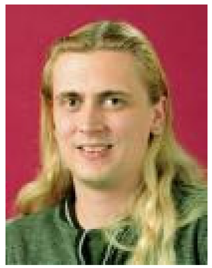
What is the quantum state?

Jonathan Barrett
QISW, Oxford, March 2012

Matt Pusey



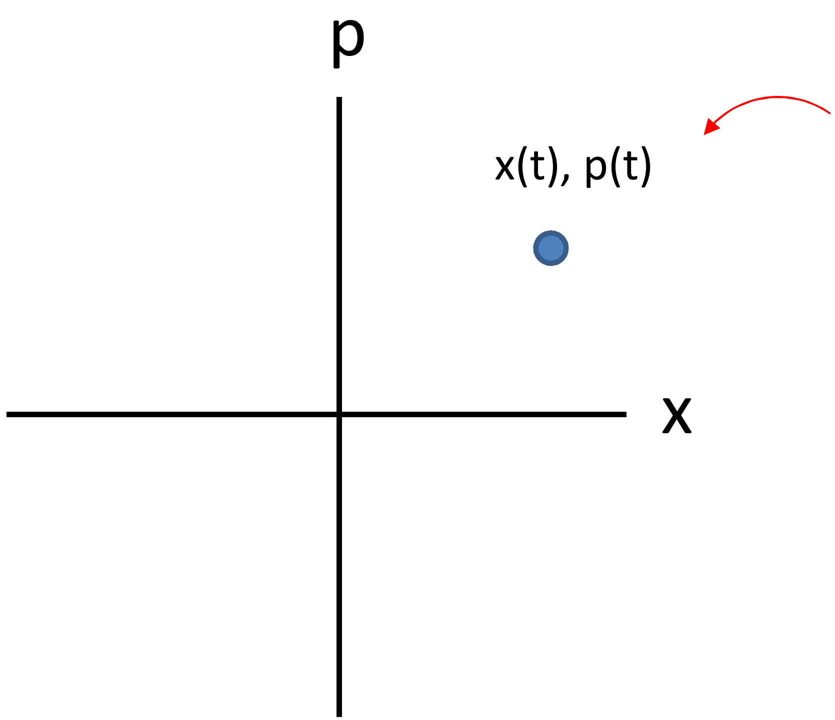
Terry Rudolph

But our present QM formalism is not purely epistemological; it is a peculiar mixture describing in part realities of Nature, in part incomplete human information about Nature --- all scrambled up by Heisenberg and Bohr into an omelette that nobody has seen how to unscramble. Yet we think that the unscrambling is a prerequisite for any further advance in basic physical theory. **For, if we cannot separate the subjective and objective aspects of the formalism, we cannot know what we are talking about; it is just that simple.**

E. T. Jaynes

Classical Mechanics

- Consider a single particle in 1 dimension.
- Particle has position and momentum. State of particle is completely detemined by the values of x,p.
- Other physical properties of the particle are functions of x,p, e.g., energy H(x,p).



State of system at time t is a point in phase space.

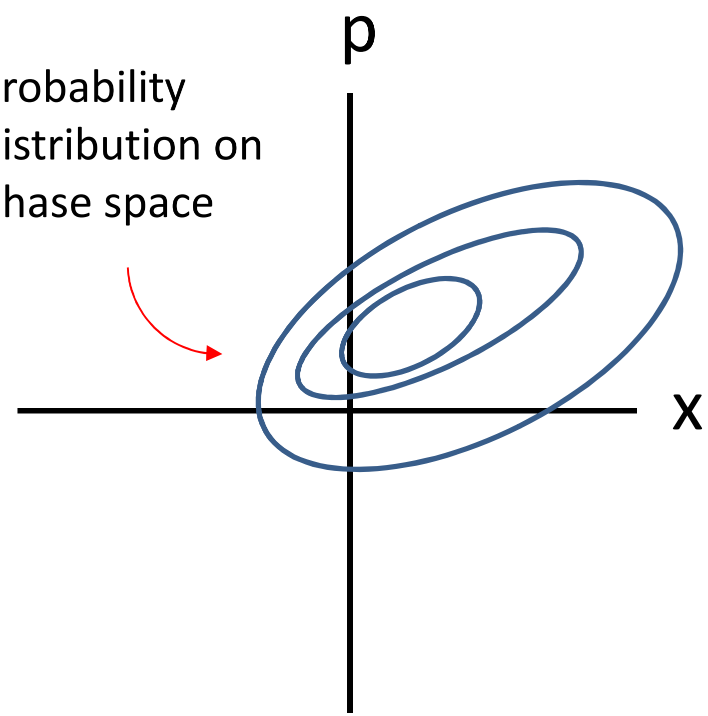
Motion determined by Hamilton's equations

