# Don't sit on the fence: A static analysis approach to automatic fence insertion



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```
1 volatile int data = 0, flag = 0;
3 void* thread_0 (void* arg) {
4     data = 1;
5     flag = 1;
6}
```

8 void\* thread\_1 (void\* arg) { 9 int flag\_set = flag; 10 int data\_upd = data; 11 assert( flag\_set  $\Rightarrow$  data\_upd ); 12 }



Sequential Consistency: (SC; interleavings semantics) data=1; ??

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Sequential Consistency: (SC; interleavings semantics)

data=1; flag=1; flag\_set=1; data\_upd=1; ?? ? data=1; flag\_set=0; flag=1; data\_upd=1; - data=1; flag\_set=0; data\_upd=1; flag=1; (| flag\_set=0; data=1; data\_upd=1; flag=1; /| flag\_set=0; data=1; flag=1; data\_upd=1; flag=1; /| flag\_set=0; data=1; flag=1; data\_upd=1; /|

Note that flag\_set⇒data\_upd is always verified

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On Power or ARM, assertion failed!





On Power or ARM, assertion failed! ?? ? Modern multicore architectures (e.g. x86, ARM) (| weaker than Sequential Consistency /|\ i.e., can exhibit more behaviours than SC / | \ Writes (amongst other things) can be reordered | \

by the processor for performance reasons

```
3 void* thread_0 (void* arg) {
4     data = 1;
5     __asm__ ("lwsync");
6     flag = 1;
7 }
```

```
9 void* thread_1 (void* arg) {
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```

Concurrent programming with **interleavings** in mind is simpler than with processor specifics



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Placing synchronisation mechanisms (fences, dependencies) to restore SC



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# Concurrent programming with **interleavings** in mind is simpler than with processor specifics

 $\label{eq:placing} \begin{array}{l} \mbox{Placing synchronisation mechanisms (fences, dependencies) to restore SC \end{array}$ 

Weak behaviours no longer permitted



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Soundly: all the weak behaviours forbidden



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Concurrent programming with **interleavings** in mind is simpler than with processor specifics

Placing synchronisation mechanisms (fences, dependencies) to restore SC

Weak behaviours no longer permitted

Soundly: all the weak behaviours forbidden

**Adequately**: only preventing reorderings of accesses affecting semantics



We offer a tool, musketeer, that

 $\bullet$  analyses concurrent C programs for TSO/x86 to Power/ARM

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- synthesises fences and dependencies for restoring SC

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- $\bullet$  analyses concurrent C programs for TSO/x86 to Power/ARM
- synthesises fences and dependencies for restoring SC
- minimises the runtime impact of these fences
- performs a source-to-source transformation in an **automated** manner

· \_-\ (| \ |\ /\/\/\/\/\/\/ | | | | | | | | |

• Inserting manually requires lots of work



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- The semantics of memory fences can be subtle



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- Architectures allow different reorderings



restored order	ARM fences	Power fences
store-read	dsb	sync
store-store	dsb	sync, lwsync
read-store	dsb, dp	sync, lwsync, dp
read-read	dsb, dp, bcc;isb	sync, lwsync, dp, bcc;isync

Figure: Effect of memory barriers on delayed pairs of events [Alglave et al., CAV'10].

! \~ (| \\_\\_\_ \\ /\\_\_ /

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- Some types of fences are cheaper than others (lwsync, isync...)


# Why optimising? (our motivation)

- Fences are slow!
- Benign reorderings (i.e., which do not affect the semantics) improve performance
- Some types of fences are cheaper than others (lwsync, isync...)

- O original
- M musketeer (our tool)
- $\mathbf{P}$  pensieve
- V volatile (unsound)
- E escape analysis
- ${\rm H}$  heap and static mem.





```
\begin{array}{l} 3 \text{ void}* \text{ thread_0 (void}* \text{ arg}) \left\{ \\ 4 \quad data = 1; \\ 5 \quad flag = 1; \\ 6 \right\} \end{array} 
\begin{array}{l} 8 \text{ void}* \text{ thread_1 (void}* \text{ arg}) \left\{ \\ 9 \quad \text{int flag_set} = flag; \\ 10 \quad \text{int data_upd} = data; \\ 11 \quad assert ( \quad flag_set \Rightarrow data_upd ); \\ 12 \right\} \end{array} 
\left. \begin{array}{l} (4) \text{ Wdata1} \\ (4) \text{ Wdat
```







```
3 void* thread_0 (void* arg) {
                                               (4) Wdata1
                                                                    (9) Rflag1
 4 data = 1;
5 flag = 1;
6}
8 void* thread_1 (void* arg) {
   int flag_set = \hat{f}lag;
9
10
  int data_upd = data;
    assert ( flag_set \Rightarrow data_upd );
11
12 }
                                               (5) Wflag1
                                                                   (10) Rdata0
```













