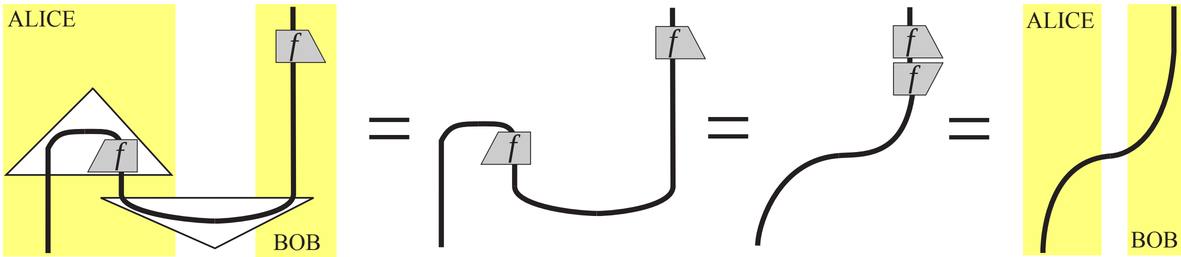
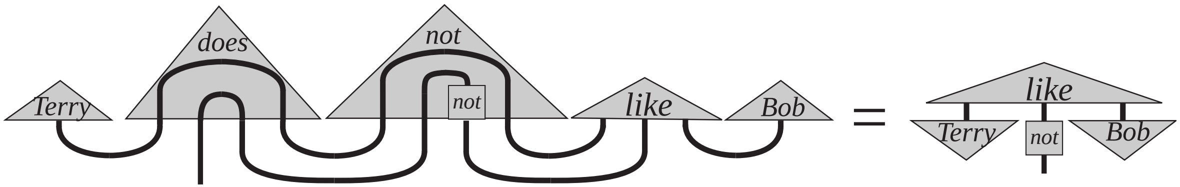
Graphical and Automated Reasoning for Quantum Algorithms and Protocols

QISW — Mar. 2012





Bob Coecke

Oxford University, Computer Science, Quantum Group

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Schrödinger (1935): the stuff which is the true soul of quantum theory is **'how quantum systems compose'**.

$$\frac{\texttt{tensor product structure}}{\texttt{the other stuff}} = \textcolor{red}{?}$$

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$\Rightarrow$ ***Framework for Generalized Process Theories***

$\Rightarrow$ ***Operational bones for Quantum Foundations***

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1. Game plan: Which assumptions (i.e. which structure) on $\otimes$ is needed to deduce **physical phenomena**?

2. Additional question: Does such an interaction structure appear elsewhere in **"our classical reality"**?

Outcome 1a: “Sheer ratio of results to assumptions” confirms that we are probing something very essential.

Hans Halvorson (2010) Editorial to: *Deep Beauty: Understanding the Quantum World through Mathematical Innovation*, Cambridge University Press.

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Outcome 1b: Exposing this structure has already helped to **solve open problems elsewhere**.

Outcome 1c: **Simple intuitive (but rigorous) diagrammatic language**, meanwhile adopted by others:

"... we join the ***quantum picturalism*** revolution [1]"

Lucien Hardy (2010) *A formalism-local framework for general probabilistic theories including quantum theory.* arXiv:1005.5164

---

[1] Coecke (2010) *Quantum picturalism.* Contemporary Physics **51**, 59–83. `arXiv:0908.1787` (survey)

— R. Duncan & S. Perdrix (2010) *Rewriting measurement-based quantum computations with generalised flow.* **ICALP**.

⇒ **Ross Duncan's talk**

— B. Coecke & A. Kissinger (2010) *The compositional structure of multipartite quantum entanglement.* **ICALP**. arXiv:1002.2540.

⇒ **DEMO**

— C. Horsman (2011) *Quantum picturalism for topological cluster-state computing.* **NJP**. arXiv:1101.4722.