$$
\begin{aligned}
\dot{q} &= \frac{\partial H}{\partial p} \\
\dot{p} &= -\frac{\partial H}{\partial q}
\end{aligned}
$$

Liouville Mechanics

• Sometimes we don't know the exact microstate of a classical system.
• The information we have defines a probability distribution ρ over phase space.
• ρ is not a physical property of the particle. The particle occupies a definite point in phase space and does not care what probabilities I have assigned to different states.

Probability distribution on phase space



Evolution of the probability distribution is given by the Liouville equation:

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + \sum_{i=1}^{n} \left(\frac{\partial \rho}{\partial q^i} \dot{q}^i + \frac{\partial \rho}{\partial p_i} \dot{p}_i \right) = 0.$$

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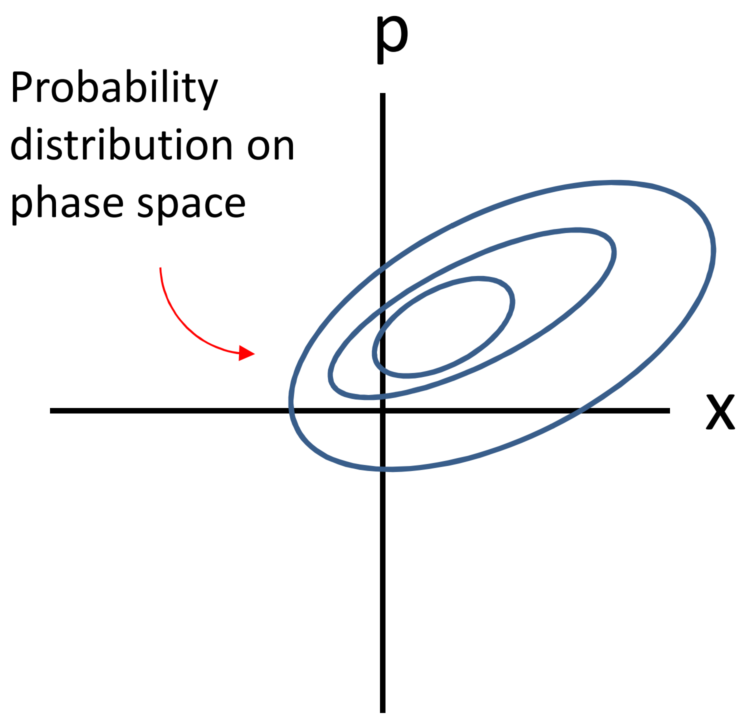


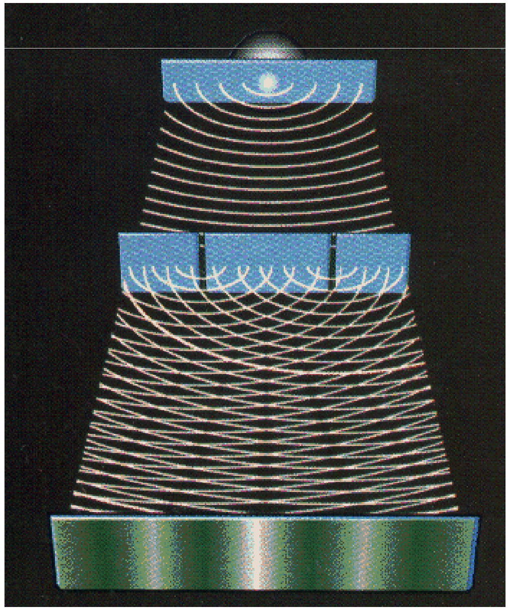
Terminology:

(x,p) *ontic state*
ρ *epistemic state*

What is the quantum state?