• Under SC, orders enforced by the processor;





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• Under SC, orders enforced by the processor; (5) < (9)





• Under SC, orders enforced by the processor; (4) < (5) < (9)





• Under SC, orders enforced by the processor; (10) < (4) < (5) < (9)





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contradiction in the global-happen-before;





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cycle existence  $\Rightarrow$  execution impossible







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• Under Power, some orders are relaxed;



/1\





contradiction in the global-happen-before;

cycle existence  $\Rightarrow$  execution impossible

• Under Power, some orders are relaxed; (9)<(5)<(10)<(4)

no critical cycle  $\Rightarrow$  execution possible







• We target all the cycles for SC that are not cycles for Power





- We target all the cycles for SC that are not cycles for Power
- We modify the code so that they would also be cycles for Power

## Automatic source-to-source transformation



## Experiments

- 1. classic examples: mutex algorithms (Dekker, Lamport, Szymanski...)
- 2. parametric examples
- 3. Debian executables (>700): in particular, memcached (high-performance caching system used e.g. by Facebook)

			×86		Power	
	LoC	build time	fences	time	fences	time
memcached	9944	59.3s	3	13.9s	70	89.9s
lingot	2894	56.8s	0	5.3s	5	5.3s
weborf	2097	65.8s	0	0.7s	0	0.7s
timemachine	1336	25.4s	2	0.8s	16	0.8s
see	2626	51.6s	0	1.4s	0	1.5s
blktrace	1567	41.7s	0	6.5s	-	timeout
ptunnel	1249	2.1s	2	95.0s	-	timeout
proxsmtpd	2024	53.5s	0	0.1s	0	0.1s
ghostess	2684	51.7s	0	25.9s	0	25.9s
dnshistory	1516	107.4s	1	29.4s	9	64.9s

## Performance impact on memcached



Average overhead of execution time after fence insertion

	memcached on x86	memcached on ARM	pfscan on x86	pfscan on ARM	
(0)	[29.461; 29.699]	[16.248; 16.553]	[15.013; 15.034]	[19.066; 19.096]	
(M)	[29.776; 30.049]	[16.748; 17.023]	[15.073; 15.097]	[19.101; 19.128]	
(P)	[30.259; 30.517]	[16.795; 17.063]	[15.083; 15.107]	[19.411; 19.437]	
(V)	N/A	[15.074; 15.097]	N/A	[15.083; 15.107]	
(E)	[34.631; 34.883]	[17.402; 17.636]	[15.086; 15.118]	[19.659; 19.684]	
(H)	[38.751; 39.050]	[18.916; 19.098]	[15.270; 15.293]	[34.422; 34.457]	
Confidence intervals for N=100 $\alpha$ =5%					

We also computed Student's T-tests for checking statistical significance

## Limitations of musketeer

• musketeer works on a sound over-approximation



## Limitations of musketeer

- musketeer works on a sound over-approximation
  - no evaluation of the branching conditions
  - no synchronisation analysis
  - abstraction of the loops



# Summary of other existing tools

	tool	group	architecture
ĺ	blender, fender	(Kuperstein et al.)	x86 RMO
	memorax	(Abdulla et al.)	x86
precise	offence	(Alglave et al.)	x86 Power
	remmex	(Linden and Wolper)	x86, PSO
	trencher	(Bouajjani et al.)	×86
dynamic {	dfence	(Liu et al.)	×86





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April 4th 2014 A <u>fixed version</u> of the tool is now available Jan 20th 2014 Comparison with other static approaches for Debian experiments Dec 4th 2013 Release of the tool

#### Tool manual

#### Here is the manual of the tool musketeer.



### Experimental Results

Here are all our experimental data, for both the parametric and Debian benchmarks. We also implemented other static approaches and <u>compared them</u> to *musketeer* on the *Debian* experiments.