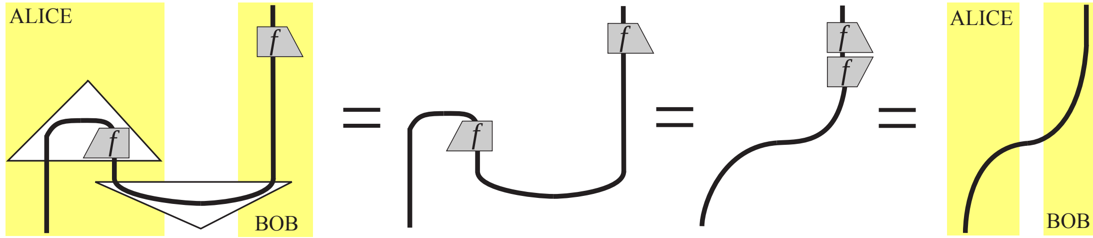
— S. Boixo & C. Heunen (2012) *Entangled and sequential quantum protocols with dephasing.* **PRL**. arXiv:1108.3569

— B. Coecke, R. Duncan, A. Kissinger & Q. Wang (2012) *Strong complementarity and non-locality in categorical quantum mechanics*. **LiCS**. arXiv:1203.4988

⇒ **Aleks Kissinger's talk**

Outcome 2a:

Behaviors of matter:



Abramsky & Coecke (2004) *A categorical semantics of quantum protocols*. LiCS'04. `arXiv:quant-ph/0402130`

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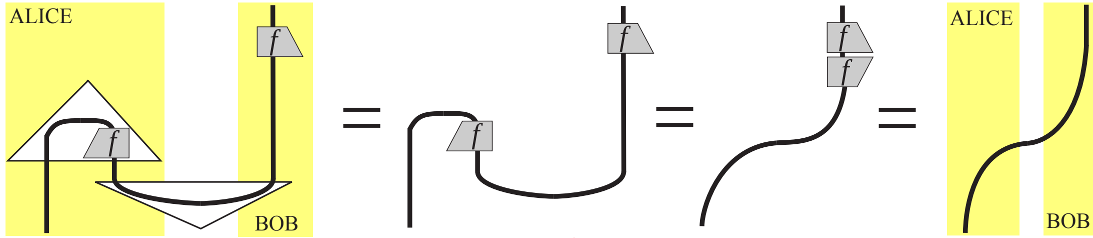
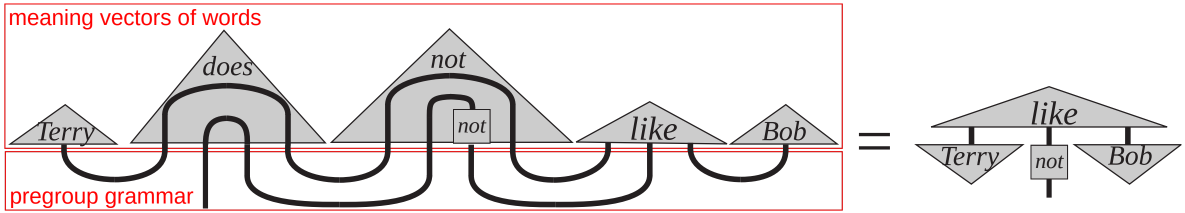


Meaning in language:



(December 2010)

Coecke, Sadrzadeh & Clark (2010) *Mathematical Foundations for a Compositional Distributional Model of Meaning*. `arXiv:1003.4394`

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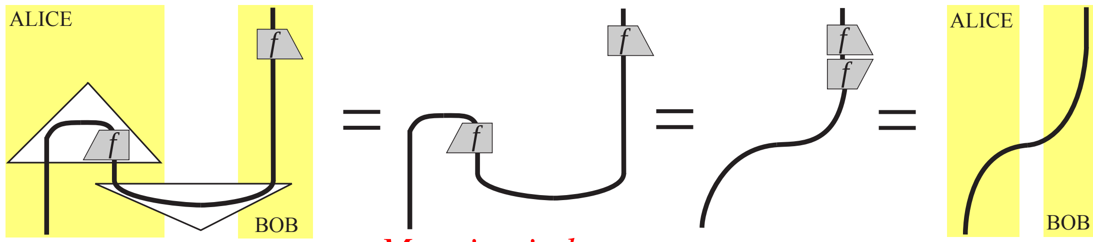

