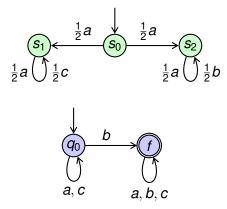
Radu Grigore Stefan Kiefer

#### Concur 2018 Beijing, 4 September 2018

### Labelled Markov Chains and DFAs



We are interested in safety specs only.

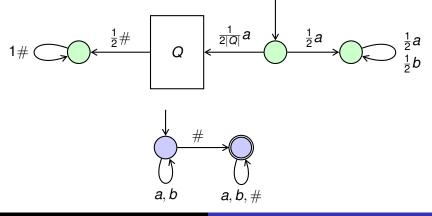
Some pairs (system, spec) are diagnosable, some are not.

# Diagnosability is PSPACE-complete

Theorem (cf. Bertrand, Haddad, Lefaucheux, 2014)

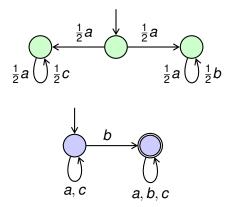
Diagnosability is PSPACE-complete.

Proof sketch. Reduce from universality of NFA where all states are initial and accepting.



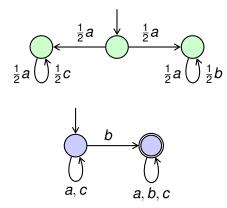
We don't insist on diagnosability.

A (selective) monitor is feasible if the probability of giving a verdict is as high as for the monitor that observes everything.



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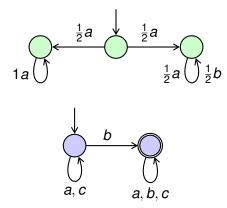
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Consider observation prefix  $a \perp a$ 

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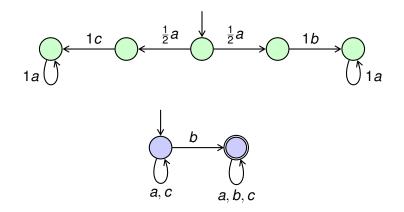
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#### Consider observation prefix $a \perp a$

## Cost of a monitor

 $egin{aligned} & C_{
ho} \ := \ ext{number of observations that } 
ho \ ext{makes} \ ( ext{random var.}) \ & c_{ ext{inf}} \ := \ \inf_{ ext{feasible } 
ho} \mathbb{E}[C_{
ho}] \end{aligned}$ 

#### Proposition

If (system, spec) is diagnosable then  $c_{inf} < \infty$ .

Proof sketch. Eagerly observe everything until a verdict can be given. Then stop observing.

#### Theorem

It is PSPACE-complete to check whether  $c_{inf} < \infty$ .

Proof similar to PSPACE-completeness of diagnosability.

## Cost of a monitor

 $C_{
ho} :=$  number of observations that ho makes (random var.)  $c_{inf} := \inf_{\text{feasible } 
ho} \mathbb{E}[C_{
ho}]$ 

#### Theorem

It is undecidable to check whether  $c_{inf} < 3$ .

Proof sketch. Reduce from the problem whether a given probabilistic automaton accepts some word with prob  $> \frac{1}{2}$ . Hard to get right.

## Cost of a monitor

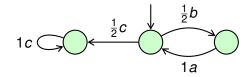
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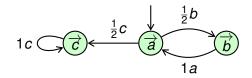
#### Theorem

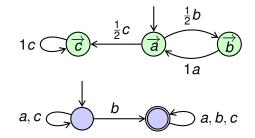
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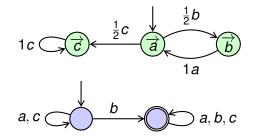
Proof sketch. Reduce from the problem whether a given probabilistic automaton accepts some word with prob  $> \frac{1}{2}$ . Hard to get right.

"Computing an optimal monitor" is also hard.





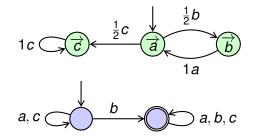




#### Proposition

In the non-hidden case we always have diagnosability.

Proof sketch. Observe everything and follow along in the DFA until a bottom SCC of the product has been reached.