Ontic ?

- A quantum wave function is a *real physical wave*.
- Quantum interference most easily understood this way.
- Defined on configuration space ??



What is the quantum state?

Epistemic ?

- A quantum state encodes an experimenter's *knowledge* or *information* about some aspect of reality.

Arguments for ψ being epistemic

Collapse! ⟶ just Bayesian updating

The wave function is not a thing which lives in the world. It is a tool used by the theory to make those inferences from the known to the unknown. Once one knows more, the wave function changes, since it is only there to reflect within the theory the knowledge one assumes one has about the world.
-----Bill Unruh

Arguments for ψ being epistemic

- Non-orthogonal quantum states cannot reliably be distinguished – just like probability distributions.

- Quantum states are exponential in the number of systems – just like probability distributions.

- Quantum states cannot be cloned, can be teleported etc – just like probability distributions.

I will show that...

- If ψ merely represents information about the objective physical state of a system, then predictions are obtained that contradict quantum theory.

In more detail, suppose that...

- A system has an ontic state -- an objective physical state, independent of the experimenter, and independent of which measurement is performed. Call this state λ.

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$$\Pr(k|M,\lambda)$$

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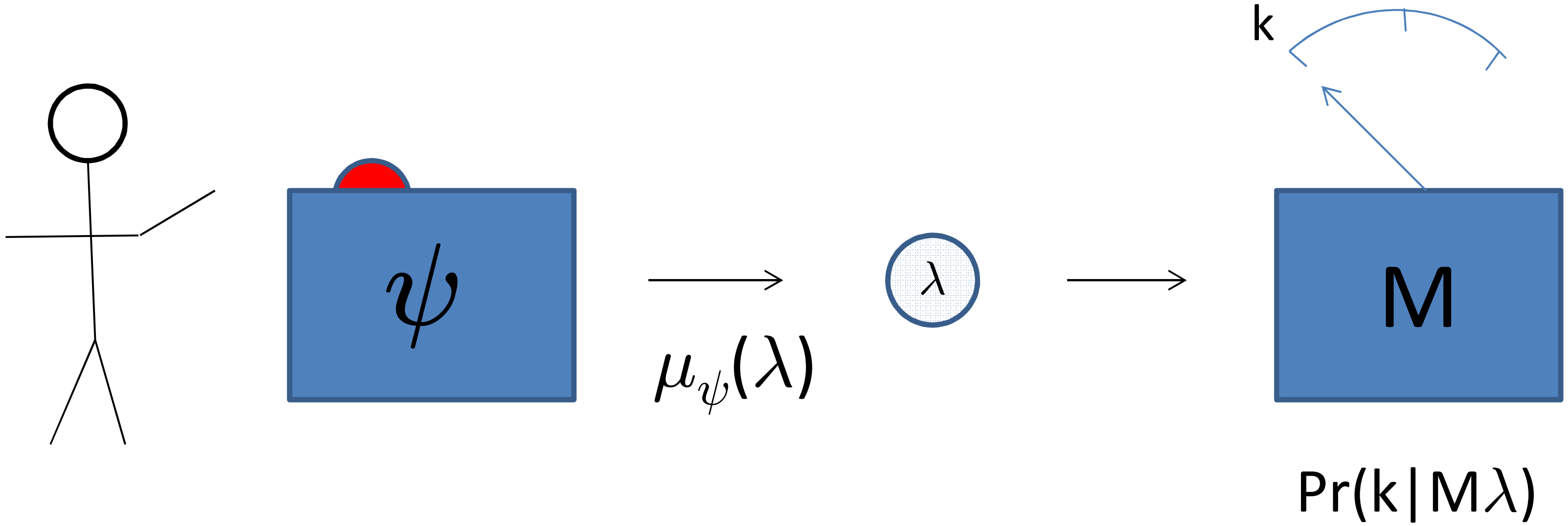
Pr(k|M,λ)

