Machine learning - HT 2016
11. Reinforcement Learning

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Textbooks

- An Introduction to Reinforcement Learning. Richard Sutton and Andrew Barto. *MIT Press, 1998*
  - Available online (html format)
  - Draft second edition
- Algorithms for Reinforcement Learning. Csaba Szepesvári. *Morgan and Claypool, 2010*
  - Available online
  - Terse and mathematical
- Course by David Silver (lectures on youtube)

Outline

Overview of reinforcement learning.

- Formulation - difference from other learning paradigms
- Markov Decision Processes
- Reward, Return, Value function, Policy
- Algorithms for policy evaluation and optimisation

Reinforcement Learning

How is RL different from other paradigms in ML?

- No supervisor, only a reward signal
- Unlike unsupervised learning not really looking for hidden structure; goal is to maximise reward
- Feedback may be delayed, long-term effects of actions
- Data is sequential and not i.i.d.; time plays an important role
- Tradeoff between exploration and exploitation

Examples of Reinforcement Learning

- Beat world champion at Go
- Fly helicopter and perform stunts [video]
- Make(?) money on the stock market
- Make robots walk
- Play video games

Reward

- A reward R_t at time t is a scalar signal
- Indicates performance of agent at time t
- Agent's goal is to maximise cumulative reward

Reward Hypothesis

All goals can be described by the maximisation of expected cumulative reward

Examples of Reward

Playing Go

- \$1 million for winning
- -\$1 million for losing

Flying a helicopter

- Positive reward for doing tricks
- Negative reward for crashing

Investing on the stock market

- \$\$\$

Reinforcement Learning: Sequential Decision Making

Goal: Select actions to maximise total future reward

Actions have long term consequences

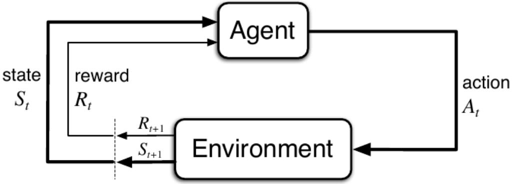
Reward may be delayed

At times, it may be imperative to sacrifice immediate reward to get long-term reward

Examples

- Blocking an opponent's move, sacrificing a rook
- Financial investment
- Refuelling a helicopter
- Getting oxygen in seaquest [video]

Agent and Environment



- t denotes discrete time
- At time step t the agent does the following:
  - Receive reward R_t (from the previous step)
  - Observe state S_t
  - Execute action A_t
- At time step t the environment "does" the following:
  - Update state to S_{t+1} based on action A_t
  - Emit reward R_{t+1}

Markov Decision Process (MDP)

A Markov decision process (MDP) is a tuple $\langle \mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R}, \gamma \rangle$

- $\mathcal{S}$ is a finite set of states
- $\mathcal{A}$ is a finite set of actions
- $\mathcal{P}$ is a state transition probability matrix

$$\mathcal{P}^a_{s,s'} = \Pr[S_{t+1} = s' \mid S_t = s, A_t = a]$$

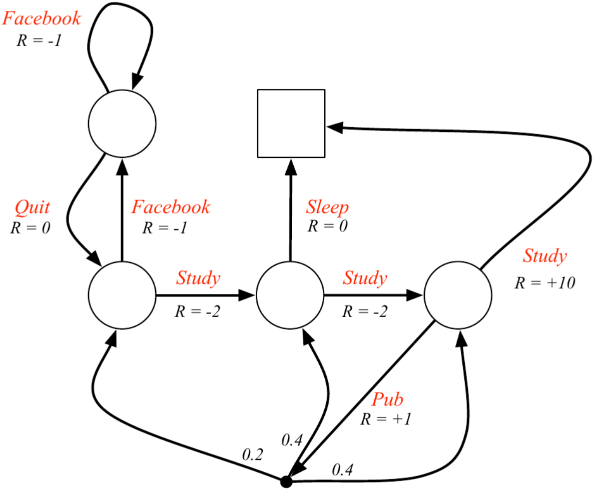
- $\mathcal{R}$ is a reward function

$$\mathcal{R}^a_s = \Pr[R_{t+1} = r \mid S_t = s, A_t = a]$$

- $\gamma \in [0, 1]$ is a discount factor

Are real-world problems really Markovian?