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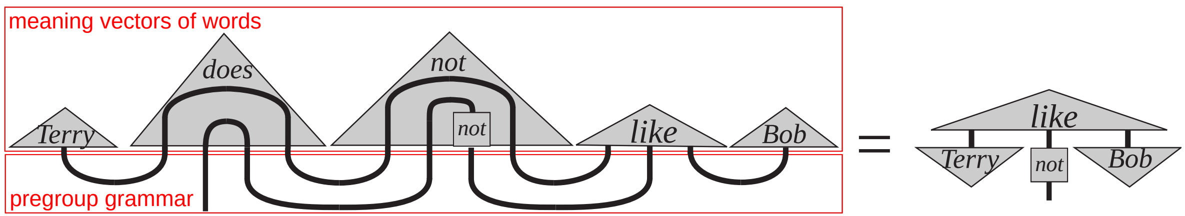
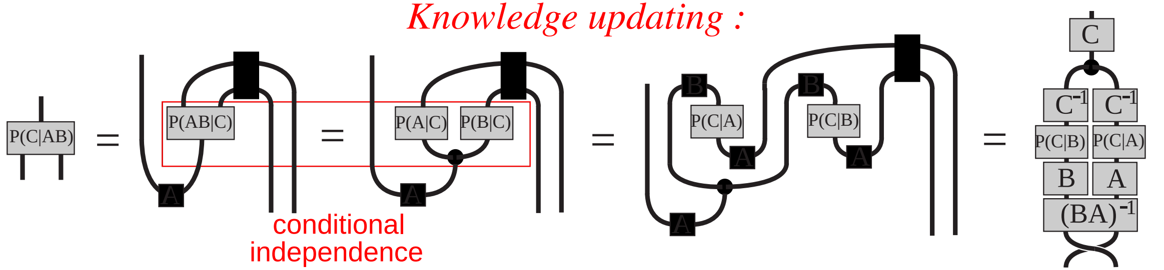
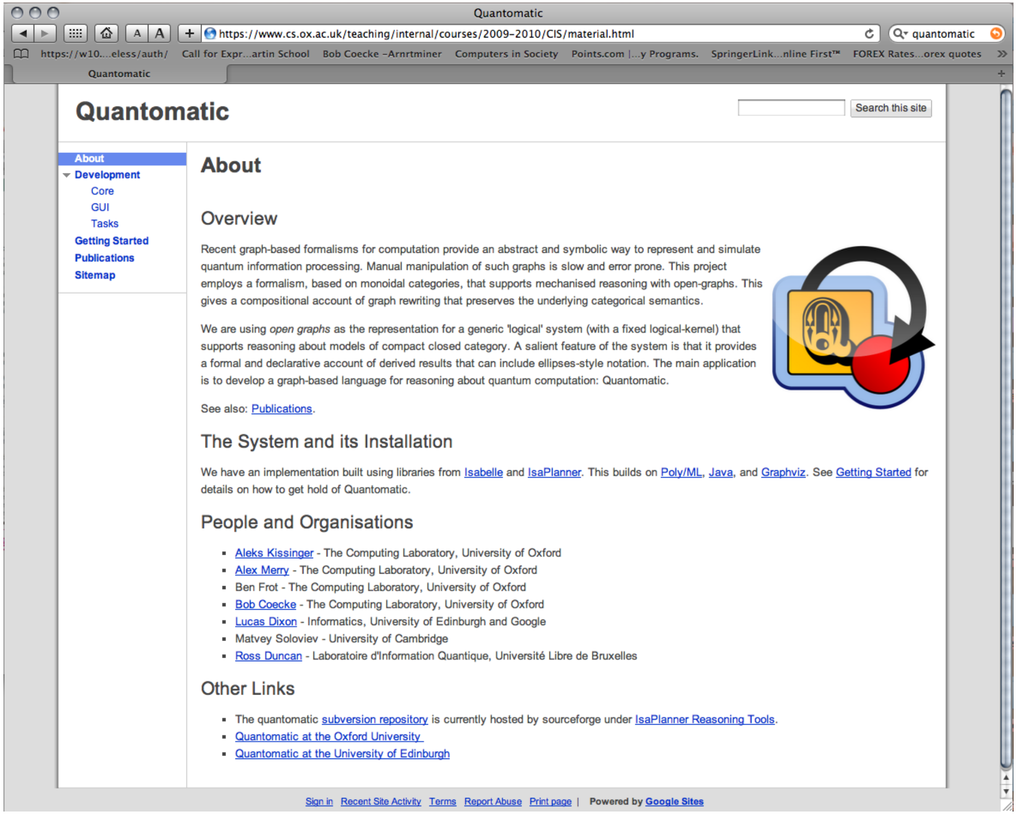


Knowledge updating :



Coecke & Spekkens (2011) *Picturing classical and quantum Bayesian inference*. Synthese. arXiv:1102.2368

Outcome 2b: The structure is a **true (quantum) logic**:



Quantomatic

https://www.cs.ox.ac.uk/teaching/internal/courses/2009-2010/CIS/material.html

Quantomatic

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Getting Started
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About

Overview

Recent graph-based formalisms for computation provide an abstract and symbolic way to represent and simulate quantum information processing. Manual manipulation of such graphs is slow and error prone. This project employs a formalism, based on monoidal categories, that supports mechanised reasoning with open-graphs. This gives a compositional account of graph rewriting that preserves the underlying categorical semantics.