- A quantum state ψ describes an experimenter's information about λ

ψ corresponds to a distribution $\mu_\psi(\lambda)$



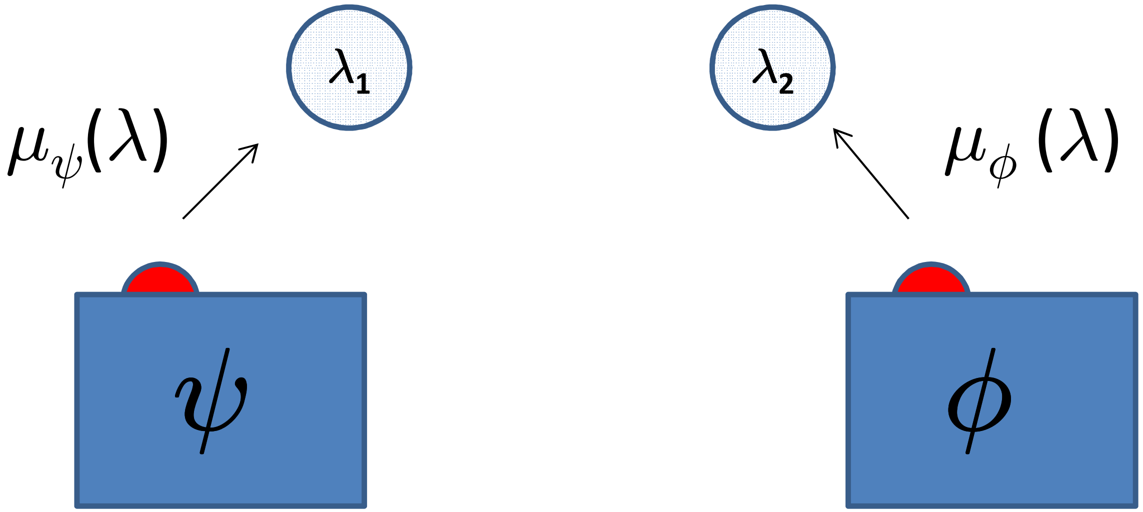
Recover quantum predictions:

$$|\langle\psi|k\rangle|^2 = \int Pr(k|M,\lambda)\mu_\psi(\lambda)d\lambda$$

So far these assumptions are similar to those of Bell's theorem...
But I will not assume locality. Instead assume

Preparation independence

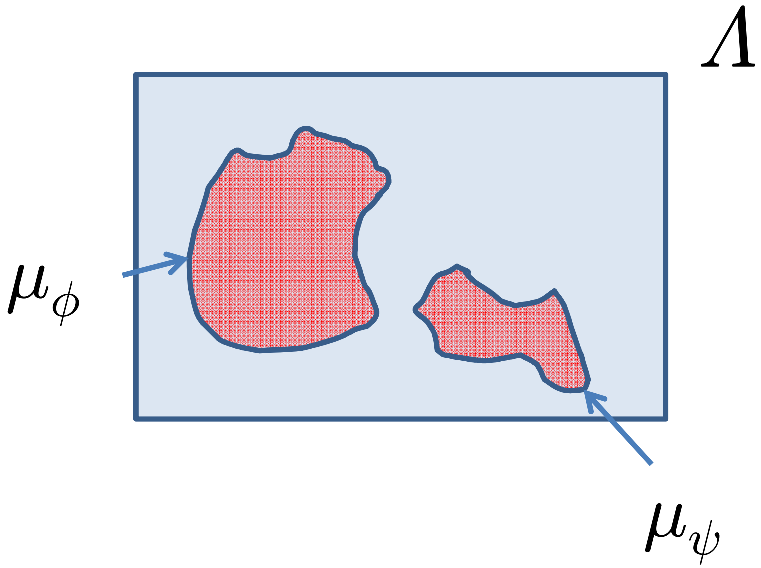
- Consider independent preparations, of quantum states ψ and ϕ, producing $\psi \otimes \phi$



- Overall distribution is $\mu_{\psi\otimes\phi}(\lambda_1,\lambda_2) = \mu_\psi(\lambda_1)\times \mu_\phi(\lambda_2)$

