathbb{E}, [A]\}$$

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$$\frac{\vec{a} \mid \Gamma \vdash M : \mathbb{E}}{\vec{a} \mid \Gamma \vdash \texttt{raise}\ M : A} \qquad \frac{\vec{a} \mid \Gamma \vdash M : \mathbb{E} \qquad \vec{a} \mid \Gamma \vdash N_1, N_2 : A}{\vec{a} \mid \Gamma \vdash \texttt{try}\ N_1\ \texttt{handle}\ M \Rightarrow N_2 : A}$$

$$\frac{\vec{a} \mid \Gamma \vdash M : [A]}{\vec{a} \mid \Gamma \vdash\ !M : A} \qquad \frac{\vec{a} \mid \Gamma \vdash M : [A] \qquad \vec{a} \mid \Gamma \vdash N : A}{\vec{a} \mid \Gamma \vdash M := N : \mathbb{1}}$$

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$$\frac{}{\vec{a}\mid\Gamma\vdash\dot{a}:\mathbb{E}}\ {\scriptstyle \dot{a}\in\vec{a}\ \wedge\ \dot{a}\in\mathbb{A}_{\mathbb{E}}}$$

$$\frac{}{\vec{a}\mid\Gamma\vdash\ddot{a}:[A]}\ {\scriptstyle \ddot{a}\in\vec{a}\ \wedge\ \ddot{a}\in\mathbb{A}_{[A]}}$$

$$\frac{\vec{a}a\mid\Gamma\vdash M:B}{\vec{a}\mid\Gamma\vdash\nu a.M:B}$$

$$\frac{\vec{a}\mid\Gamma\vdash M:A_\nu\quad\vec{a}\mid\Gamma\vdash N:A_\nu}{\vec{a}\mid\Gamma\vdash[M=N]:\mathbb{N}}\ A_\nu\in\{\mathbb{E},[A]\}$$

$$\frac{\vec{a}\mid\Gamma\vdash M:\mathbb{E}}{\vec{a}\mid\Gamma\vdash\mathtt{raise}\ M:A}$$

$$\frac{\vec{a}\mid\Gamma\vdash M:\mathbb{E}\qquad\vec{a}\mid\Gamma\vdash N_1,N_2:A}{\vec{a}\mid\Gamma\vdash\mathtt{try}\ N_1\ \mathtt{handle}\ M\Rightarrow N_2:A}$$

$$\frac{\vec{a}\mid\Gamma\vdash M:[A]}{\vec{a}\mid\Gamma\vdash\,!M:A}$$

$$\frac{\vec{a}\mid\Gamma\vdash M:[A]\qquad\vec{a}\mid\Gamma\vdash N:A}{\vec{a}\mid\Gamma\vdash M:=N:\mathbb{1}}$$

Terms form a nominal set

Operational semantics

Values: $V ::= a \mid x \mid \lambda x.M \mid \langle V, V' \rangle \mid \texttt{skip} \mid \hat{n}$

Mixed environments: $P ::= \epsilon \mid a, P \mid \ddot{a} :: V, P \quad (a, \ddot{a} \notin \texttt{dom}(P))$

- Call-by-value β-rules

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Mixed environments: $P ::= \epsilon \mid a, P \mid \ddot{a} :: V, P \quad (a, \ddot{a} \notin \texttt{dom}(P))$

- Call-by-value β-rules
- $P \models \nu a.M \longrightarrow P, a \models M$
- $P \models [a = a] \longrightarrow P \models \hat{0}$
- $P \models [a = b] \longrightarrow P \models \hat{1}$
- update and dereferencing rules

... exceptions

- $P \models \texttt{try}\ V\ \texttt{handle}\ \dot{a} \Rightarrow N \longrightarrow P \models V$
- $P \models \texttt{try}\ (\texttt{raise}\ \dot{a})\ \texttt{handle}\ \dot{a} \Rightarrow N \longrightarrow P \models N$
- $P \models \texttt{try}\ (\texttt{raise}\ \dot{b})\ \texttt{handle}\ \dot{a} \Rightarrow N \longrightarrow P \models \texttt{raise}\ \dot{b}$
- $P \models \mathrm{Z}[\texttt{raise}\ \dot{a}] \longrightarrow P \models \texttt{raise}\ \dot{a}$

... exceptions

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- $P \models \texttt{try}\ (\texttt{raise}\ \dot{b})\ \texttt{handle}\ \dot{a} \Rightarrow N \longrightarrow P \models \texttt{raise}\ \dot{b}$

- $P \models \mathrm{Z}[\texttt{raise}\ \dot{a}] \longrightarrow P \models \texttt{raise}\ \dot{a}$

