MSc in Computer Science
MSc in Mathematics and the Foundations of Computer Science

Michaelmas Term 2017

Foundations of CS

Exercise class 4

1. Show that there is a language E such that both E and E^c (the complement of E) have non-empty intersection with every infinite semi-decidable set.

2. Show that every infinite semi-decidable language has an infinite decidable subset.

3. For each of the following problems, say whether they are decidable, semi-decidable, or not semi-decidable. Prove your answer. In the case of a decidable problem a proof could mean some pseudo-code, or a reduction. In the case of a semi-decidable set, a proof would mean pseudo-code for a recognizer or enumerator, plus a reduction that shows undecidability. For problems that are not even semi-decidable, a proof could be (e.g.) by arguing that the complement is semi-decidable, but not decidable, using a reduction.

   (a) $\{\langle G \rangle : G$ is a context-free grammar and G generates some string $\}$ (i.e. $L(G) \neq \emptyset$).

   (b) $\{\langle M \rangle : M$ accepts some input w using at most $|w|^2$ many steps $\}$ (where $|w|$ denotes the length of string w)

   (c) $\{\langle M \rangle : M$ accepts some input using at most 200 steps $\}$

   (d) $\{\langle M \rangle : M$ accepts exactly two strings $\}$

4. A two-stack NPDA is like a NPDA, ... but with two stacks. The actions depend on the current input symbol the top of each stack and the control state, and an action gives a new control state and an action on each stack. For example, a transition $(q, c, A, B) \to (q', C, D)$ says that if the machine is in state q reading symbol c, with the top of Stack 1 being A and the top of Stack 2 being B, then the new state is q', the top of Stack 1 is swapped with C and the top of Stack 2 is swapped with D. A transition $(q, c, \epsilon, B) \to (q', C, D)$ would say that if the machine is in state q reading symbol c, with the top of Stack 2 being B, then the new state is q', C is pushed onto the top of Stack 1 and the top of Stack 2 is swapped with D.

   Let $NONEMP_{2NPDA} = \{\langle N \rangle : N$ is a 2NPDA that accepts some string $\}$

   Show that the halting problem can be reduced to $NONEMP_{2NPDA}$, and hence $NONEMP_{2NPDA}$ is undecidable.

   Hint: Given a Turing Machine configuration uqv (where the tape configuration is uv, the control state is q, and the head is at the beginning of v), let one of the stacks store u and the other v.