We are using *open graphs* as the representation for a generic 'logical' system (with a fixed logical-kernel) that supports reasoning about models of compact closed category. A salient feature of the system is that it provides a formal and declarative account of derived results that can include ellipses-style notation. The main application is to develop a graph-based language for reasoning about quantum computation: Quantomatic.

See also: Publications.

The System and its Installation

We have an implementation built using libraries from Isabelle and IsaPlanner. This builds on Poly/ML, Java, and Graphviz. See Getting Started for details on how to get hold of Quantomatic.

People and Organisations

- Aleks Kissinger - The Computing Laboratory, University of Oxford
- Alex Merry - The Computing Laboratory, University of Oxford
- Ben Frot - The Computing Laboratory, University of Oxford
- Bob Coecke - The Computing Laboratory, University of Oxford
- Lucas Dixon - Informatics, University of Edinburgh and Google
- Matvey Soloviev - University of Cambridge
- Ross Duncan - Laboratoire d'Information Quantique, Université Libre de Bruxelles

Other Links

- The quantomatic subversion repository is currently hosted by sourceforge under IsaPlanner Reasoning Tools.
- Quantomatic at the Oxford University
- Quantomatic at the University of Edinburgh

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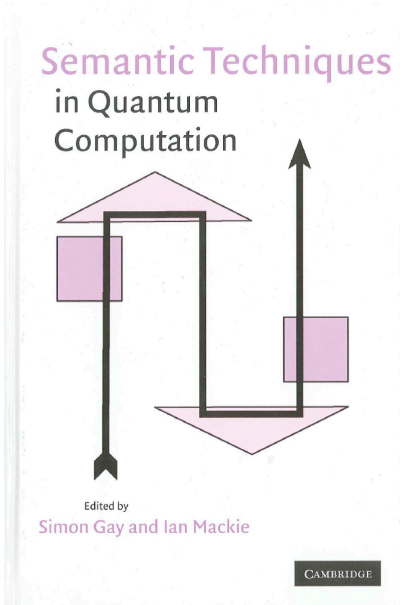
Lucas Dixon, Ross Duncan, Ben Frot, Aleks Kissinger, Alex Merry

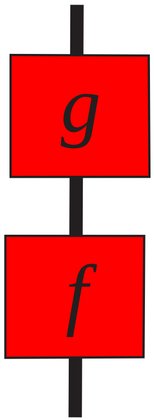
A MINIMAL LANGUAGE FOR QUANTUM PROCESSES

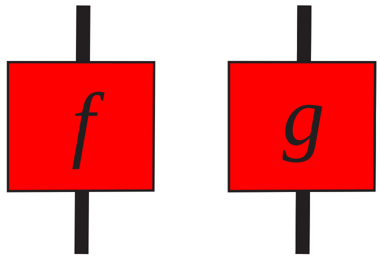


Samson Abramsky & Coecke (2004) *A categorical semantics for quantum protocols*. In: IEEE-LiCS'04. `quant-ph/0402130`

Coecke (2005) *Kindergarten quantum mechanics*. `quant-ph/0510032`

— *graphical notation for processes* —

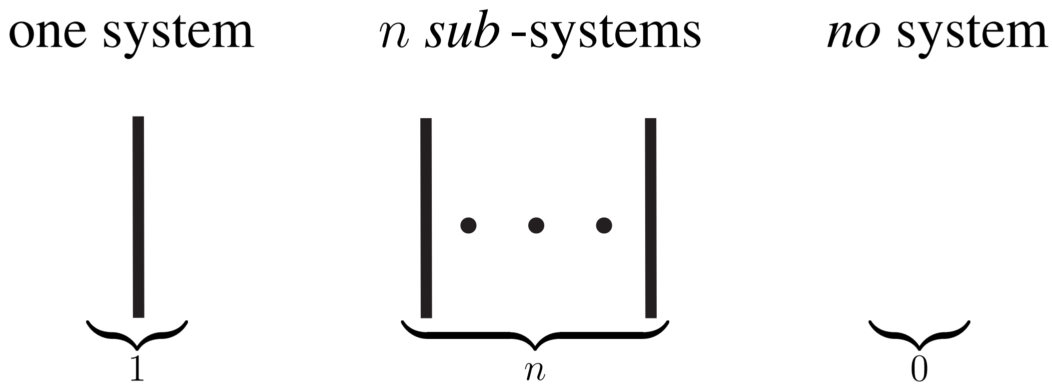
$g \circ f \equiv$ 

$f \otimes g \equiv$ 

Roger Penrose (1971) *Applications of negative dimensional tensors*.
In: Combinatorial Mathematics and its Applications. Academic Press.