- $$\frac{P \models M \longrightarrow P' \models M'}{P \models \mathrm{E}[M] \longrightarrow P' \models \mathrm{E}[M']}$$

$$\mathrm{E} ::= \mathrm{Z} \mid \texttt{try}\ \_\ \texttt{handle}\ \dot{a} \Rightarrow N$$

... exceptions

- $P \models \texttt{try}\ V\ \texttt{handle}\ \dot{a} \Rightarrow N \longrightarrow P \models V$
- $P \models \texttt{try}\ (\texttt{raise}\ \dot{a})\ \texttt{handle}\ \dot{a} \Rightarrow N \longrightarrow P \models N$
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- $\dfrac{P \models M \longrightarrow P' \models M'}{P \models \mathrm{E}[M] \longrightarrow P' \models \mathrm{E}[M']}$

$\mathrm{E} ::= \mathrm{Z} \mid \texttt{try}\ \_\ \texttt{handle}\ \dot{a} \Rightarrow N$

Observational approximation

$M \lessapprox N$ iff, for any variable- and name-closing context $\mathrm{C} : \mathbb{N}$,

$$\exists P'.(\models \mathrm{C}[M] \twoheadrightarrow P' \models \hat{0}) \implies \exists P''.(\models \mathrm{C}[N] \twoheadrightarrow P'' \models \hat{0})$$

Examples

$$\lambda f.\, 0 \;\ncong_{(A_\nu \to \mathbb{N}) \to \mathbb{N}}\; \nu a.\nu b.\lambda f.\, [fa \Leftrightarrow fb]$$

$$\nu a.\lambda f.\, [fa \Leftrightarrow fa] \;\ncong_{(A_\nu \to \mathbb{N}) \to \mathbb{N}}\; \nu a.\nu b.\lambda f.\, [fa \Leftrightarrow fb]$$

$$\lambda f.\, \texttt{stop} \;\ncong_{(\mathbb{1} \to \mathbb{1}) \to \mathbb{1}}\; \lambda f.\, f\, \texttt{skip}\, ;\, \texttt{stop}$$

Categorical semantics

We describe an abstract categorical model for the $\nu\varepsilon\rho$-calculus. We use:

- a computational *monad* T,
- a family of local-state *comonads* $Q = (Q^{\vec{a}})_{\vec{a}\in\mathbb{A}^\#}$,

and obtain:

$$[\![\vec{a} \mid \Gamma \vdash M : A]\!] : Q^{\vec{a}}[\![\Gamma]\!] \longrightarrow T[\![A]\!]$$

Specifications

Cartesian $\mathcal{M}$, monad (T, η, μ), comonads $(Q^{\vec{a}}, \varepsilon, \delta)_{\vec{a} \in \mathbb{A}^\#}$. Also:

Strong monad, with exponentials

- $\tau : A \times TB \longrightarrow T(A \times B)$, $\quad \mathsf{ev}^T : A \times TB^A \longrightarrow TB$.

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Product comonad

- $\zeta : A \times Q^{\vec{a}} B \xrightarrow{\cong} Q^{\vec{a}}(A \times B)$ $\quad$ (so $Q^{\vec{a}} A \cong Q^{\vec{a}} 1 \times A$),

$$Q^{\epsilon} 1 = 1 \,, \qquad Q^{\vec{a}} = Q^{\pi \circ \vec{a}} \,.$$

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Name-equality

- $\mathsf{eq} : Q^a 1 \times Q^a 1 \longrightarrow \mathbb{N}$,

$$\begin{array}{ccccc}
Q^a 1 & \xrightarrow{\ \Delta\ } & Q^a 1 \times Q^a 1 & \xleftarrow{\ \langle \frac{ab}{a}, \frac{ab}{b} \rangle\ } & Q^{ab} 1 \\
{\scriptstyle !}\downarrow & & \downarrow{\scriptstyle \mathsf{eq}} & & \downarrow{\scriptstyle !} \\
1 & \xrightarrow{\ \tilde{0}\ } & \mathbb{N} & \xleftarrow{\ \tilde{1}\ } & 1
\end{array}$$

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Name-discard

- $\frac{\vec{a}}{\vec{a}'} : Q^{\vec{a}} \longrightarrow Q^{\vec{a}'}$, whenever $\vec{a}'\subseteq\vec{a}$,

$$\frac{\vec{a}}{\epsilon}=\varepsilon\,,\qquad\qquad \frac{\vec{a}}{\vec{a}''};\frac{\vec{a}''}{\vec{a}'}=\frac{\vec{a}}{\vec{a}'}\,.$$

