Word order alternation in Sanskrit via precyclicity in pregroups

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Outline

• Residuated Monoids and Pregroups
• Cyclicity and Precyclicity
• Pregroups Grammars
• The Basic Grammar of Sanskrit
• Word Order Alternation
Residuated Monoids

A residuated monoid $M$ is a partially ordered monoid where the multiplication has a left and a right adjoint.

$$(M, \leq, \cdot, 1, /, \backslash)$$

$$b \leq a \backslash e \iff a \cdot b \leq e \iff a \leq e/b$$

Monoidal Closed Category
A pregroup $P$ is a partially ordered monoid where each element has a left and a right adjoint.

\[(P, \cdot, 1, \leq, (\cdot)^l, (\cdot)^r)\]

\[p^l \cdot p \leq 1 \leq p \cdot p^l\]

\[p \cdot p^r \leq 1 \leq p^r \cdot p\]
Some Properties

Adjoint are order reversing \( p \leq q \implies q^l \leq p^l \)

Opposite adjoints cancel out \( (p^l)^r = (p^r)^l = p \)

Non-opposite adjoints don’t \( \cdots, p^{ll}, p^l, p, p^r, p^{rr}, \cdots \)

Multiplication is self adjoint \( (p \cdot q)^l = q^l \cdot p^l \)
Translation

An element $x$ of a residuated monoid is translated into an element of a pregroup $t(x)$ as follows

\[
\begin{align*}
t(1) &= 1 \\
t(a \cdot b) &= t(a) \cdot t(b) \\
t(a \backslash b) &= t(a)^r \cdot t(b) \\
t(a / b) &= t(a) \cdot t(b)^l
\end{align*}
\]
Cyclicity

A partially ordered monoid is cyclic whenever we have an element $c$ such that for all $a, b$ we have

$$a \cdot b \leq c \iff b \cdot a \leq c$$

If the monoid is residuated, the above becomes equivalent to the following

$$c/a = a\backslash c$$
Dualization

An element $d$ of a residuated monoid is dualizing whenever for all $a, b$ we have

$$(d/a)\setminus d = a = d/(a\setminus d)$$

If the dualizing element is furthermore cyclic, we obtain

$$d/(d/a) = a = (a\setminus d)\setminus d$$
Example

In a residuated lattice monoid (quantale), the bottom of the lattice is used to define a notion of negation.

\[\neg^r a := a \setminus \bot \quad \neg^l a := \bot / a\]

Bottom is cyclic. \[\neg^l a = \neg^r a\]

Bottom is dualizing. \[\neg^r \neg^r a = a \quad \neg^l \neg^l a = a\]

Bottom is cyclic and dualizing. \[\neg^r \neg^l a = a \quad \neg^l \neg^r a = a\]
**Proposition**. In any pregroup, the following hold

\[
\begin{align*}
1 \leq a \cdot b & \quad \overset{(ll)}{\Rightarrow} \quad 1 \leq b \cdot a^{ll} \quad \overset{(rr)}{\Rightarrow} \quad 1 \leq b^{rr} \cdot a \\
1 \leq a \cdot b & \quad \overset{(ll)}{\Rightarrow} \quad 1 \leq b \cdot a^{ll} \quad \overset{(rr)}{\Rightarrow} \quad 1 \leq b^{rr} \cdot a
\end{align*}
\]

**Movement**

**Observation.** In any pregroup, \( l \) is dualizing. If the pregroup is proper, \( l \) is not cyclic.
Pregroup Grammars

A pregroup grammar for a language $L$ is a free pregroup $T(B)$ freely generated over a set $B$ of basic grammatical types of $L$ together with a relation $D$ over the vocabulary $Z$ of $L$ and $T(B)$.

$$D \subseteq Z \times T(B)$$

A string of words of $L$ is a grammatical sentence, whenever the multiplication of the types of the words is below the type $s$.

$$w_1 w_2 \cdots w_n \quad \text{with} \quad t_i \in D(w_i)$$

$$t_1 \cdot t_2 \cdot \cdots \cdot t_n \leq s$$
Precyclic Pregroup Grs

We refer to \( b\alpha^\ll \) and \( b^\rr a \) as precyclic permutations of \( ab \).

