CS 174 Midterm 2 (Fall 2011) E. Mossel.

Name: ____________________

SID number: ____________________

Instructions:

- Use a black/blue pen. Do not use pencils.
- Write your name and SID number on every page of the exam.
- No written materials are allowed.
- Do not use calculators/computers/cellphones or other electronic devices.
- Explain carefully all of your steps and state results that you are using (You do not have to prove any fact that was proven either in class on in the book).
- The grade of the midterm will be $\min(\sum of point, 100)$

For grading only:

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Problem 1. (30 points)

Consider the three state Markov chain with the following transition matrix where $0 < p < 1/3$:

$$P = \begin{pmatrix} 1-2p & p & p \\ p & 1-2p & p \\ p & p & 1-2p \end{pmatrix}$$

Find a simple expression for $P^t_{i,j}$ for all states i, j and all $t \geq 1$.

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Problem 2 (30 points)

Suppose we have 3 numbers chosen independently and uniformly at random.

- The first is chosen from $\{0, \ldots, M_1 - 1\}$.
- The second is chosen from $\{0, \ldots, M_2 - 1\}$.
- The third is chosen from $\{0, \ldots, M_3 - 1\}$.

Show that we can extract on average at least $\lfloor \log_2 M_1 M_2 M_3 \rfloor - 1$ independent unbiased bits from them. (you will get 10 points if you show that we can extract on average at least $\lfloor \log_2 M_1 M_2 M_3 \rfloor - 3$ independent unbiased bits from them; hint for the harder case: try to think what to do with 2 numbers first)

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Problem 3 (30 points)

Consider the following Markov chain taking values $1, 2, 3, 4$.

$$X_{t+1} = \begin{cases} X_t & \text{with probability } 0.5 \\ 2X_t \bmod 5 & \text{with probability } 0.25 \\ 3X_t \bmod 5 & \text{with probability } 0.25 \end{cases}$$

a - 5pt. Write the transition matrix of the chain.

b - 5pt. Is the chain ergodic?

c - 20pt. Show that $\lim_{k\to\infty} P[X_k = i] = 1/4$ for $1 \leq i \leq 4$.

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Problem 4 (40 points)

Let $G = (V, E)$ be a simple graph of maximal degree d and $n = |V|$. Given an assignment of colors $Red, Blue, Yellow$ to the vertices of G, call a vertex v *lonely* if it has a different color than *all* of its neighbors.

a - 20pt. Find a randomized algorithm that assigns the colors $Red, Yellow, Blue$ to the vertices of G and such that at least $n \times (\frac{2}{3})^{d+1}$ of the vertices are lonely. The expected running time should be $O(n3^d)$.

b - 20t. Find a deterministic algorithm that assigns the colors $Red, Yellow, Blue$ to the vertices of G and such that at least $n \times (\frac{2}{3})^{d+1}$ of the vertices are lonely. The running time should be $O(n3^d)$.

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