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Name-creation

- $\mathtt{nu}^{\vec{a}a} : Q^{\vec{a}} \longrightarrow TQ^{\vec{a}a}$,

$$\begin{array}{ccc} Q^{\vec{a}}A & \xrightarrow{\langle \mathtt{id},\mathtt{nu}\rangle} & Q^{\vec{a}}A\times TQ^{\vec{a}a}A \\ \downarrow{\scriptstyle \mathtt{nu}} & & \downarrow{\scriptstyle \tau} \\ TQ^{\vec{a}a}A & \xrightarrow{T\langle\frac{\vec{a}a}{\vec{a}},\mathtt{id}\rangle} & T(Q^{\vec{a}}A\times Q^{\vec{a}a}A) \end{array} \qquad \begin{array}{ccc} Q^{\vec{a}'}A & \xrightarrow{\mathtt{nu}^{\vec{a}'a}} & TQ^{\vec{a}'a}A \\ \downarrow{\scriptstyle \frac{\vec{a}'}{\vec{a}}} & & \downarrow{\scriptstyle T\frac{\vec{a}'a}{\vec{a}a}} \\ Q^{\vec{a}}A & \xrightarrow{\mathtt{nu}^{\vec{a}a}} & TQ^{\vec{a}a}A \end{array}$$

... references and exceptions

E.g. for exceptions:

$$\texttt{inx} : Q^{\dot{a}}1 \longrightarrow TA\,,$$

$$\texttt{hdl} : Q^{\dot{a}}1 \times TA \times TA \longrightarrow TA\,,$$

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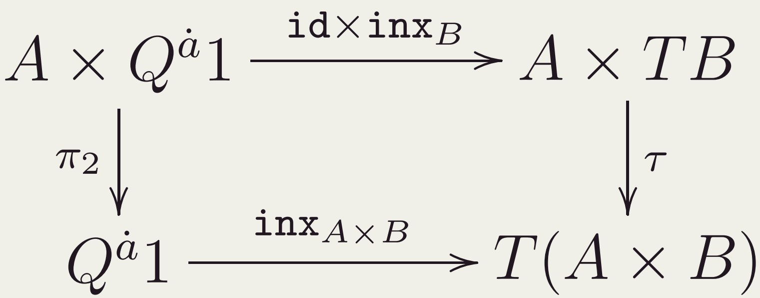

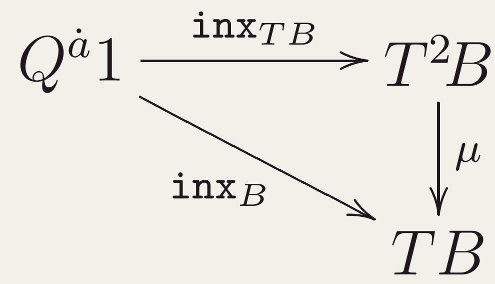

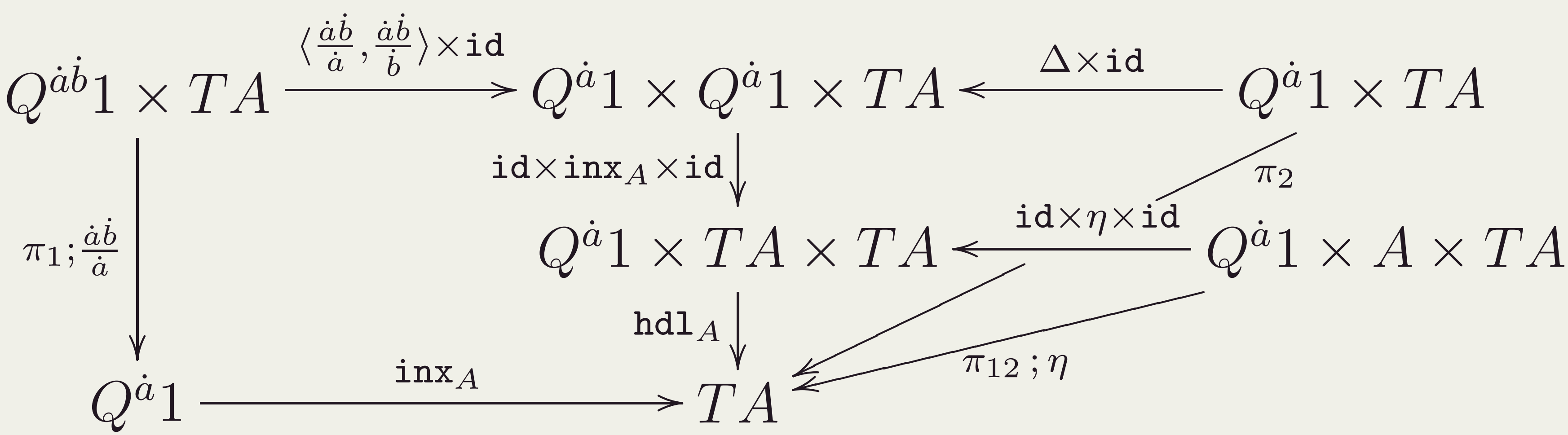

Nominal games

The concrete model

Nominal games are:

- call-by-value [HY99],
- stateful [Ong02],
- nominal [AGMOS04].

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- call-by-value [HY99],
- stateful [Ong02],
- nominal [AGMOS04].

Games start on the left; arenas start with a PA; ...

Moves are attached with state; there are state-change conditions on plays; ...

Arenas, plays, strategies are cast in nominal sets; state consists of names; name-change conditions; ...

Formally

An ***arena*** $A \triangleq (M_A, I_A, \vdash_A, \lambda_A)$ is given by:

- A nominal set M_A of moves,
- A nominal subset $I_A \subseteq M_A$ of *initial* moves,
- A nominal *justification* relation $\vdash_A \subseteq M_A \times (M_A \setminus I_A)$,
- A nominal *labeling* function $\lambda_A : M_A \to \{O, P\} \times \{A, Q\}$,

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such that:

- initial moves are P-Answers, and $O \vdash_A P \vdash_A O$,
- O-moves justify P-moves, and viceversa,
- Answers justify Questions.

Basic constructions

$$1 \triangleq \{* : PA\}\,, \quad \mathbb{N} \triangleq \{n : PA \mid n \in \omega\}\,, \quad \mathbb{A}^{\vec{a}} \triangleq \{\pi \circ \vec{a} : PA\}$$

Basic constructions

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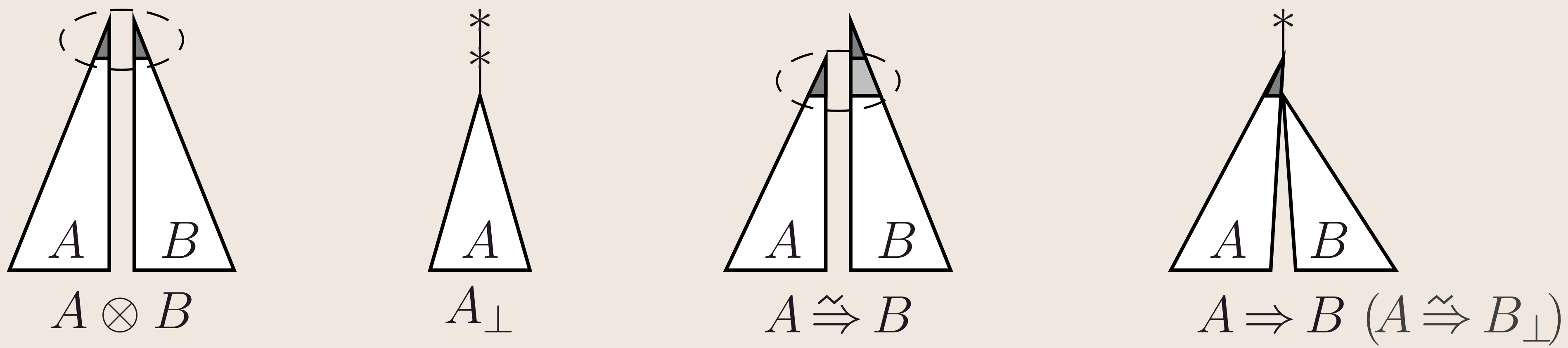

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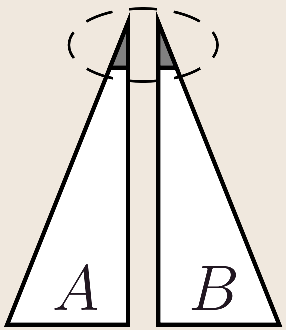
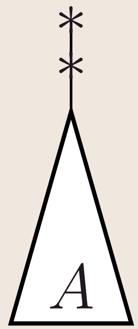
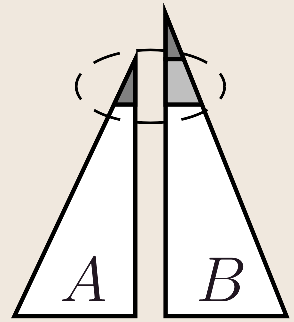


