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Pitfalls in the use of Parallel Inference for the Dirichlet Process

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Outline

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- The Dirichlet process
- Parallel inference
- Non-approximate parallel inference in the Dirichlet process
- What can go wrong
- How can we try to fix it
- Conclusions

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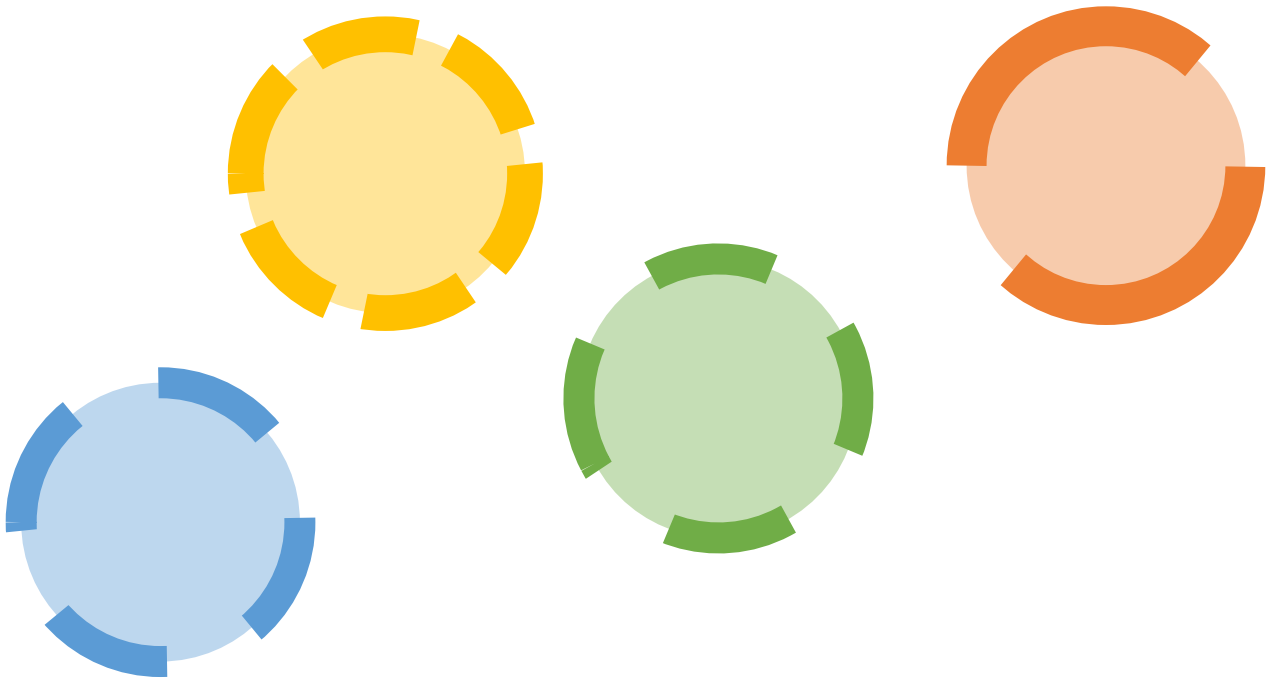
Sampling from the Dirichlet process – the Chinese restaurant process



- A restaurant with 4 tables and 2, 4, 4, and 6 customers sitting around each one

Real world applications – Natural Language Processing

- Language modelling
  - A derivative model (the Hierarchical Pitman–Yor process) was shown to correspond to the state-of-the-art in language modelling
- Machine Translation
  - Used to obtain state-of-the-art results in Bayesian word alignment
- Working with huge datasets (tens of GBs)
- Development cycle taking weeks at a time
- Usually using small values for the concentration parameter ($\alpha = 0.1$ is common)

- Inference is slow!
- A common problem with non-parametric techniques
- Possible solutions:
  - Variational inference - an approximate approach
  - Parallel MCMC inference

Parallel inference

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Given a network with many nodes (computers in a network or cores in a cluster), we would like to have an inference that:

- distributes the computational load evenly across the nodes,
- scales favourably with the number of nodes,
- has low overhead in the global steps,
- and converges to the true posterior distribution

Parallel inference in the DP

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- Approximate parallel inference (Asuncion, Smyth, and Welling [2008])
  - Gives slower convergence (Williamson et al., [2013])
- Non-approximate parallel inference using a re-parametrisation of the Dirichlet process
  - Recently suggested, independently, by Lovell, Adams, and Mansingka [2012] and Williamson, Dubey, and Xing [2013]