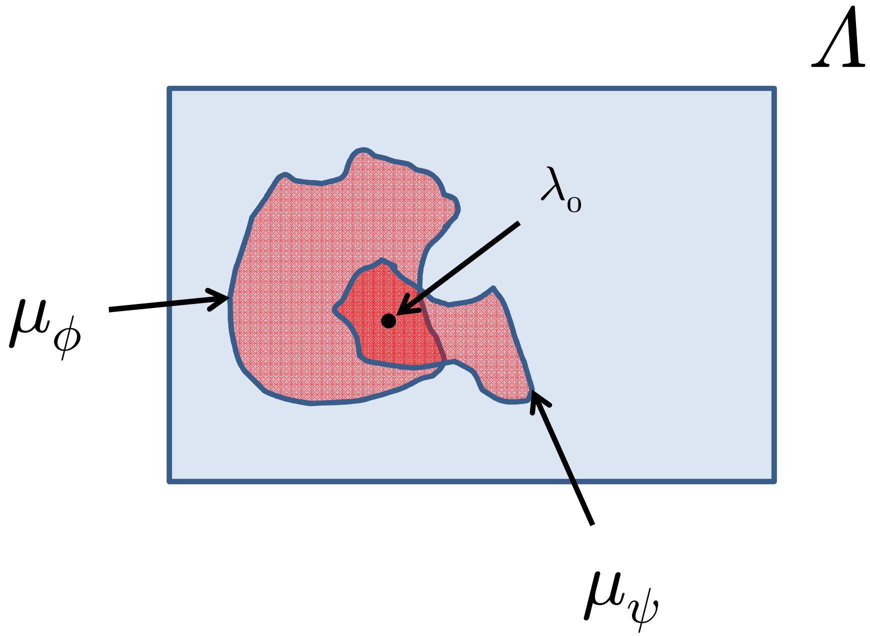
The ψ-ontic case

Suppose that for every pair of distinct quantum states ϕ and ψ, the distributions μ_ϕ and μ_ψ do not overlap:



- The quantum state can be inferred from the ontic state.
- The quantum state is a **physical property** of the system, and is not mere information.

The ψ-epistemic case



- μ_ϕ and μ_ψ can overlap.
- Given the ontic state λ_0 above, cannot infer whether the quantum state ϕ or ψ was prepared.

These distinctions were first made rigorously by:

Harrigan and Spekkens, Found. Phys. 40, 125 (2010).
L. Hardy, priv. comm.

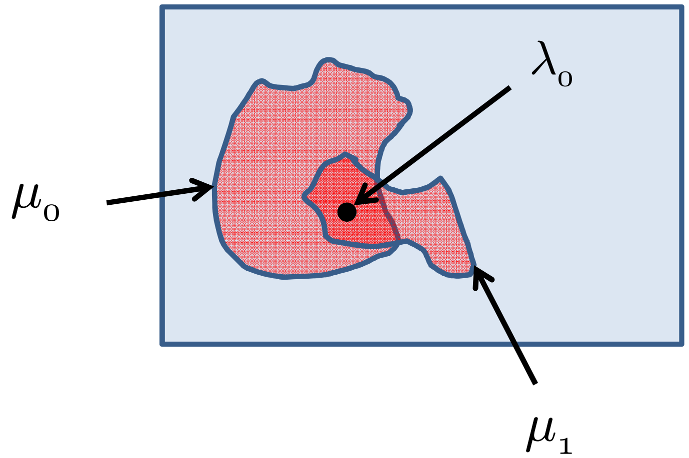
See also:

Montina, Phys. Rev. A 77, 022104 (2008).