André Joyal & Ross Street (1991) *The geometry of tensor calculus* I.
Advances in Mathematics **88**, 55–112.

— *kinds of systems* —



— ***merely a new notation?*** —

$$(g \circ f) \otimes (k \circ h) = (g \otimes k) \circ (f \otimes h)$$

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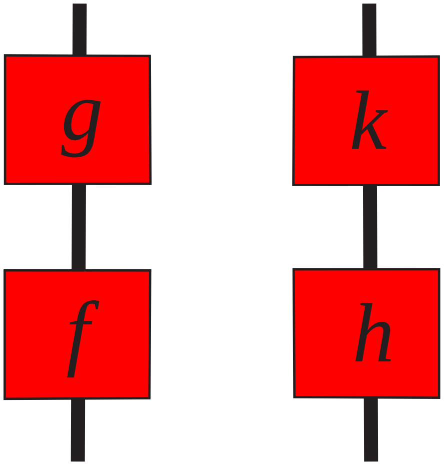

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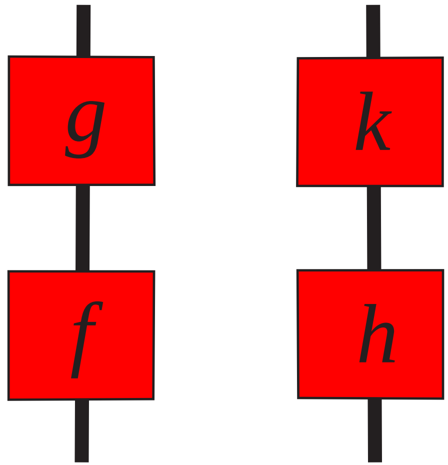

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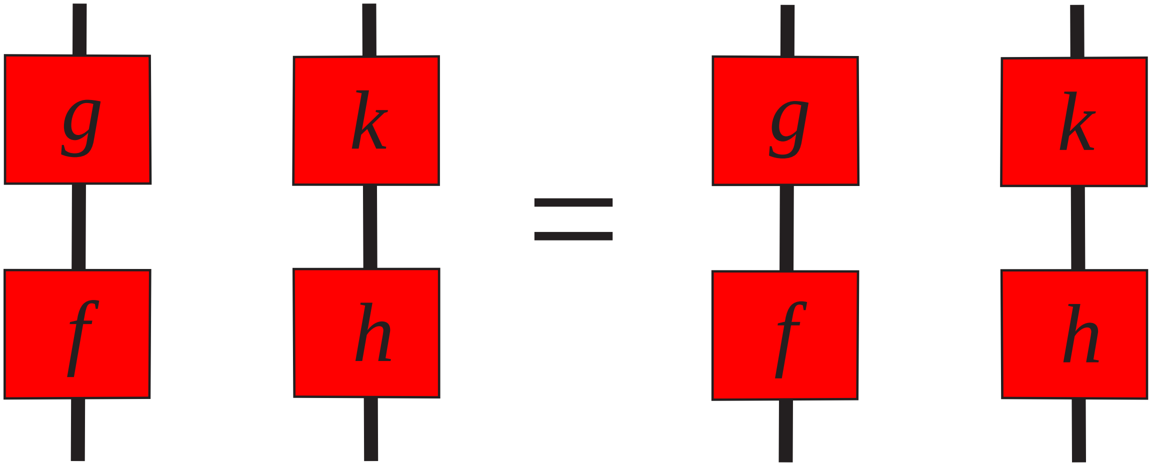

