MSc in Computer Science
MSc in Mathematics and the Foundations of Computer Science

Michaelmas Term 2017

Foundations of CS

Exercise class 3

1. Give a context-free grammar for $\{w \; : \; w \in \{0,1\}^*, w = w^R$ and length of w is even $\}$, where w^R is the reversal of string w.

   That is, w is an even-length palindrome.

2. (this is Sipser 2.22, page 132) Let $C = \{x\#y \; : \; x \in \{0,1\}^*$ and $x \neq y\}$

   Show that C is a context-free language.

3. Show the the context-free languages are closed under union.

4. Use the pumping lemma for context-free languages to show that the language $\{0^n 1^n 0^n 1^n \; : \; n \geq 0\}$ is not context-free. (this is Sipser Problem 2.30 a, page 133)

5. Show that the context-free languages are not closed under complement (recall that the complement of a language L is the set of strings that are not in L). If you wish, you can make use of any result proven in lectures, or you can use the result of the problem above, even if you have not solved it.

6. Give implementation-level descriptions of Turing machines that decide the language $\{w \; : \; w$ contains twice as many 0's as 1's$\}$, over the alphabet $\{0,1\}$.

   This is Sipser, 3.8 b page 162; you might want to look at the solution for 3.8 a to get an idea of the level of detail expected.

7. Give an algorithm (described informally) that takes as input an NPDA A and determines whether the language of A is nonempty.

   You should argue for the correctness of your algorithm, but you do not need to give a full formal proof of correctness. You can make use of constructions given in lecture if needed.

8. This is Sipser, problem 3.12, page 163

   A *Turing Machine with left reset* is similar to an ordinary Turing Machine, but the transition function has the form:

   $\delta : Q \times \Gamma \to Q \times \Gamma \times \{R, RESET\}$

   If $\delta(q,a) = (r, b, RESET)$ when the machine is in state q reading an a, the machine's head jumps to the left-hand end of the tape after it writes b on the tape and enters state r. Note that these machines do not have the usual ability to move the head one symbol left. Show that Turing Machines with left reset recognize the class of Turing-recognizable languages.

9. This is Sipser, problem 3.13, page 163

   A *Turing Machine with "stay put" instead of left* is similar to an ordinary Turing Machine, but the transition function has the form $\delta : Q \times \Gamma \to Q \times \Gamma \times \{R, S\}$

   At each point the machine can move its head right or let it stay in the same position. Show that this Turing Machine variant is *not* equivalent to the usual version. What class of languages do these machines recognize?