Parallel inference in the DP

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Given $\alpha > 0$ and base distribution H

$$\gamma \sim \text{Dir}_K(\alpha\mu_1, ..., \alpha\mu_K)$$
$$G_k \sim \text{DP}(\alpha\mu_k, H)$$
$$G = \sum_{k=1}^{K} \gamma_k G_k$$

for some given $(\mu_k)_1^K$ where $\sum_{k=1}^K \mu_k = 1$ and $\mu_k \geq 0$ (one would usually choose $\mu_1 = \frac{1}{K}, ..., \mu_K = \frac{1}{K}$)

- We sample γ to decide how much data to send to each node,
- perform DP inference independently on each one,
- and collect the results at the end

Parallel inference in the DP

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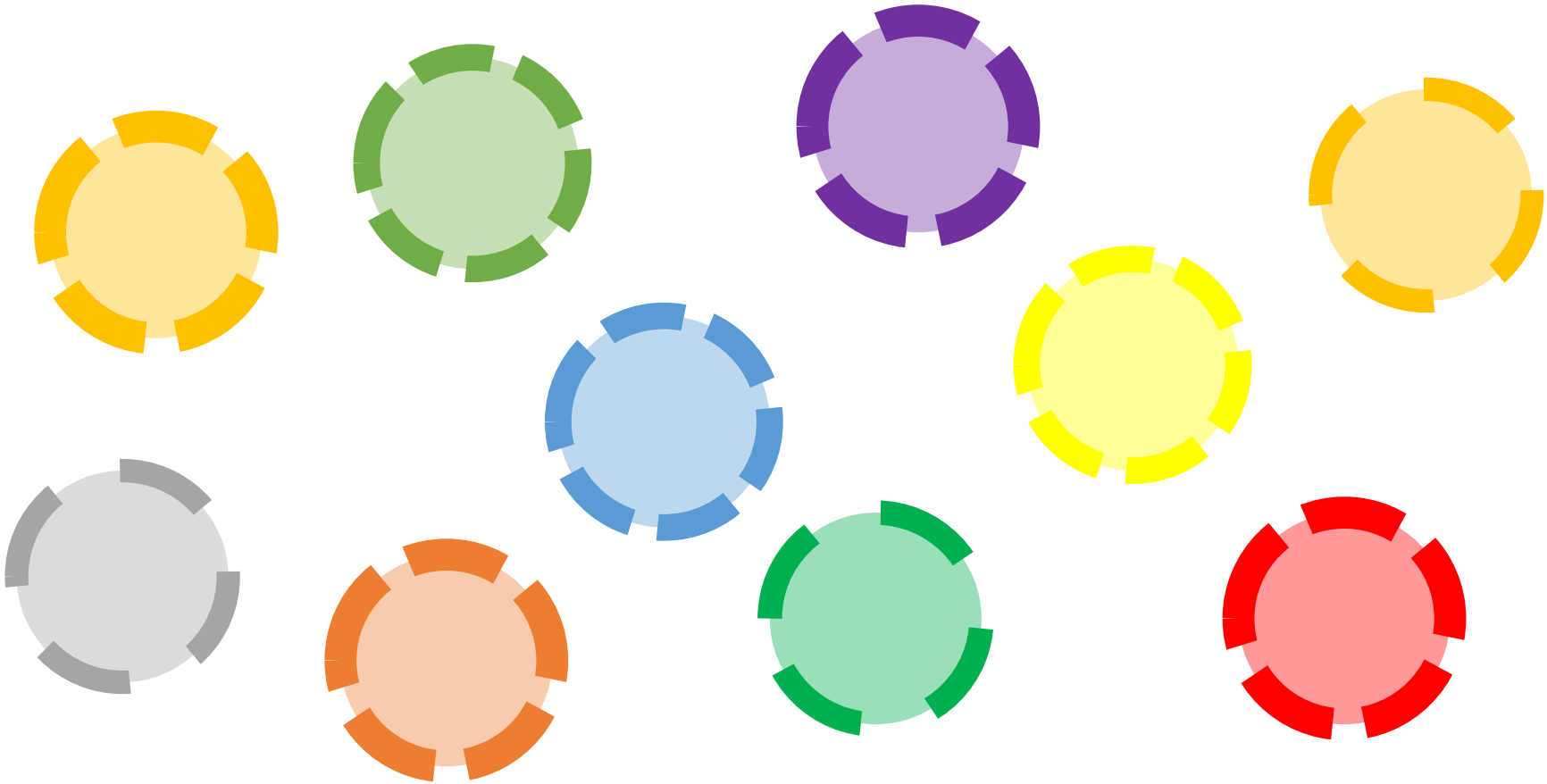
For a network with 10 nodes we split the data using a sample from a Dirichlet distribution with 10 components:



- Each table corresponding to a single node and each customer to a data point sent to that node

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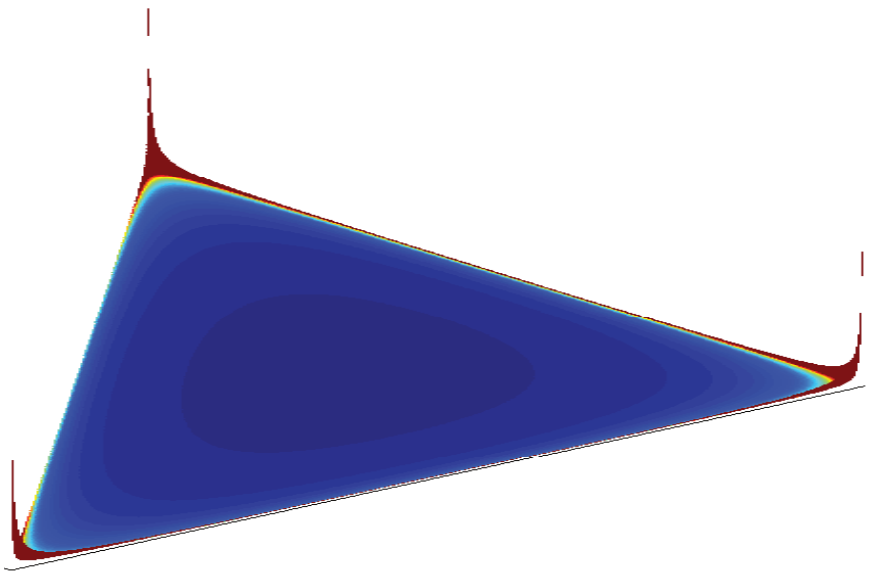
However...

- Samples from the Dirichlet distribution with parameter smaller than 1 have most of the mass concentrated around the corners of the simplex



However...

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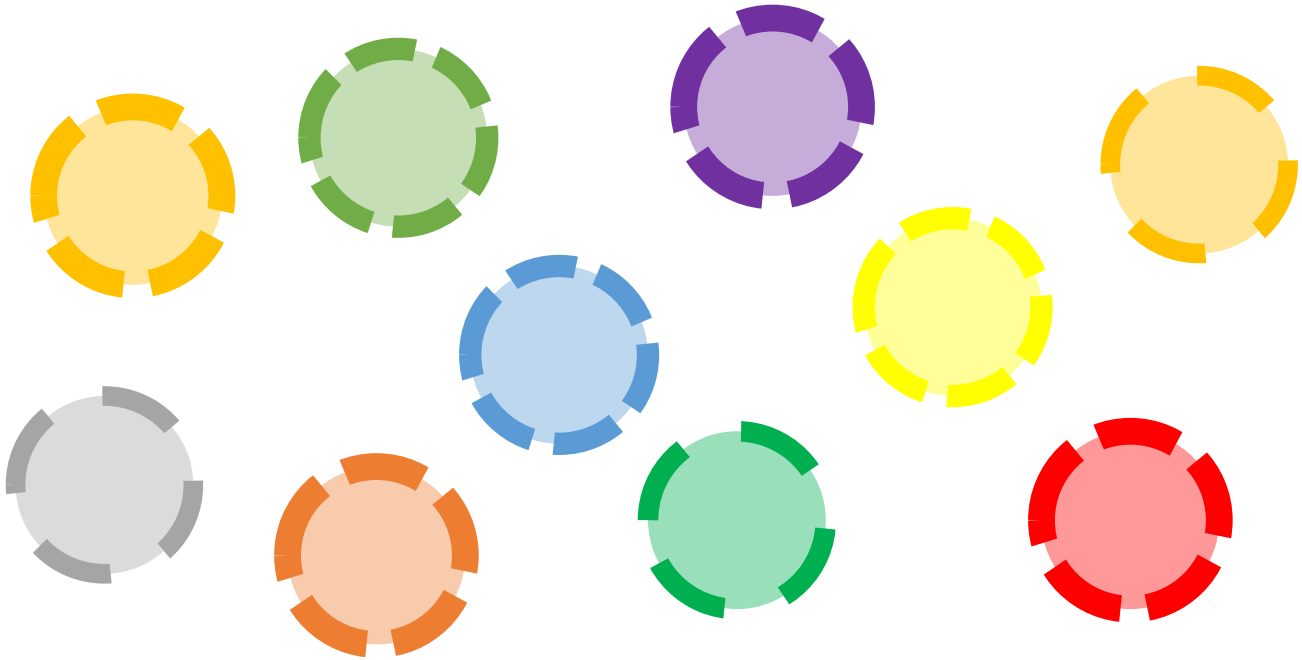
- and in the limit of K we obtain samples from the Dirichlet process with parameter α:

$$\mathrm{DP}(\alpha)$$

- This means that the expected number of nodes used is the same as the expected number of tables in a restaurant with parameter α
  - (we can augment the number of nodes by sending multiple jobs to the same machine)

However...

Actual samples from a Dirichlet process with 50 data points don't look like this:



- The expected number of tables in a restaurant with n customers is given by

$$\alpha \log(n)$$

However...

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So a sample from a Dirichlet process with 50 data points would look more like this:

Which means that only a constant number of nodes, dependent on the number of data points, would be used.

However...

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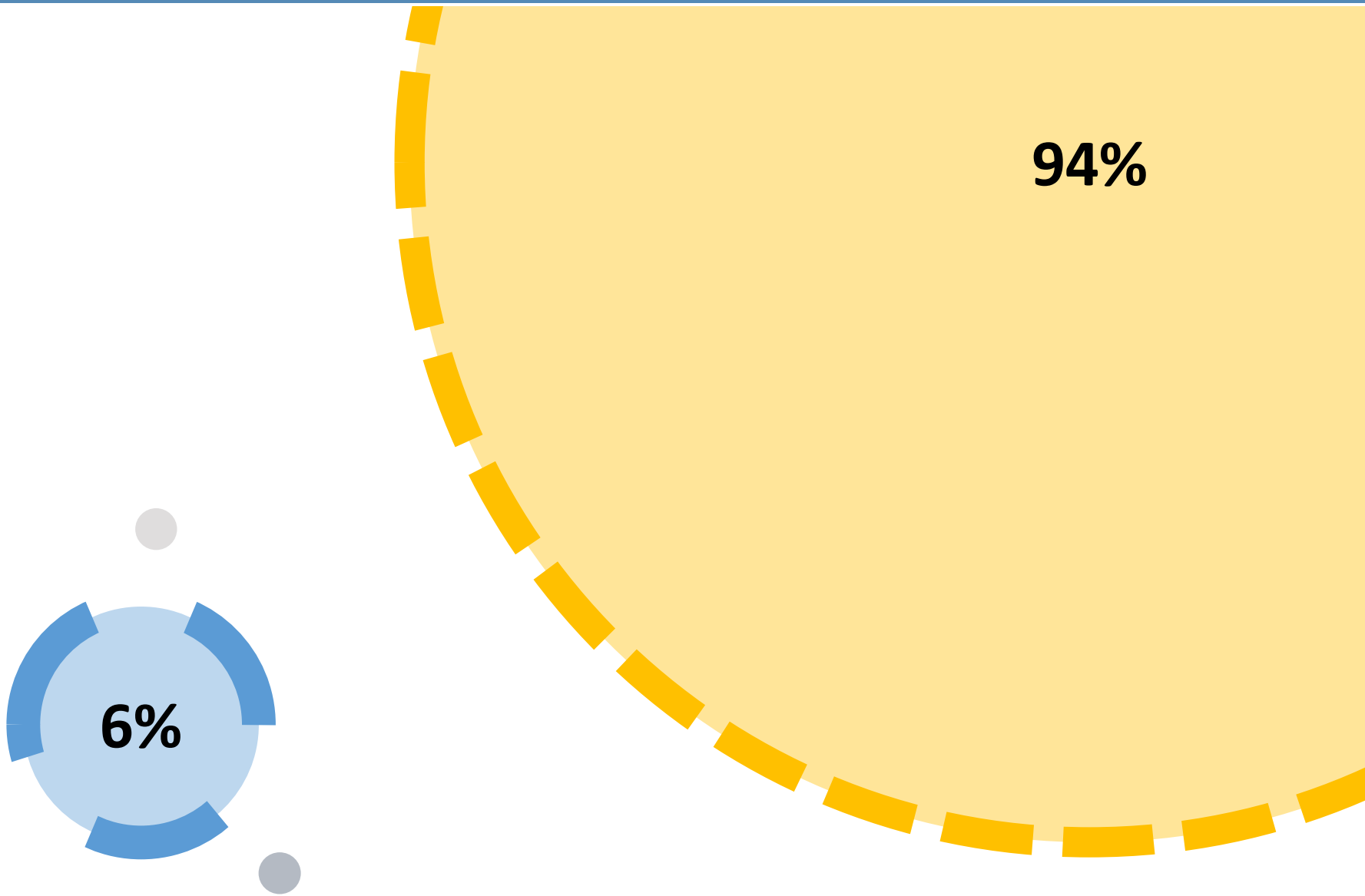
Even worse, the sizes of the different tables follows an exponential decay, so the the number of customers sitting next to each table would actually be