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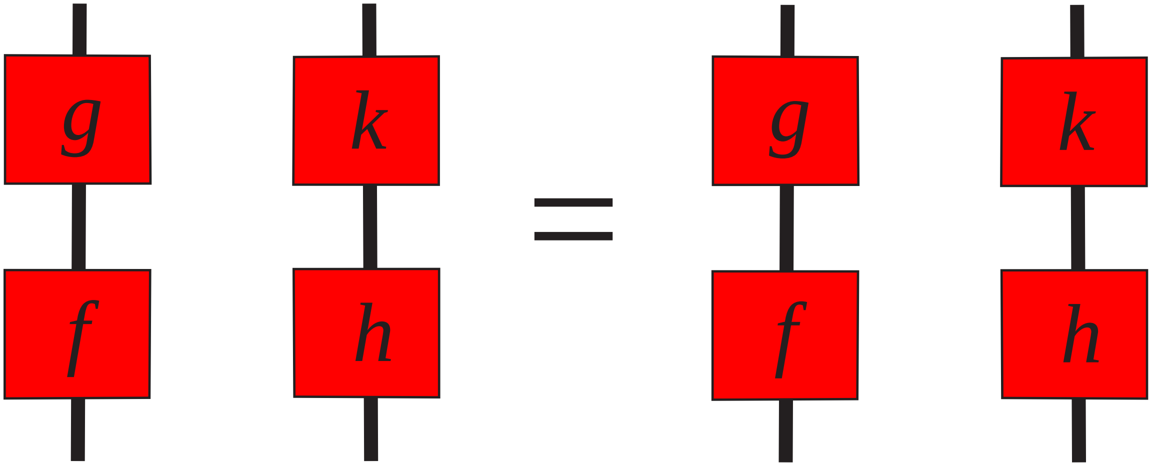


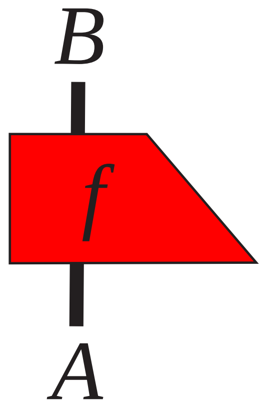
peel potato and then fry it,
while,
clean carrot and then boil it

=

peel potato while clean carrot,
and then,
fry potato while boil carrot

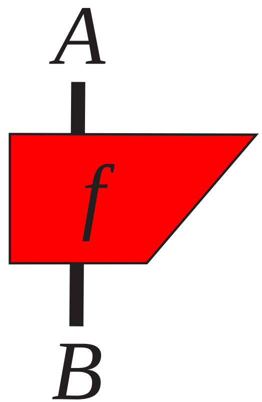
— *adjoint* —

$$f : A \to B$$



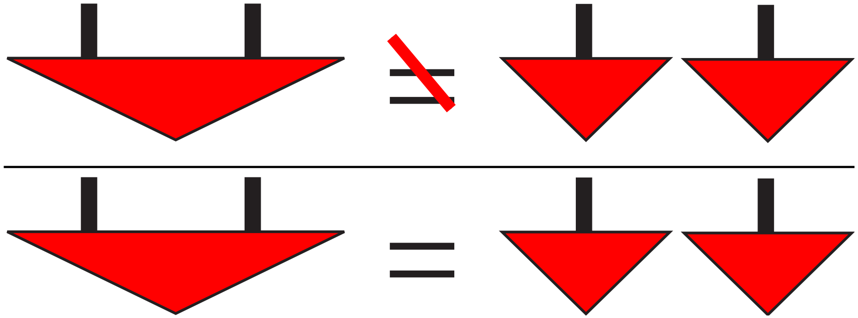
— *adjoint* —

$$f^{\dagger} : B \to A$$



— *asserting (pure) entanglement* —

$$\frac{quantum}{classical} =$$



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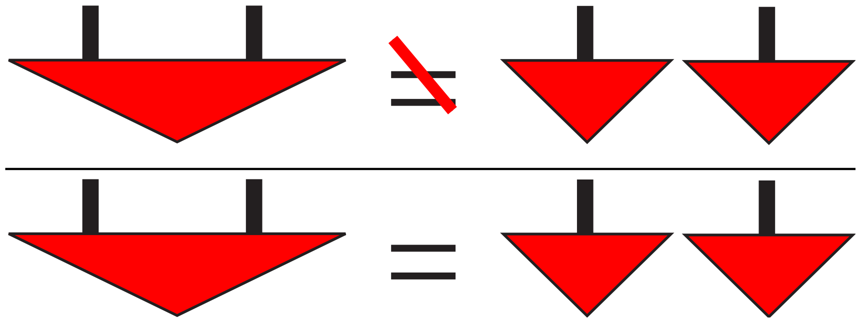
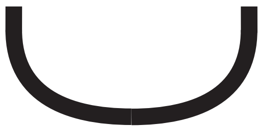