A precyclic pregroup grammar is a pregroup grammar with a set of transformations as follows

\[
\begin{align*}
(ll)\text{-transformation} & : A \leq B(ab)C \quad \overset{(ll)}{\sim} \quad A \leq B(b\alpha^\ll)C \\
(rr)\text{-transformation} & : A \leq B(ab)C \quad \overset{(rr)}{\sim} \quad A \leq B(b^\rr a)C
\end{align*}
\]

for each precyclic permutation of \( ab \).
Sanskrit

**Origin**: 2000 BC, Hindi hymns

**Grammar**: 2-4 BC, *Panini*, Rigid set of rules

19th century, *Apte*, European-style

20th century, *Staal*, Parse Trees

21th century, *Gillon*, Generative Rules
Basic Fragment

Sentence Structure

Compounds (two cases)

Inflection (see paper)

Word Order (three cases)
Sentence Structure

Basic types \[ \{ \pi, o, p, n, s \} \]

Subject - Subject-Enlargement - Verb - Object - Object-Enlargement - Adverb

\[
\begin{align*}
\pi & \quad (\pi^r \pi) & \quad (\pi^r s o^l) & \quad o & \quad o^r o & \quad s^r s
\end{align*}
\]

Ramah from the old city saw Súbhadrā (a) beautiful woman.

Ramah \( \text{kapiJjalArma} \) \( \text{apasyat} \) Súbhadrā \( \text{maJjunAzI} \)

\[
\begin{align*}
\pi & \quad (\pi^r \pi) & \quad (\pi^r s o^l) & \quad o & \quad (o^r o)
\end{align*}
\]

Copulars (verb \textit{aste}) are treated differently, see paper.
Compounds

\[ \pi^r \pi \pi^l \cdots \pi^l \]

\[ \pi^r \cdot \pi^r \cdot \pi^r \pi \]

\[ \text{luke} \]

<table>
<thead>
<tr>
<th>Case</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominative</td>
<td>([N_1 \text{ atamane}] [N \text{ pada}])</td>
</tr>
<tr>
<td>Accusative</td>
<td>(\pi \leq \pi^r \pi)</td>
</tr>
</tbody>
</table>

\[ \text{aluke} \]

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<tbody>
<tr>
<td>Nominative</td>
<td>([N \text{ sarva}] [N \text{-jna}])</td>
</tr>
<tr>
<td>Accusative</td>
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</tr>
</tbody>
</table>

Sunday, 25 May 14
**Word Order**

**Staal’s constraints.** Word order is free among the branches of the same constituent.

<table>
<thead>
<tr>
<th>Permutation</th>
</tr>
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<tbody>
<tr>
<td>( \pi^r s \sigma^{(ll)} \overset{\sim}{\rightarrow} s\pi^l )</td>
</tr>
<tr>
<td>( \pi^r s o^l \sigma^{(rr)} \overset{\sim}{\rightarrow} o^r \pi^r s )</td>
</tr>
</tbody>
</table>

Ramah Govindam apasyat.

\[
\pi \quad o \quad (\pi^r s o^l) \overset{(rr)}{\sim} \pi \quad o \quad (o^r \pi^r s) \leq s
\]

apasyat Govindam Ramah.

\[
(\pi^r s o^l) \quad o \quad \pi \quad \leq (\pi^r s) \quad \pi \overset{\ll}{\sim} (s \pi^l) \quad \pi \leq s
\]
Word Order

Gillon’s conjecture. More complex sentences allow for other types of alternation.

\[(\pi^r \pi) \quad (\pi^r s \pi^l) \quad o \quad \pi \leq (\pi^r \pi)(\pi^r s)\pi \overset{(ll)}{\sim} (\pi\pi^l)(\pi^r s)\pi \]

\[(\pi^l \pi^l \pi^l \pi^l \pi) \pi \overset{(ll)}{\sim} (\pi\pi^l)(\pi^r s)^{ll} \leq \pi(\pi^r s)^{ll} \]

\[(\pi^r s)\pi \overset{(ll)}{\sim} (s\pi^l)\pi \leq s \]
Future Work

Sequent Calculus

Precyclic Permutations

Precyclic Transformations