$$C, \quad Cq, \quad Cq^2, \quad Cq^3, \quad \cdots$$

for $q = \dfrac{\alpha}{1+\alpha}$ and $C = \dfrac{1}{1+\alpha}$, so an actual sample would be...

However...



for $n = 50$ data points and $\alpha = 0.1$.

However...

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So for $n = 50$ data points and $\alpha = 0.1$ the parallel inference would send 94% of the data to a single machine.

- Sampling from the *finite* **Dirichlet distribution** with K components (nodes in a network) and different parameter values we get a load balance:

# of nodes	$\alpha = 0.1$	$\alpha = 2$
$K = 10^1$	94%, 6%, 0%, 0%, ...	54%, 23%, 12%, 6%, ...
$K = 10^2$	94%, 6%, 0%, 0%, ...	48%, 22%, 12%, 7%, ...
$K = 10^3$	94%, 6%, 0%, 0%, ...	48%, 21%, 12%, 7%, ...
$K = 10^4$	94%, 6%, 0%, 0%, ...	48%, 21%, 12%, 7%, ...
$K = 10^5$	94%, 6%, 0%, 0%, ...	48%, 21%, 12%, 7%, ...

Figure: Average load on each node in decreasing order

However...

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And in general, for a Dirichlet process with parameter α, 95% of the data for would be sent to

$$\approx \frac{1.3}{\log(\alpha + 1) - \log(\alpha)}$$

nodes,

- independently of the size of the dataset,
- independently of the number of nodes in the network,
- and dependent only on the **parameter used to model the data**

What can we do?

- We can try to initialise the sampler near the posterior
- We might want to use the Pitman–Yor process to distribute the data
- We could use approximate inference with Metropolis–Hastings corrections
- We can develop better approximate inference approaches
- Don’t use the Dirichlet process
- Use a partly-parametric partly-non-parametric approach

What can we do?

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We can try to initialise the sampler near the posterior

- When we know the data has **many clusters** which are **evenly balanced**
- Initialise the sampler randomly with many evenly sized clusters
- ... however still doesn't answer many real-world cases
- ... and the distribution of the clusters between the nodes has the same skewed balance

What can we do?

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We could use the Pitman–Yor process to distribute the data

- This would yield

$$\propto n^{\alpha}$$

  number of tables used

- ... but still an exponential decay in the sizes of the tables

What can we do?

We could also use approximate inference with Metropolis–Hastings corrections

- Was suggested by Doshi, Knowles, Mohamed, and Ghahramani [2009] for the Indian-buffet process
- We sample from the Markov chain and use Metropolis–Hastings corrections to discard some of the samples
- Recently implemented by Chang and Fisher III [2013]
- ... but we should be careful with the global step overhead

What can we do?

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Develop better approximate inference

- Current approach uses Gibbs sampling after distributing the data evenly across the different nodes and in the global step we sync. the state of the nodes (Asuncion, Smyth, and Welling [2008])
- Was reported by Williamson et al., [2013] to have slow convergence

What can we do?

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Don’t use the Dirichlet process

- Recently shown that the Dirichlet process is inconsistent in the number of cluster
- An alternative distribution for clustering has been suggested: using a Poisson distribution mixture of Dirichlet distributions
- Might open the door for more efficient parallel inference

What can we do?

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And finally, use a partly-parametric partly-non-parametric mixture of the Dirichlet-K distribution and Dirichlet process for clustering

- Would allow us to use an unbounded number of clusters with at least K clusters
- The distribution of the load would be partly-balanced

Conclusions —

- Scaling up inference for the Dirichlet process is still an open problem
- ... which has to be solved if we want it to be used in industry and real-world applications!