⇒ introduce ‘parallel wire’ between systems:

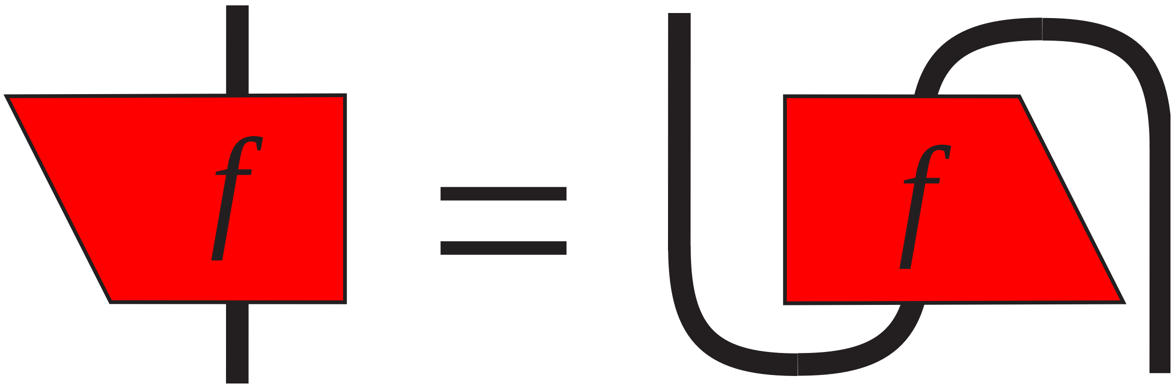


subject to: only topology matters!

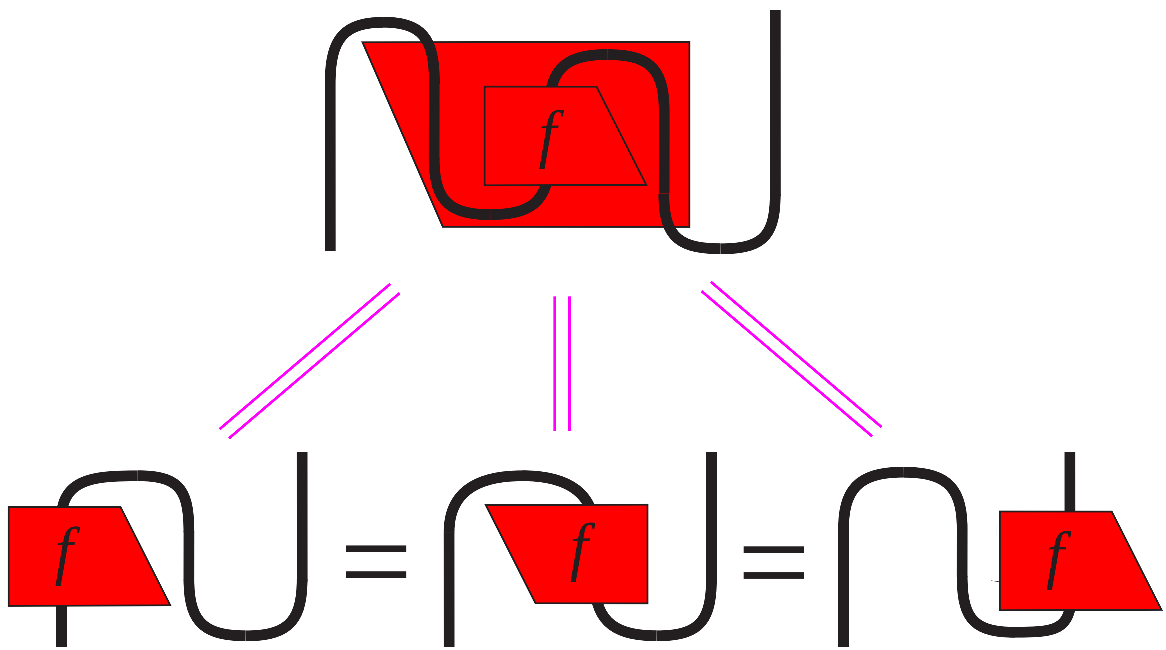
— *quantum-like* —

E.g.