## www.cprover.org/wmm/musketeer

- tool and manual
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## Alternative static methods

- (O) original (no additional fence inserted)
- (M) musketeer (our tool)
- $\bullet$  (P) pensieve (in essence: all the pairs with communications)
- (E) escape analysis (in essence: all the pairs)
- $\bullet~({\rm H})$  fences around heap and static variable accesses

# Statistical evaluation of the experiments (1/2)

	memcached on x86	memcached on ARM	pfscan on x86	pfscan on ARM
(0)	[29.461; 29.699]	[16.248; 16.553]	[15.013; 15.034]	[19.066; 19.096]
(M)	[29.776; 30.049]	[16.748; 17.023]	[15.073; 15.097]	[19.101; 19.128]
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Confidence intervals for N=100,  $\alpha$ =5%

	memcached on x86	memcached on ARM	pfscan on x86	pfscan on ARM	
(M) vs. (P)	5.008	0.440	1.158	32.576	
Student T-test of $({\rm M})$ vs. $({\rm P})$ for N=100, $\alpha{=}5\%$					

t-value with 198 degrees of freedom at  $\alpha{=}5\%$  is 1.972

# Statistical evaluation of the experiments (2/2)

	stack on x86	stack on ARM	queue on x86	queue on ARM
(0)	[9.757; 9.798]	[11.291; 11.369]	[11.947; 11.978]	[20.441; 20.634]
(M)	[9.818; 9.850]	[11.316; 11.408]	[12.067; 12.099]	[20.687; 20.857]
(P)	[10.077; 10.155]	[11.995; 12.109]	[13.339; 13.373]	[22.035; 22.240]
(V)	N/A	[11.779; 11.834]	N/A	[21.334; 21.526]
(E)	[11.316; 11.360]	[13.071; 13.200]	[13.949, 13.981]	[22.722; 22.903]
(H)	[12.286; 12.325]	[14.676; 14.844]	[14.941, 14.963]	[25.468; 25.633]

Confidence intervals for data structure experiments for N=100,  $\alpha{=}5\%$ 

## Variables and cost function

variables: [type of fence]<sub>[edge]</sub>

$$\begin{split} \sum_{e \in \mathsf{po}_s^+} d\mathtt{p}_e * \mathsf{cost}(d\mathtt{p}) + \sum_{e' \in \mathsf{po}_s} d\mathtt{p}_{e'} * \mathsf{cost}(d\mathtt{p}) + \mathtt{l}\mathtt{w}\mathtt{f}_{e'} * \mathsf{cost}(\mathtt{l}\mathtt{w}\mathtt{f}) \\ + \mathtt{c}\mathtt{f}_{e'} * \mathtt{cost}(\mathtt{c}\mathtt{f}) + \mathtt{f}_{e'} * \mathtt{cost}(\mathtt{f}) + \mathtt{b}\mathtt{r}_{e'} * \mathtt{cost}(\mathtt{b}\mathtt{r}). \end{split}$$



## Constraints

- Constraints ensure that each relevant delayed pair is prevented ⇒ soundness.
- Suppose first that relaxed pairs are in pos.



- $\min \quad \mathrm{dp}_{(i,j)} + \mathrm{dp}_{(f,g)} + 3 * (\mathtt{f}_{(i,j)} + \mathtt{f}_{(f,g)}) + 2 * (\mathtt{lwf}_{(i,j)} + \mathtt{lwf}_{(f,g)})$
- $\begin{array}{ll} \textbf{s.t.} & \mbox{delay}\;(f,g) \colon \;\; \mbox{dp}_{(f,g)} + \mbox{lwf}_{(f,g)} + \mbox{f}_{(f,g)} \geq 1 \\ & \mbox{delay}\;(i,j) \colon \;\; \mbox{dp}_{(i,j)} + \mbox{lwf}_{(i,j)} + \mbox{f}_{(i,j)} \geq 1 \end{array}$

## Constraints: Entangled Cycles

• Variables are shared between the constraints. A fence between a pair can affect another pair.



$$\begin{array}{ll} \min & \mathrm{dp}_{(i,j)} + \mathrm{dp}_{(f,g)} + \mathrm{dp}_{(k,l)} + 3 * (\mathrm{f}_{(i,j)} + \mathrm{f}_{(f,g)} + \mathrm{f}_{(k,l)}) \\ & + 2 * (\mathrm{lwf}_{(i,j)} + \mathrm{lwf}_{(f,g)}) + \mathrm{lwf}_{(k,l)} \end{array}$$

 $\begin{array}{ll} \text{s.t.} & \mbox{delay}\;(f,g)\colon \;\; \mbox{dp}_{(f,g)} + \texttt{lwf}_{(f,g)} + \texttt{f}_{(f,g)} \geq 1 \\ & \mbox{delay}\;(i,j)\colon \;\; \mbox{dp}_{(i,j)} + \texttt{lwf}_{(i,j)} + \texttt{f}_{(i,j)} \geq 1 \\ & \mbox{delay}\;(k,l)\colon \;\; \mbox{dp}_{(k,l)} + \texttt{lwf}_{(k,l)} + \texttt{f}_{(k,l)} \geq 1 \end{array}$ 

Constraints: relaxed pairs are in  $po_s^+$ 

 Relaxed pairs can be in po<sup>+</sup><sub>s</sub>. To entangle cycles, we represent each po<sub>s</sub> in the relaxed po<sup>+</sup><sub>s</sub>.