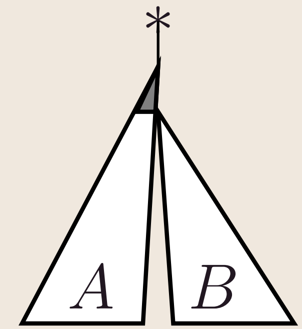
$A \otimes B$



$A_\perp$

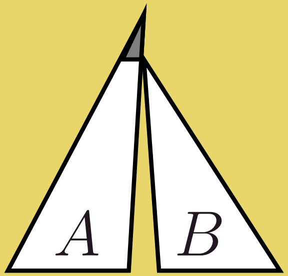


$A \mathrel{\tilde{\Rightarrow}} B$

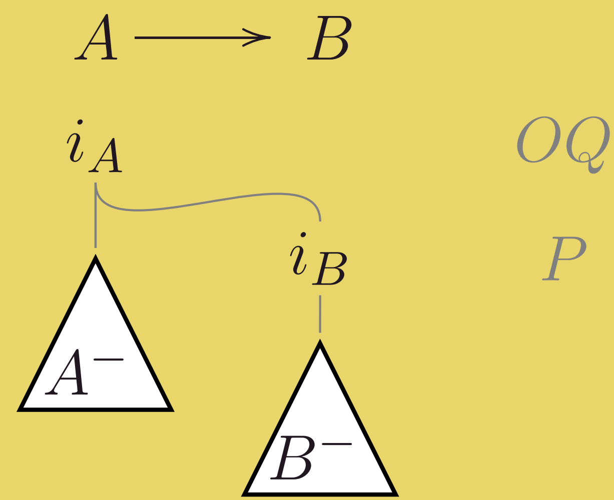


$A \Rightarrow B \; (A \mathrel{\tilde{\Rightarrow}} B_\perp)$

Prearenas



$A \to B$



A ***move-with-names*** of a prearena A is: $m^{\vec{a}}$

- $\vec{a} \in \mathbb{A}^{\#}$ *–local state*
- $m \in M_A$

Plays

A ***move-with-names*** of a prearena A is: $m^{\vec{a}}$

$\vec{a} \in \mathbb{A}^{\#}$ *–local state*
$m \in M_A$

A ***play*** on A is a *justified sequence* of moves-with-names satisfying *Vis* and *WB*, and also:

- O-moves do not change the local state, and initial moves have empty state,
- P-moves can add fresh names to the local state,
- P-moves must add fresh names to the local state in order to use them.

Examples

- Summary

Nominal computation

The $\nu\varepsilon\rho$-calculus

Nominal games
- The concrete model
- Formally
- Basic constructions
- Plays
- Examples
- Strategies
- Innocent?
- The intensional model
- 'Examples'
- 'Handling'
- Restriction, quotienting and FA

Further directions

$$\frac{\mathbb{A}_{\mathbb{E}}}{\dot a} \quad (= \mathbb{A}^{\dot a}) \qquad PA$$

$$\frac{\mathbb{A}_{\mathbb{E}} \Rightarrow \mathbb{A}_{\mathbb{E}}}{*} \qquad PA$$

$$\dot a \qquad OQ$$

$$\dot b \qquad PA$$

Examples

$$\frac{\mathbb{A}_{\mathbb{E}}}{\dot{a}} \quad (= \mathbb{A}^{\dot{a}}) \qquad PA$$