Example: Student MDP



Source: David Silver

Components of an RL agent

Goal: Maximise expected cumulative discounted future reward

$$\text{Expected Return} := G_t = \mathbb{E}[\sum_{j=1}^{\infty} \gamma^{j-1} R_{t+j}]$$

Model: Agent's representation of the environment (transitions)

Policy: How the agent chooses actions given a state

Value Function: How much long term reward can be achieved from a given state

Why discount?

Why should we consider discounted reward rather than just add up?

- Mathematically more convenient formulation to deal with
- Don't have infinite returns because of positive reward cycles in MDP
- Captures the idea that future may be "uncertain"
- Bird in hand vs two in bush (especially true for monetary reward)
- Can use $\gamma = 1$ if MDP is episodic

Model

- A model helps predict future states and rewards given current state and action
- $\mathcal{P}$ (stochastically) determines the next state

$$\mathcal{P}^a_{s,s'} = \Pr[S_{t+1} = s' \mid S_t = s, A_t = a]$$

- $\mathcal{R}$ (stochastically) determines the immediate reward

$$\mathcal{R}^a_s = \Pr[R_{t+1} = r \mid S_t = s, A_t = a]$$

- Model-based reinforcement learning (planning)
- Model-free reinforcement learning (trial and error)

Policy

- A policy describes the agent's behaviour or strategy
- Map each state to an action
- Deterministic Policy: $a = \pi(s)$
- Stochastic Policy: $\pi(a \mid s) = \Pr[A_t = a \mid S_t = s]$

Reinforcement Learning: Prediction and Control

Prediction

- Policy is fixed
- Evaluate the future reward (return) from each state

Control

- Find the policy that maximises future reward

Value Function

- Value function defines expected future reward
- Useful for defining quality (goodness/badness) of a given state
- Useful to select a suitable action (policy improvement)

$$v_\pi(s) = \mathbb{E}_\pi[R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \cdots \mid S_t = s]$$

Learning vs Planning

Two fundamental problems in sequential decision making

Reinforcement Learning

- Environment is unknown (observed through trial and error)
- Agent interacts with the environment
- Agent improves policy/strategy/behaviour through this interaction

Planning

- Model of the environment is known
- Agent does not play directly with the environment
- Compute good (optimal) policy/strategy through simulation, reasoning, search

Value Function and Action-Value Function

State-Value Function

- The *state-value function* $v_\pi(s)$ for an MDP is the expected return starting from state s, and then following policy π

$$v_\pi(s) = \mathbb{E}_\pi[G_t \mid S_t = s]$$

Action-Value Function

- The *action-value function* $q_\pi(s, a)$ for an MDP is the expected return starting from state s, taking action a, and then following policy π

$$q_\pi(s, a) = \mathbb{E}_\pi[G_t \mid S_t = s, A_t = a]$$

Bellman Expectation Equation

State-Value Function

- The *state-value function* satisfies the fixed-point equation. It can be decomposed into reward at current time, plus the discounted value at the successor state.

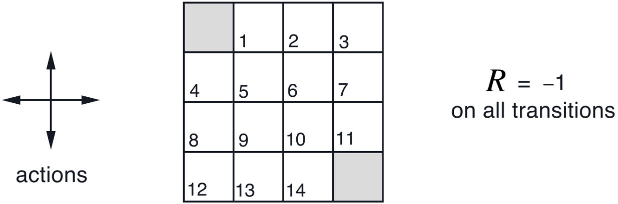
$$v_\pi(s) = \mathbb{E}_\pi[R_{t+1} + \gamma v_\pi(S_{t+1}) \mid S_t = s]$$

Action-Value Function

- The *action-value function* also satisfies a similar relationship.

$$q_\pi(s, a) = \mathbb{E}_\pi[R_{t+1} + \gamma q_\pi(S_{t+1}, A_{t+1}) \mid S_t = s, A_t = a]$$