A no-go theorem

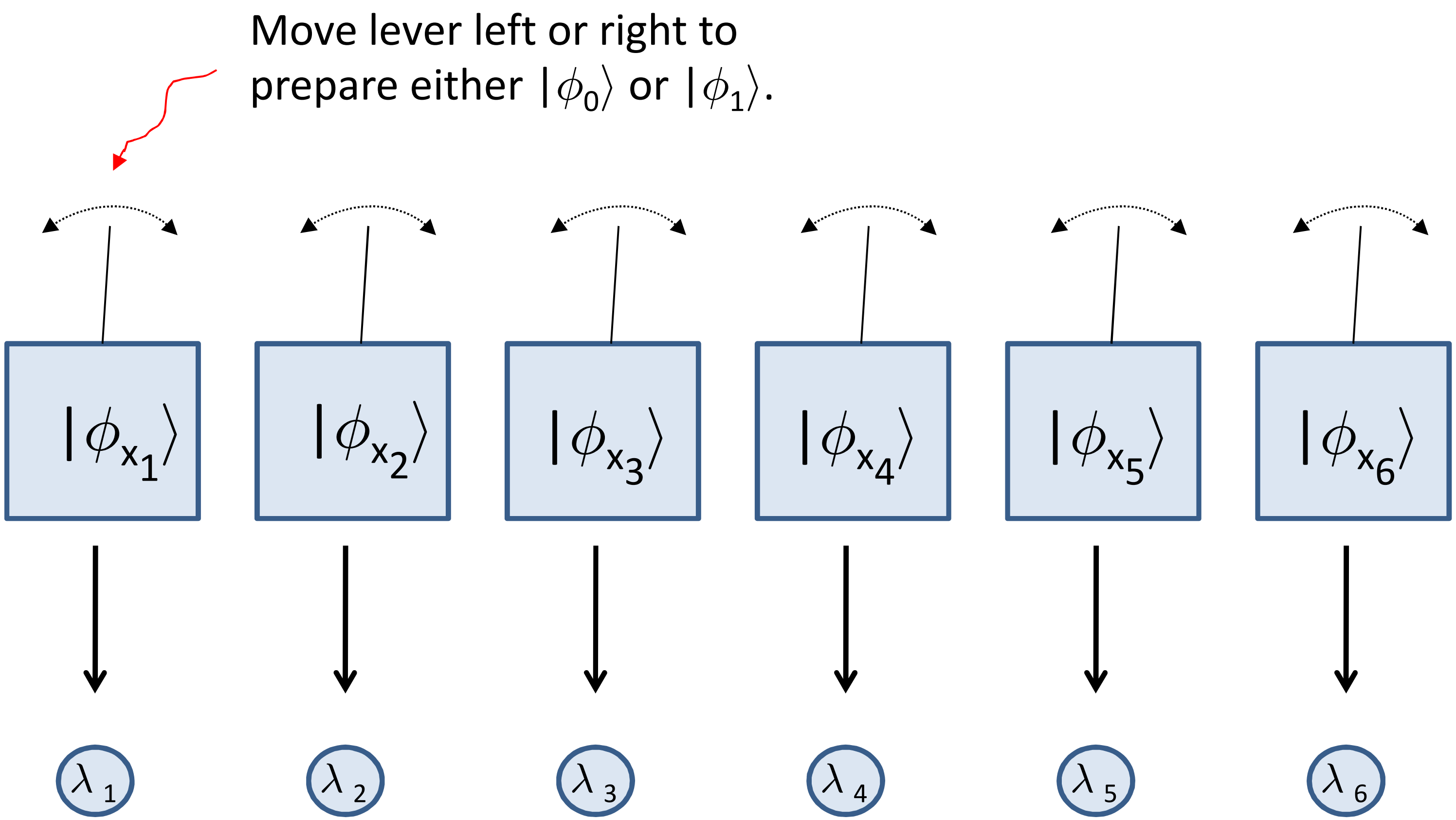
Suppose there are distinct quantum states ϕ_0 and ϕ_1, and an ontic state λ_0 such that:

$\Pr(\lambda_0 \mid \phi_0) \geq q > 0,$
$\Pr(\lambda_0 \mid \phi_1) \geq q > 0.$

