Word order alternation in Sanskrit via precyclicity in pregroups

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Outline

- Residuated Monoids and Pregroups
- Cyclicity and Precyclicity
- Pregroups Grammars
- The Basic Grammar of Sanskrit
- Word Order Alternation

Residuated Monoids

A residuated monoid M is a partially ordered monoid where the multiplication has a left and a right adjoint.

$$(M, \leq, \cdot, 1, /, \backslash)$$

$$b \leq a \backslash e \quad \Leftrightarrow \quad a \cdot b \leq e \quad \Leftrightarrow \quad a \leq e/b$$

Monoidal Closed Category

Pregroups

A pregroup P is a partially ordered monoid where the each element has a left and a right adjoint.

$$(P, \cdot \ , 1, \leq, ()^l, ()^r)$$

$$p^l \cdot p \leq 1 \leq p \cdot p^l \qquad\qquad p \cdot p^r \leq 1 \leq p^r \cdot p$$

Compact Closed Category

Some Properties

Adjoint are order reversing $p \leq q \implies q^l \leq p^l$

Opposite adjoints cancel out $(p^l)^r = (p^r)^l = p$

Non-opposite adjoints don't $\cdots, p^{ll}, p^l, p, p^r, p^{rr}, \cdots$

Multiplication is self adjoint $(p \cdot q)^l = q^l \cdot p^l$

Translation

An element *x* of a residuated monoid is translated into an element of a pregroup *t(x)* as follows

$$t(1) = 1$$

$$t(a \cdot b) = t(a) \cdot t(b)$$

$$t(a \backslash b) = t(a)^r \cdot t(b)$$

$$t(a/b) = t(a) \cdot t(b)^l$$

Cyclicity

A partially ordered monoid is *cyclic* whenever we have an element *c* such that for all *a*,*b* we have

$$a \cdot b \leq c \implies b \cdot a \leq c$$

If the monoid is residuated, the above becomes equivalent to the following

$$c/a \ = \ a\backslash c$$

Dualization

An element *d* of a residuated monoid is *dualizing* whenever for all *a,b* we have

$$(d/a)\backslash d \;=\; a \;=\; d/(a\backslash d)$$

If the dualizing element is furthermore cyclic, we obtain

$$d/(d/a) \;=\; a \;=\; (a\backslash d)\backslash d$$

Example

In a residuated lattice monoid (quantale), the bottom of the lattice is used to define a notion of negation.

$$\neg^r a := a \setminus \bot \qquad \neg^l a := \bot / a$$

Bottom is cyclic. $\neg^l a = \neg^r a$

Bottom is dualizing. $\neg^r \neg^r a = a \qquad \neg^l \neg^l a = a$

Bottom is cyclic and dualizing. $\neg^r \neg^l a = a \qquad \neg^l \neg^r a = a$

Precyclicity

Observation. *In any pregroup 1 is dualizing. If the pregroup is proper, 1 is not cyclic.*

Proposition. *In any pregroup, the following hold*

$$a \cdot b \leq 1 \overset{(ll)}{\Longrightarrow} b^{ll} \cdot a \leq 1 \qquad a \cdot b \leq 1 \overset{(rr)}{\Longrightarrow} b \cdot a^{rr} \leq 1$$

$$1 \leq a \cdot b \overset{(ll)}{\Longrightarrow} 1 \leq b \cdot a^{ll} \qquad 1 \leq a \cdot b \overset{(rr)}{\Longrightarrow} 1 \leq b^{rr} \cdot a$$

Movement

Pregroup Grammars

A pregroup grammar for a language ***L*** is a free pregroup *T(**B**)* freely generated over a set ***B*** of basic grammatical types of ***L*** together with a relation ***D*** over the vocabulary ***Z*** of ***L*** and *T(**B**)*.

$$\mathbf{D} \subseteq \mathbf{Z} \times \mathrm{T}(\mathbf{B})$$

A string of words of ***L*** is a grammatical sentence, whenever the multiplication of the types of the words is below the type *s*.

$w_1 w_2 \cdots w_n$ $\qquad$ $t_i \in D(w_i)$

$$t_1 \cdot t_2 \cdot \ \cdots \ \cdot t_n \leq s$$

Precyclic Pregroup Grs

We refer to ba^{ll} and $b^{rr}a$ as *precyclic permutations* of ab.