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Proof sketch. Observe everything and follow along in the DFA until a bottom SCC of the product has been reached.

#### Key Observation

In the non-hidden case, maximum procrastination is optimal.

## Non-Hidden Case

The optimal monitor acts as follows:

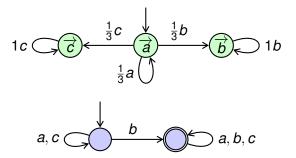
- Compute k, the minimum number of observations such that skipping k observations leads to confusion.
- Skip k-1 observations, and then make 1 observation.
- Goto 1.

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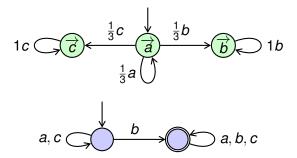


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Goto 1.



Here  $k = \infty$ . So, choose k very large.

At every stage the monitor has a belief  $\{(s_1, q_1), \dots, (s_m, q_m)\}$ about where the product MC × DFA is.

We might have m > 1 but all  $(s_i, q_i)$  in the belief must be language equivalent in a certain DFA.

To compute

$$c_{inf} := \inf_{\text{feasible } 
ho} \mathbb{E}[C_{
ho}]$$

one can set up and solve a small linear equation system. (A belief with  $k = \infty$  has an expected cost of 1.)

#### Theorem

In the non-hidden case one can compute c<sub>inf</sub> in polynomial time.

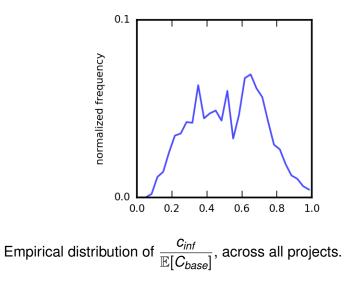
# Experiments

We have shown: maximal procrastination is optimal. How much better is maximal procrastination than the baseline?

We took 11 open-source Java projects among those most forked on GitHub, totaling 80,000 Java methods.

- On each, we ran the Facebook Infer static analyzer to compute a symbolic flowgraph (SFG) → skeleton for MC
- For each MC skeleton we sampled transition probabilities from Dirichlet distributions. (The optimal monitor is independent of those transition probabilities.)
- We considered a fixed safety property about iterators.
- On the remaining methods we computed *c<sub>inf</sub>* using Gurobi.
- Our implementation is in a fork of Infer, on GitHub.

	Project Size			Monitors			$\frac{c_{inf}}{\mathbb{E}[C_{base}]}$	
Name	Methods	SFGs	LOC	Count	Avg-Size	Max-Size	Med	GAvg
tomcat	26K	52K	946K	343	69	304	0.53	0.50
okhttp	ЗK	6K	49K	110	263	842	0.46	0.42
dubbo	8K	16K	176K	91	111	385	0.53	0.51
jadx	4K	9K	48K	204	96	615	0.58	0.50
RxJava	12K	45K	192K	83	41	285	0.52	0.53
guava	22K	43K	1218K	1126	134	926	0.41	0.41
clojure	5K	19K	66K	219	120	767	0.44	0.44
AndroidUtilCode	ЗK	7K	436K	39	89	288	0.66	0.58
leakcanary	1K	1K	11K	12	79	268	0.66	0.59
deeplearning4j	21K	40K	408K	262	51	341	0.58	0.58
fastjson	2K	7K	47K	204	63	597	0.59	0.53



# **Related Work**

#### Can faults in a given system be diagnosed?

- diagnosability; originally for finite non-stochastic systems [SSLST, 1995]
- polynomial-time, but exponentially-sized monitors

#### Diagnosability in stochastic systems (labelled MCs)

- since [Thorsley, Teneketzis, 2005]
- many different notions of diagnosability
- most of them PSPACE-complete [Bertrand, Haddad, Lefaucheux, 2014]

#### Selective monitoring

- best-effort monitoring with a specified overhead budget, e.g., [Arnold, Vechev, Yahav, 2008]
- RVSE [SBSGHSZ, 2011] also computes a probability that the program run is faulty
- our approach is opposite: no compromises on precision