Evaluating a Random Policy in the Small Gridworld



- No discounting, $\gamma = 1$
- States 1 to 14 are not terminal, the grey state is terminal
- All transitions have reward -1, no transitions out of terminal states
- If transitions lead out of grid, stay where you are
- Policy: Move north, south, east, west with equal probability

Policy Evaluation

0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0

$k = 0$

0.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	0.0

$k = 1$

0.0	-1.7	-2.0	-2.0
-1.7	-2.0	-2.0	-2.0
-2.0	-2.0	-2.0	-1.7
-2.0	-2.0	-1.7	0.0

$k = 2$

0.0	-2.4	-2.9	-3.0
-2.4	-2.9	-3.0	-2.9
-2.9	-3.0	-2.9	-2.4
-3.0	-2.9	-2.4	0.0

$k = 3$

0.0	-6.1	-8.4	-9.0
-6.1	-7.7	-8.4	-8.4
-8.4	-8.4	-7.7	-6.1
-9.0	-8.4	-6.1	0.0

$k = 10$

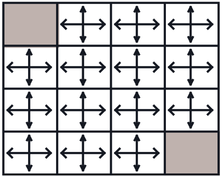
0.0	-14.	-20.	-22.
-14.	-18.	-20.	-20.
-20.	-20.	-18.	-14.
-22.	-20.	-14.	0.0

$k = \infty$

Policy Improvement

$k = 0$

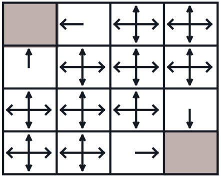
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0



Random Policy

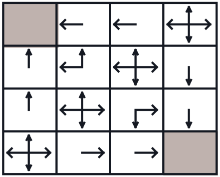
$k = 1$

0.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	0.0



$k = 2$

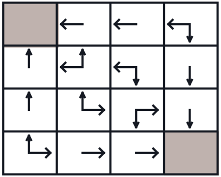
0.0	-1.7	-2.0	-2.0
-1.7	-2.0	-2.0	-2.0
-2.0	-2.0	-2.0	-1.7
-2.0	-2.0	-1.7	0.0



Policy Improvement

$k = 3$

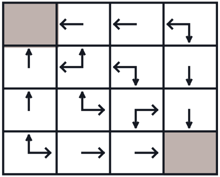
0.0	-2.4	-2.9	-3.0
-2.4	-2.9	-3.0	-2.9
-2.9	-3.0	-2.9	-2.4
-3.0	-2.9	-2.4	0.0



Optimal Policy

$k = 10$

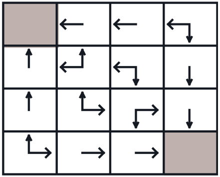
0.0	-6.1	-8.4	-9.0
-6.1	-7.7	-8.4	-8.4
-8.4	-8.4	-7.7	-6.1
-9.0	-8.4	-6.1	0.0



Optimal Policy

$k = \infty$

0.0	-14.	-20.	-22.
-14.	-18.	-20.	-20.
-20.	-20.	-18.	-14.
-22.	-20.	-14.	0.0



Optimal Policy

Bellman Optimality Equations

State-Value Function

- The *optimal state-value function* satisfies the fixed point equation.

$$\begin{aligned} v_*(s) &= \max_{a \in \mathcal{A}} \mathbb{E}[R_{t+1} + \gamma v_*(S_{t+1}) \mid S_t = s, A_t = a] \\ &= \max_{a \in \mathcal{A}} q_*(s, a) \end{aligned}$$

Action-Value Function

- The *optimal action-value function* also satisfies a fixed point equation.

$$\begin{aligned} q_*(s, a) &= \mathbb{E}[R_{t+1} + \gamma v_*(S_{t+1}) \mid S_t = s, A_t = a] \\ &= \mathbb{E}[R_{t+1} + \gamma \max_{a' \in \mathcal{A}} q_*(S_{t+1}, a') | S_t = s, A_t = a] \end{aligned}$$