A *precyclic pregroup grammar* is a pregroup grammar with a set of transformations as follows

$$(ll)\text{-}\mathbf{transformation} \quad A \leq B(ab)C \overset{(ll)}{\rightsquigarrow} A \leq B(ba^{ll})C$$
$$(rr)\text{-}\mathbf{transformation} \quad A \leq B(ab)C \overset{(rr)}{\rightsquigarrow} A \leq B(b^{rr}a)C$$

for each precyclic permutation of ab.

Sanskrit

Origin: 2000 BC, Hindi hymns

Grammar: 2-4 BC, *Panini*, Rigid set of rules

19th century, *Apte*, European-style

20th century, *Staal*, Parse Trees

21th century, *Gillon*, Generative Rules

Basic Fragment

Sentence Structure

Compounds (two cases)

Inflection (see paper)

Word Order (three cases)

Sentence Structure

Basic types $\{\pi, o, p, n, s\}$

Subject	Subject-Enlargement	Verb	Object	Object-Enlargement	Adverb
π	$(\pi^r \pi)$	$(\pi^r s o^l)$	o	$o^r o$	$s^r s$

Ramah	from the old city	saw	Súbhadrā	(a) beautiful woman.
Ramah	kapiJjalArma	apasyat	Súbhadrā	maJjunAzI
π	$(\pi^r \pi)$	$(\pi^r s\, o^l)$	o	$(o^r o)$

Copulars (verb aste) are treated differently, see paper.

Compounds

luke

$$\pi^r \pi \underbrace{\pi^l \cdots \pi^l}_{k-1}$$

aluke

$$\underbrace{\pi^r \cdots \pi^r}_{k-1} \pi^r \pi$$

$[[_{N1}$ atamane$]$	$[_N$ pada$]]$	$\rightarrow$	$[_N$ atamanepada$]$
$\pi^r\ \pi\ \pi^l$	π	$\leq$	$\pi^r\ \pi$
oneself	voice		voice for oneself

$[_N[_N$ sarva$]$	$[_N$ -jna$]]$	$\rightarrow$	$[_N$ sarvajna$]$
π	$\pi^r\pi^r\pi$	$\leq$	$\pi^r\pi$
all	knowing		omniscient

Word Order

***Staal's* constraints.** Word order is free among the branches of the same constituent.

[Ramah] [apasyat Govindam.]

Permutation
$\pi^r s \overset{\sigma(ll)}{\rightsquigarrow} s\pi^l$
$\pi^r s\, o^l \overset{\sigma(rr)}{\rightsquigarrow} o^r \pi^r s$

Ramah Govindam apasyat.

$$\pi \quad o \quad (\underline{\pi^r s o^l}) \overset{(rr)}{\rightsquigarrow} \pi\ o\ (o^r \pi^r s) \leq s$$

apasyat Govindam Ramah.

$$(\pi^r\ \mathrm{s}\ o^l) \quad \mathrm{o} \quad \pi \quad \leq \underline{(\pi^r s)}\ \pi \overset{(ll)}{\rightsquigarrow} (\mathrm{s}\ \pi^l)\ \ \pi \leq \mathrm{s}$$

Word Order

Gillon's conjecture. More complex sentences allow for other types of alternation.

Rama	from the old city	apasyat	Govinda.
Ramah	kapiJjalArma	saw	Govindam.

from the old city	saw	Govinda	Rama.
kapiJjalArma	apasyat	Govindam	Ramah.

$$(\pi^r \pi) \quad (\pi^r s o^l) \quad o \quad \pi \quad \leq \underline{(\pi^r \pi)}(\pi^r s)\pi \overset{(ll)}{\rightsquigarrow} (\pi\pi^l)\underline{(\pi^r s)\pi}$$

$$\overset{(ll)}{\rightsquigarrow} (\pi\pi^l)\pi(\pi^r s)^{ll} \leq \underline{\pi(\pi^r s)^{ll}}$$

$$\overset{(rr)}{\rightsquigarrow} (\underline{\pi^r s})\pi \overset{(ll)}{\rightsquigarrow} (s\pi^l)\pi \leq s$$

Future Work

Sequent Calculus

Precyclic Permutations

Precyclic Transformations