Modelling Biomechanics: applications of the FE method on nonlinear problems
The finite element method is very commonly used in structural problems, and theories of solid mechanics can be used to predict deformation or stresses. Most materials have a linear relationship between stress and strain, for small enough strains. For hard objects (metals, bone, etc), the theory of linear elasticity suffices.
Modelling biomechanics: Nonlinear stress-strain relationships

In general the relationship between stress and strain is much more complex.

Stress is a **nonlinear** function of strain, but can also depend on the history of the loading / unloading (hysteresis). The stress-strain relationship may differ in different directions in the material (**isotropic vs anisotropic**)

**The general theory of nonlinear elasticity has to be used when predicting soft-tissue deformation**
Modelling biomechanics: stages

- **Generate mesh**
  - usually generated using a set of MR images
  - images need to segmented before mesh generation

- **Set up equations to be solved**
  - time-dependent / static / quasi-static?
  - stress-strain relationship: nonlinear – which form? anisotropic?
  - forces – gravity?

- **Apply appropriate boundary conditions**
  - usually need to apply a fixed displacement boundary condition somewhere
  - fixed or zero external stress boundary conditions elsewhere
  - can be very difficult to determine appropriate boundary conditions..

- **Numerically compute solution**
  - develop or use a finite element solver, which will have to include a nonlinear solver
Breast cancer diagnosis is hindered by the fact that the breast shape varies dramatically between mammography ultrasound and MRI.

**Mammography:** breast is compressed (several directions are possible)
**MRI:** Patient lies on back
**Ultrasound:** Patient sits up
**Surgery (biopsy):** Patient lies on back

**Goal:** develop a 3D computational model of the breast from MR images, which can be deformed to simulate other scenarios and aid diagnosis.
Predicting tumour location in the breast: mesh generation
Predicting tumour location in the breast: CC to MLO mammogram simulation
Predicting tumour location in the breast: CC to MLO mammogram simulation
Electrical activation of cardiac myocytes induces tension generation – electro-mechanics. Deformation of cardiac tissue has an influence on electrical activity.

Whilst a great deal of insight can be gained into cardiac activity / pathology / effect of drugs using an electro-physiological tissue model, electro-mechanical models are also of great use.

Difficulties in simulating cardiac electro-mechanics:
- highly nonlinear (computationally intensive)
- highly anisotropic tissue
- active tension
- residual stress
- suitable boundary conditions

LeGrice et. al., Laminar structure of the heart: Ventricular myocyte arrangement and connective tissue architecture in the dog, 1995
Cardiac electro-mechanical modelling

ECG & Membrane Potential of Ventricular Cell

- Phase 1: Transient efflux of K⁺
- Phase 2: Influx of Ca²⁺ and Na⁺
- Phase 0: Fast Na⁺-influx
- Phase 3: Efflux of K⁺ > influx of Ca²⁺ and Na⁺
- Phase 4: Na⁺-K⁺-pump

Myocardial cell
- Na⁺: 140 mmol/L
- K⁺: 4 mmol/L
- Ca²⁺: 2 mmol/L

Extracellular fluid

Threshold:
- Absolute refractory period
- Relative refractory period

Steep phase 0 means rapid depolarisation

Fig. 11-2
Cardiac electro-mechanical modelling

- Tissue-level conduction (PDEs)
- Tissue-level deformation (PDEs of nonlinear elasticity)
- Cell-level activation (ODEs)
- Cell-level contraction (ODEs)

Electro-physiology: monodomain or bidomain equations

Deformation

M.E.F.: Fibre stretch or stretch-rate
Cardiac electro-mechanical modelling

Model developed at INRIA, Sophia Antipolis, France

Model developed at Lausanne, Switzerland and Milan, Italy
Cardiac electro-mechanical modelling

Modelling study – Niederer et al., *Length-dependent tension in the failing heart and the efficacy of cardiac resynchronization therapy*, 2011.

Cardiac Resynchronization Therapy (CRT) is a method of treating heart failure that involves implantation of a pacemaker which resynchronizes activation-contraction

This study investigated the efficacy of CRT and suggested a link between CRT-efficacy and Frank-Starling mechanism