## Constraints: Cumulativity

• External rf can be reordered. Only lwsync and sync can fix.

 $\min \quad \mathrm{dp}_{(i,j)} + \mathrm{dp}_{(f,g)} + 3 * \big( \mathtt{f}_{(i,j)} + \mathtt{f}_{(f,g)} \big) + 2 * \big( \mathtt{lwf}_{(i,j)} + \mathtt{lwf}_{(f,g)} \big)$ 

$$\begin{array}{lll} \textbf{s.t.} & \mbox{delay}\;(f,g)\colon\; \mbox{dp}_{(f,g)} + \mbox{lwf}_{(f,g)} + \mbox{f}_{(f,g)} \geq 1 \\ & \mbox{delay}\;(i,j)\colon\; \mbox{dp}_{(i,j)} + \mbox{lwf}_{(i,j)} + \mbox{f}_{(i,j)} \geq 1 \\ & \mbox{delay}\;(j,f)\colon\; \mbox{lwf}_{(f,g)} + \mbox{f}_{(f,g)} + \mbox{lwf}_{(i,j)} + \mbox{f}_{(i,j)} + \mbox{f}_{(i,j)} \geq 1 \\ & \mbox{delay}\;(g,i)\colon\; \mbox{lwf}_{(f,g)} + \mbox{f}_{(f,g)} + \mbox{lwf}_{(i,j)} + \mbox{f}_{(i,j)} \geq 1 \\ \end{array}$$

#### Cases where musketeer is more precise than trace-based



$$\begin{array}{ll} \min & \mathrm{dp}_{(e,g)} + \mathrm{dp}_{(f,h)} + \mathrm{dp}_{(f,g)} + 3* \left( \mathbf{f}_{(e,f)} + \mathbf{f}_{(f,g)} + \mathbf{f}_{(g,h)} \right) \\ & + 2* \left( \mathrm{lwf}_{(e,f)} + \mathrm{lwf}_{(f,g)} + \mathrm{lwf}_{(g,h)} \right) \\ \mathrm{s.t.} & \text{cycle 1, delay } (e,g) \colon \ \mathrm{dp}_{(e,g)} + \mathbf{f}_{(e,f)} + \mathbf{f}_{(f,g)} + \mathrm{lwf}_{(e,f)} + \mathrm{lwf}_{(f,g)} \geq 1 \\ & \text{cycle 2, delay } (f,g) \colon \ \mathrm{dp}_{(f,h)} + \mathbf{f}_{(f,g)} + \mathbf{f}_{(g,h)} + \mathrm{lwf}_{(f,g)} + \mathrm{lwf}_{(g,h)} \geq 1 \\ & \text{cycle 3, delay } (f,g) \colon \ \mathrm{dp}_{(f,g)} + \mathbf{f}_{(f,g)} + \mathrm{lwf}_{(f,g)} \geq 1 \end{array}$$

Figure: Example of resolution with btwn.

# Complexity



$$\sum_{i=2}^{n} {n \choose i} (A_m^2)^i,$$
  
i.e.  $o(m^{2n})$ 

step	complexity
Constructing the graph	$O(\#\mathbb{E}^2)$
Finding all the cycles	$O((\#po_s + \#cmp + \#\mathbb{E}) * \#C)$ , with $\#C$ polynomial in $\#\mathbb{E}$ but exponential in $\#thds$
Constructing the ILP	linear (constant for variables, linear for constraints)
Solving ILP	NP (but fast in practice)
Inserting in the source	constant

## Additional encodings

btwn	ILP variables	number of variables	complexity
btwn1	po₅ in the criti- cal cycles	$\sum_{d \in delays} \#soc(d)$	<i>O</i> (1)
btwn <sub>2</sub>	po <sub>s</sub> in the inter- sections of pairs of critical cycles	$\sum_{d \in delays \cap \bigcup_{j \neq k} \max(ctn(C_j) \cap ctn(C_k))} \#soc(d)$	$O(\# { m cycles}^2)$
$btwn_3$	po <sup>+</sup> at the intersections of any set of critical cycles	$\leq \# \bigcup_{j \neq k} \max(ctn(\mathit{C}_j) \cap ctn(\mathit{C}_k))$	$O(2^{\# cycles})$
btwn4	po <sup>+</sup> <sub>s</sub> at the intersections of any pair of critical cycles	$\leq \# \bigcup_{j \neq k} \max(ctn(\mathit{C}_j) \cap ctn(\mathit{C}_k))$	$O(\# { m cycles}^2)$

## Evaluation of the additional encodings



repeats