# Representing and Solving Finite-Domain Constraint Problems using Systems of Polynomials (Extended Abstract)

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In this paper [1] we investigate the use of a system of multivariate polynomials to represent the restrictions imposed by a collection of constraints. A system of polynomials is said to allow a particular combination of values for a set of variables if the simultaneous assignment of those values to the variables makes all of the polynomials in the system evaluate to zero.

The use of systems of polynomials has been considered a number of times in the constraints literature, but is typically used to represent constraints on continuous variables. Here we focus on the use of polynomials to represent finite domain constraints. One advantage of representing such constraints using polynomials is that they can then be treated in a uniform way along with continuous constraints, allowing the development of very general constraint-solving techniques. Systems of polynomials can be processed by standard computer algebra packages such as Mathematica and REDUCE, so our approach helps to unify constraint programming with other forms of mathematical programming.

Systems of polynomials have been widely studied, and a number of general techniques have been developed, including algorithms that generate an equivalent system with certain desirable properties, called a Gröbner Basis. Given a Gröbner Basis, it is possible to obtain the solutions to a system of polynomials very easily (or determine that it has no solutions). A Gröbner Basis provides a convenient representation for the whole set of solutions which can be used to answer a wide range of questions about them, such as the correlations between individual variables, or the total number of solutions.

In general, the complexity of computing a Gröbner Basis for a system of polynomials is doubly-exponential in the size of the system. However, we observe that with the systems we use for constraints over finite domains this complexity is only singly exponential, and so comparable with other search techniques for constraints. Our main contributions are a family of polynomial-time algorithms, related to the construction of Gröbner Bases, which simulate the effect of the local-consistency algorithms used in constraint programming. Finally, we discuss the use of adaptive consistency techniques for systems of polynomials.

## References

1. C. Jefferson, P. Jeavons, M. J. Green, and M. R. C. van Dongen. Representing and solving finite-domain constraint problems using systems of polynomials. *Annals of Mathematics and Artificial Intelligence*, 67(3-4):359–382, 2013.