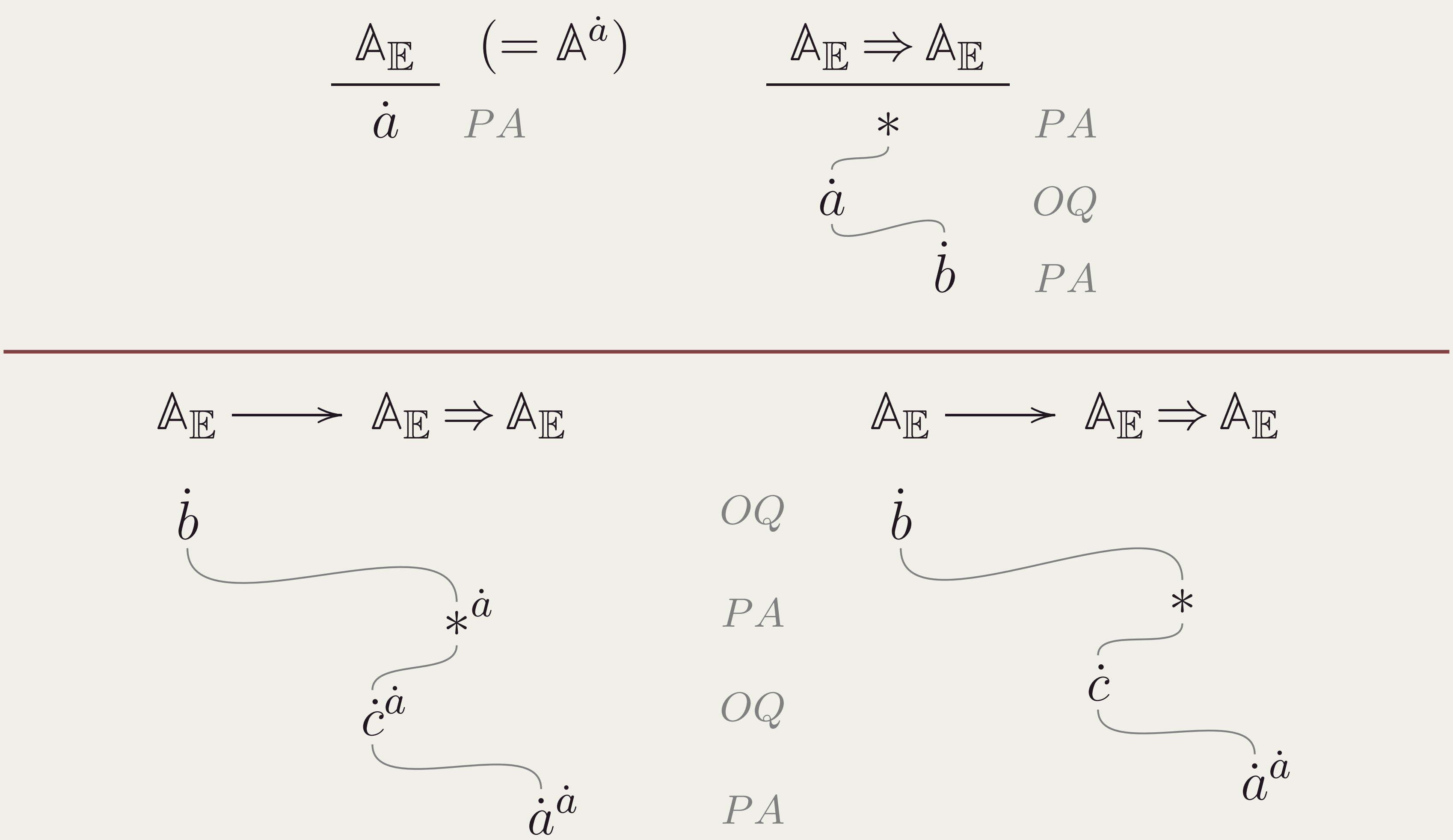
$$\frac{\mathbb{A}_{\mathbb{E}} \Rightarrow \mathbb{A}_{\mathbb{E}}}{*}$$

$*$	PA
$\dot{a}$	OQ
$\dot{b}$	PA

$$\mathbb{A}_{\mathbb{E}} \longrightarrow \mathbb{A}_{\mathbb{E}} \Rightarrow \mathbb{A}_{\mathbb{E}}$$

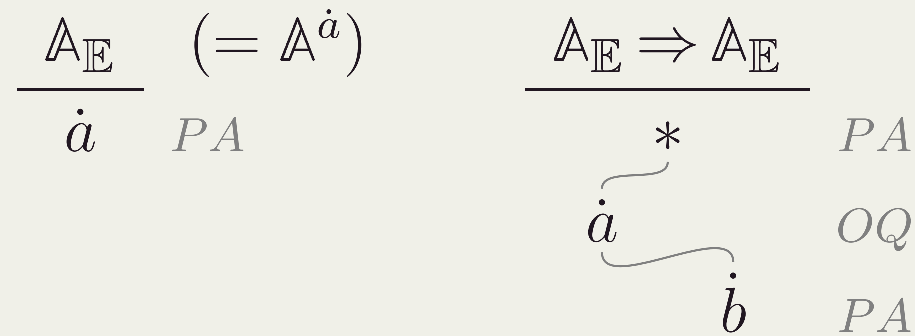
$\dot{b}$	OQ
$*^{\dot{a}}$	PA
$\dot{c}^{\dot{a}}$	OQ
$\dot{a}^{\dot{a}}$	PA