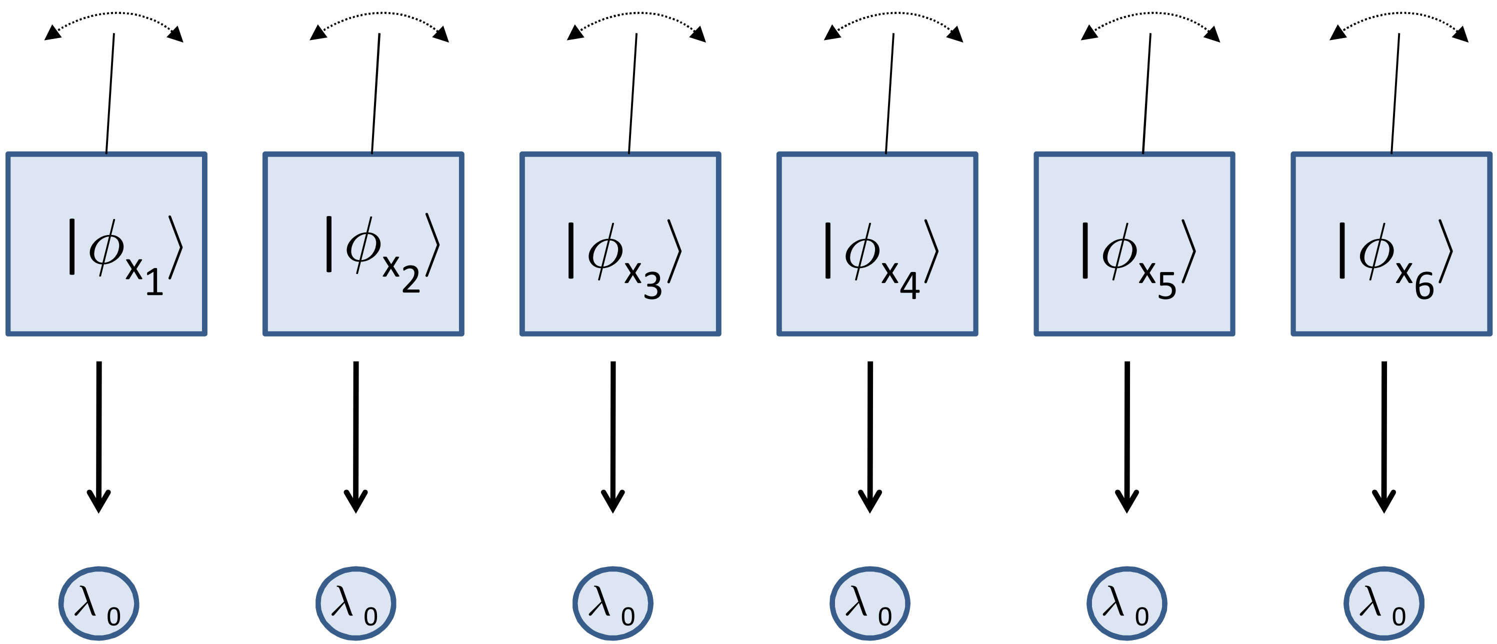


Prepare n systems independently...

- Each is prepared in either the state $|\phi_0\rangle$ or the state $|\phi_1\rangle$.
- 2^n possible joint states: $|\phi_{x_1}\rangle \otimes |\phi_{x_2}\rangle \otimes \cdots \otimes |\phi_{x_n}\rangle$



For any $|\phi_{x_1}\rangle \otimes |\phi_{x_2}\rangle \otimes \cdots \otimes |\phi_{x_n}\rangle$ there is some chance that every one of the n systems has the ontic state λ_0 .



$$\Pr(\lambda_0 \times \lambda_0 \times \cdots \times \lambda_0) \geq q^n$$

- **Now here's the problem...**

A `*PP-measurement*`
Cf Caves, Fuchs, Schack, Phys. Rev. A **66**, 062111 (2002).

- For large enough n there is an entangled measurement across the n systems, with 2^n outcomes corresponding to projectors $P_1, \ldots, P_{2^n}$ and

$$\langle\phi_0|\otimes\cdots\otimes\langle\phi_0|\otimes\langle\phi_0| \quad P_1 \quad |\phi_0\rangle\otimes\cdots\otimes|\phi_0\rangle\otimes|\phi_0\rangle = 0$$
$$\langle\phi_0|\otimes\cdots\otimes\langle\phi_0|\otimes\langle\phi_1| \quad P_2 \quad |\phi_0\rangle\otimes\cdots\otimes|\phi_0\rangle\otimes|\phi_1\rangle = 0$$
$$\vdots$$
$$\langle\phi_1|\otimes\cdots\otimes\langle\phi_1|\otimes\langle\phi_1| \quad P_{2^n} \quad |\phi_1\rangle\otimes\cdots\otimes|\phi_1\rangle\otimes|\phi_1\rangle = 0$$

- For any of the preparations there is a non-zero probability that the ontic state is $\lambda_o \times \cdots \times \lambda_o$.

