CS 174 Midterm 1 (Fall 2011) E. Mossel.

Name: ____________________

SID number: ____________________

Instructions:

- Use a black/blue pen. Do not use pencils.
- Write your name and SID number on every page of the exam.
- No written materials are allowed.
- Do not use calculators/computers/cellphones or other electronic devices.
- Explain carefully all of your steps and state results that you are using (You do not have to prove any fact that was proven either in class on in the book).
- The grade of the midterm will be $\min(\sum of point, 100)$

For grading only:

1.a

1.b

2.a

2.b

3

4.a

4.b

total

Do not write on this side.

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Problem 1. (20+20 points)

We have a function $F : \{0, ..., n-1\} \to \{0, .., m-1\}$. We know that for all $0 \leq x, y \leq n-1$ it holds that

$$F(x + y \bmod n) = (1 + F(x) + F(y)) \bmod m$$

The only way we have for evaluating F is to use a lookup table that stores the values of F. Unfortunately an Evil Adversary has changed the value of 10% (0.1 fraction) of the table entries when we were not looking.

a - 20 pt. Describe a simple algorithm that given an input z, always outputs the correct value of $F(z)$ with probability 1. Your algorithm should use as few lookups and as little computation as possible.

b - 20 pt. Answer the same question if the algorithm should return the correct value of $F(z)$ with probability at least 0.9 (your algorithm should work for every value of z regardless of what values the adversary changed)

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Problem 2 (20+20 points)

3 computer viruses are attempting to infect a computer network.

- The first virus attacks every day around 12am and has a infection probability ($0 < a < 1$).
- The second virus attacks every day around 2am and has b infection probability ($0 < b < 1$).
- The third virus attaches every day around 4am has a c infection probability ($0 < c < 1$).

a - 20 pt. Let hour 0 be defined to be 10pm on Jan 1. Assume that at hour 0 the network is not infected. Let T be the first day where at hour $24 * T$ the network is infected. Write a formula for $E[T]$ in terms of a, b and c.

b - 20 pt. Write a formula for the relative probability of each of the viruses to be the *first* to infect the network.

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Problem 3 (20 points) Let n be divisible by 3 and let T be a set of size n. A balanced 3-partition of T is a partition of T into three sets A, B, C each of size exactly $n/3$. Find an efficient algorithm that samples a uniformly chosen balanced 3-partition of T. The algorithm should run in time at most $O(n^2)$.

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Problem 4 (5+25 points)

A *cycle* in a permutation π is a set of values $x_1, x_2, \ldots, x_k$ such that the x_i are all distinct and $x_{i+1} = \pi(x_i)$ for $i = 1, \ldots, k-1$ and $x_1 = \pi(x_n)$. For each permutation π on the set $\{1, ..., n\}$ there is a partition of $\{1, ..., n\}$ into cycles $C_1, \ldots, C_r$. r is called the number of cycles of π and π is said to have r cycles.

a - 5 pt. How many permutations of $\{1, \ldots, n\}$ have exactly n cycles?

b - 25 pt. find a formula for the expected number of cycles in a uniformly chosen permutation of n elements.

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