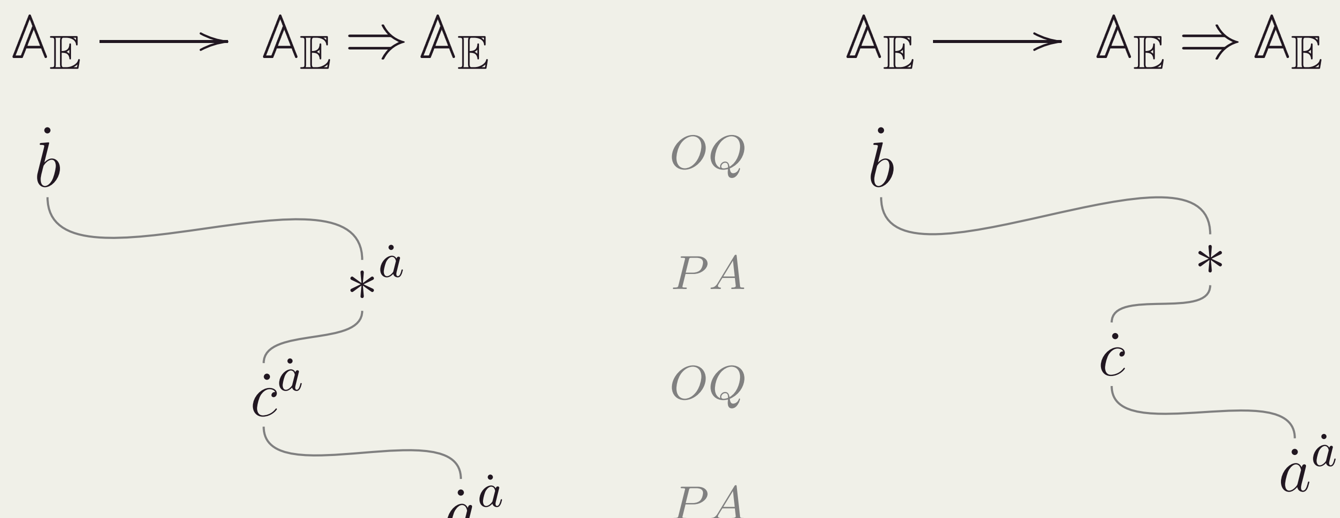
Examples



$\mathbb{A}_{\mathbb{E}}$ $(= \mathbb{A}^{\dot a})$
$\dot a$ PA
$\mathbb{A}_{\mathbb{E}} \Rightarrow \mathbb{A}_{\mathbb{E}}$
$*$ PA
$\dot a$ OQ
$\dot b$ PA
$\mathbb{A}_{\mathbb{E}} \longrightarrow \mathbb{A}_{\mathbb{E}} \Rightarrow \mathbb{A}_{\mathbb{E}}$
$\dot b$ OQ
$*^{\dot a}$ PA
$\dot c^{\dot a}$ OQ
$\dot a^{\dot a}$ PA
$\mathbb{A}_{\mathbb{E}} \longrightarrow \mathbb{A}_{\mathbb{E}} \Rightarrow \mathbb{A}_{\mathbb{E}}$
$\dot b$
$*$
$\dot c$
$\dot a^{\dot a}$

Examples





$x : \mathbb{E} \vdash \nu\dot{a}.\lambda y^{\mathbb{E}}.\dot{a}$ vs $x : \mathbb{E} \vdash \lambda y^{\mathbb{E}}.\nu\dot{a}.\dot{a}$

Strategies

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determinacy, innocence, totality.

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Let $\mathcal{V}$ be the category of (nominal) arenas and strategies.

Structure in $\mathcal{V}$:

- finite products, given by $\otimes$ and 1,
- coproducts, given by $+$,
- monadic exponentials, given by $(\_)_\perp$,
- ...

Innocent?

Previous game models of references or exceptions were based on strategies which broke innocence (amongst others) in order to express 'non-functional' behavior.

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In nominal games this is not necessary: we can build these behaviors *monadically*. E.g. for exceptions:

- The category of nominal games has coproducts,
- it also has an object of exception-names, $\mathbb{A}_{\mathbb{E}}$,
- so exceptions are modeled by $\_ + \mathbb{A}_{\mathbb{E}}$.

The intensional model

Translate types to arenas:

$$[\![\mathbb{1}]\!] = 1 \quad [\![\mathbb{N}]\!] = \mathbb{N} \quad [\![\mathbb{E}]\!] = \mathbb{A}_{\mathbb{E}} \quad [\![[A]]\!] = \mathbb{A}_{[A]} \quad [\![A \times B]\!] = [\![A]\!] \otimes [\![B]\!]$$

$$[\![A \to B]\!] = [\![A]\!] \otimes \xi \Rightarrow ([\![B]\!] + \mathbb{A}_{\mathbb{E}}) \otimes \xi \qquad \xi = \bigotimes_A \mathbb{A}_{[A]} \Rightarrow [\![A]\!]$$

– solved as a recursive domain equation.