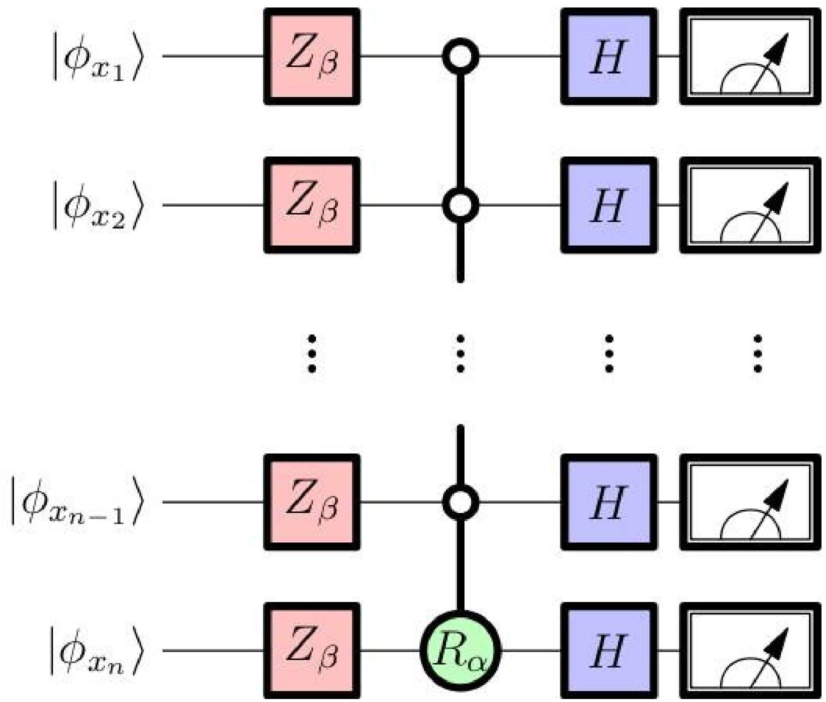
- Must have $\Pr(P_i | \lambda_o \times \cdots \times \lambda_o) = 0$ for any i. But probs must sum to 1!

The measurement

Choose n such that $2^{1/n} - 1 \leq \tan(\theta/2)$.

Wlog, write $|\phi_0\rangle = \cos(\theta/2)\ |0\rangle - \sin(\theta/2)\ |1\rangle$

$|\phi_1\rangle = \cos(\theta/2)\ |0\rangle + \sin(\theta/2)\ |1\rangle$



$$Z_\beta = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\beta} \end{pmatrix}$$

$$R_\alpha|00\cdots0\rangle = e^{i\alpha}|00\cdots0\rangle$$
$$R_\alpha|b\rangle = |b\rangle,$$

on all other basis states $|b\rangle$.

Approximate case

Suppose that in a real experiment, the measured probabilities are within ϵ of the quantum predictions. Then

$$\delta(\mu_0, \mu_1) \geq 1 - 2\sqrt[n]{\epsilon}$$

Classical trace distance

A comparison

Bell's theorem

Systems have an objective physical state

Experimenter free will

Quantum theory

Nonlocality

New theorem

Systems have an objective physical state

Preparation independence

Quantum theory

ψ-ontic

What now?

- A quantum state is not “experimenter’s information about the objective physical state of a system”.

3 possibilities

Systems don’t have “objective physical states”. Quantum state is “experimenter’s information about measurement outcomes”.

The state vector is a physical property of a quantum system.

Collapse is mysterious.
S’s cat is mysterious.

Undercut the assumptions of the theorem.

Retrocausal influences?
Relational properties?