Optimal Policy

$$\begin{aligned} \pi^*(s) &= \operatorname*{argmax}_a q_*(s, a) \\ &= \mathbb{E}[R_{t+1} + \gamma v_*(S_{t+1}) \mid S_t = s, A_t = a] \end{aligned}$$

Model-Free Reinforcement Learning

- If we know the model, based on optimality equations we can, possibly with great computational effort, solve the MDP to find an optimal policy

- In reality, we don't know the MDP but can try to learn v_* and q_* approximately through interaction

- Monte Carlo Methods

$$V(S_t) \leftarrow V(S_t) + \alpha(G_t - V(S_t))$$

- Temporal difference method, *e.g.,* TD(0)

$$V(S_t) \leftarrow V(S_t) + \alpha(R_{t+1} + \gamma V(S_{t+1} - V(S_t))$$

- Important to explore as well as exploit

Reinforcement Learning: Exploration vs Exploitation

- Learning is through trial and error
- Discovering good policy requires diverse experiences
- Should not lose too much reward during exploration
- Exploration: Discover more information about the environment
- Exploitation: Use known information to maximise reward

Examples: Exploration vs Exploitation

Video Game Playing

- In seaquest, if you never try to get oxygen, only limited potential for reward

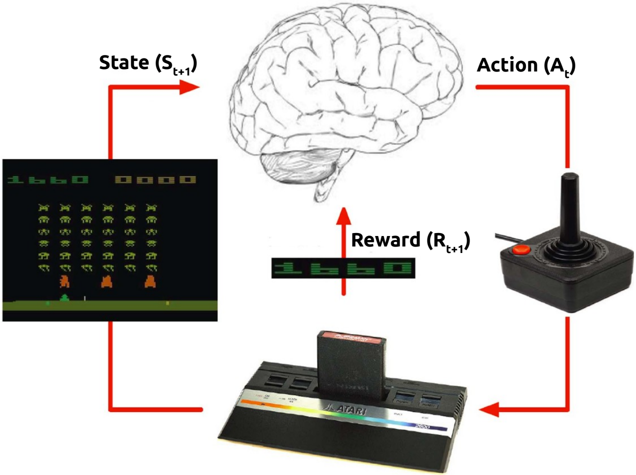
Restaurant Selection

- Try the new American burger place, or go to your favourite curry place?

Online Advertisements

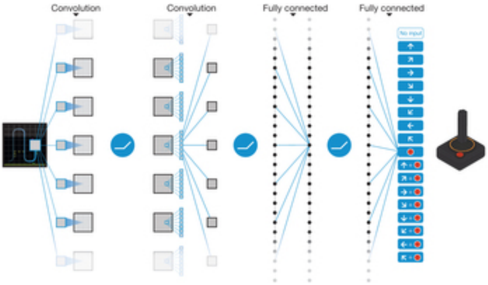
- Keep showing money-making ads or try new ones?

Very Large MDPs



Source: David Silver

Function approximation



- Approximate $q_*(s,a)$ using a convnet
- Requires new training procedures

Source: Mnih *et al.* (Nature 2015)

Summary: What we did

In the past 8 weeks we've seen machine learning techniques from Gauss to the present day

Supervised Learning

- Linear regression, logistic regression, SVMs
- Neural networks, deep learning, convolutional networks
- Loss functions, regularisation, maximum likelihood, basis expansion, kernel trick

Unsupervised Learning

- Dimensionality reduction: PCA, Johnson Lindenstrauss
- Clustering: k-means, heirarchical clustering, spectral clustering

Reinforcement Learning

- MDPs, prediction, control, Bellman equations

Summary: What we did not

ML Topics

- Boosting, bagging, decision trees, random forests
- Bayesian approaches, graphical models, inference
- Dealing with very high-dimensional data
- More than half of Murphy's book

Further Exploration

- Lots of online videos, videolectures.net, ML summer schools
- ML toolkits: torch, theano, tensor flow, sci-kit learn, R
- Conferences: NIPS, ICML, COLT, UAI, ICLR, AISTATS, ACL, CVPR, ICCV
- Arxiv: cs.LG, cs.AI, ml-news mailing list, podcasts, blogs, reddit