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$$[\![A \to B]\!] = [\![A]\!] \stackrel{\sim}{\Rightarrow} (\xi \Rightarrow ([\![B]\!] + \mathbb{A}_{\mathbb{E}}) \otimes \xi) \qquad \xi = \bigotimes_A \mathbb{A}_{[A]} \Rightarrow [\![A]\!]$$

Thus, our computation monad T is:

$$T \triangleq \xi \Rightarrow ((\_ + \mathbb{A}_{\mathbb{E}}) \otimes \xi)\,.$$

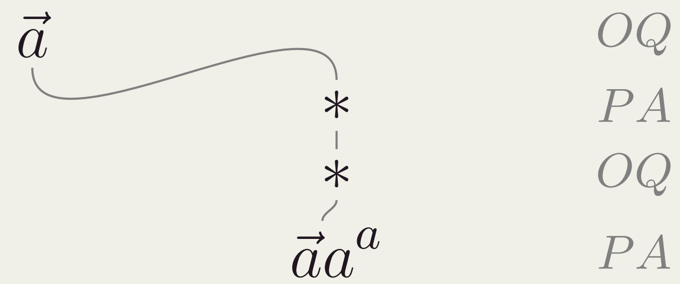
The comonads $(Q^{\vec{a}})_{\vec{a} \in \mathbb{A}^{\#}}$ are given by:

$$Q^{\vec{a}} \triangleq \mathbb{A}^{\vec{a}} \otimes \_\,.$$

Hence, we obtain a (sound) model of $\nu\varepsilon\rho$ in $\mathcal{V}$.

‘Examples’

$$\mathtt{nu}^{\vec{a}a} : \mathbb{A}^{\vec{a}} \longrightarrow (\mathbb{A}^{\vec{a}a})_{\perp}$$



‘Examples’

$$\mathtt{nu}^{\vec{a}a} : \mathbb{A}^{\vec{a}} \longrightarrow (\mathbb{A}^{\vec{a}a})_\bot$$

$\vec{a}$		OQ
	$*$	PA
	$*$	OQ
	$\vec{a}a^{a}$	PA

$$\llbracket M \rrbracket : \mathbb{A}^{\vec{a}a} \otimes \llbracket \Gamma \rrbracket \to \llbracket A \rrbracket_\bot$$

$$\llbracket \nu a.M \rrbracket : \mathbb{A}^{\vec{a}} \otimes \llbracket \Gamma \rrbracket \xrightarrow{\mathtt{nu}} (\mathbb{A}^{\vec{a}a} \otimes \llbracket \Gamma \rrbracket)_\bot \xrightarrow{\llbracket M \rrbracket_\bot ; \mathtt{dn}} \llbracket A \rrbracket_\bot$$

$(\vec{a}, i_\Gamma)$		
	$*$	
		$*$
		$*$
	$*$	
	$(\vec{a}a, i_\Gamma)^a$	
$\vdots$	$\vdots$	$\vdots$

‘Handling’

$$\texttt{hdl} : \mathbb{A}_{\mathbb{E}} \otimes (A + \mathbb{A}_{\mathbb{E}}) \otimes B \longrightarrow \mathbb{A}_{\mathbb{E}} + A + B$$

$(\dot{a}, i_A, i_B)$

i_A

$\vdots \qquad \vdots$

$(\dot{a}, \dot{b}, i_B)$

$\dot{b}$

$(\dot{a}, \dot{a}, i_B)$

i_B

$\vdots \qquad \vdots$

Restriction, quotienting and FA

– *Is the model complete?*

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In the operational semantics we have *store-discipline* and *exceptions-discipline*. This does not happen in the game model.

We therefore restrict strategies by imposing extra conditions, obtaining thus *x-tidy strategies*. Let $\chi\mathcal{T}$ be the subcategory of $\mathcal{V}$ with objects $Q^{\vec{a}}[\![A]\!]$ and arrows x-tidy strategies.

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$\chi\mathcal{T}$ is a sound model

If $\sigma : Q^{\vec{a}}[\![A]\!] \longrightarrow T[\![B]\!]$ is in $\chi\mathcal{T}$ and has *finite description* then it is definable

$$[\![M]\!] \lesssim [\![N]\!] \iff M \lessapprox N$$

Further directions

Directions in Nominal computation

- The monadic-comonadic description of the nominal effect.
- The connection between nominal and non-nominal approaches.
- Decidability issues.
- Nominal concurrency, etc.

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\+ THANKS!