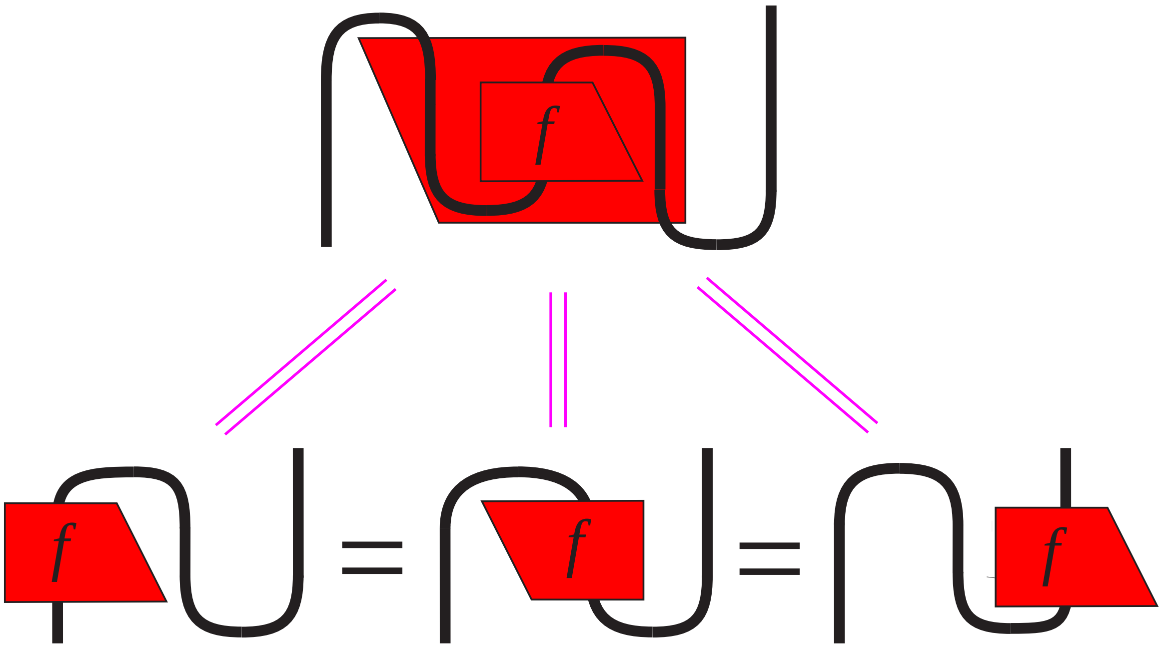
— *quantum-like* —



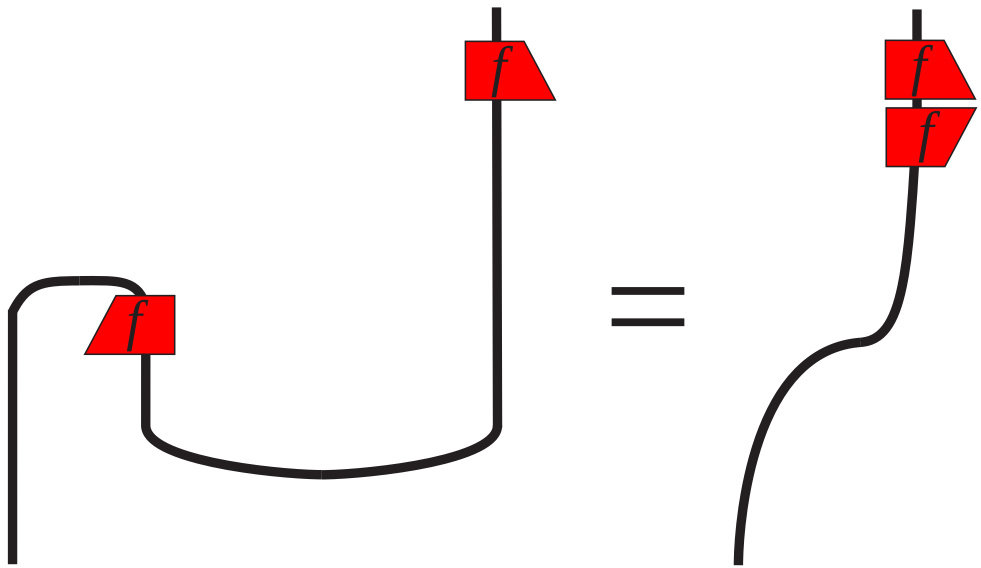
— *sliding* —



— *sliding* —



In QM: cups = Bell-states, caps =Bell-effects, π-rotations = transpose



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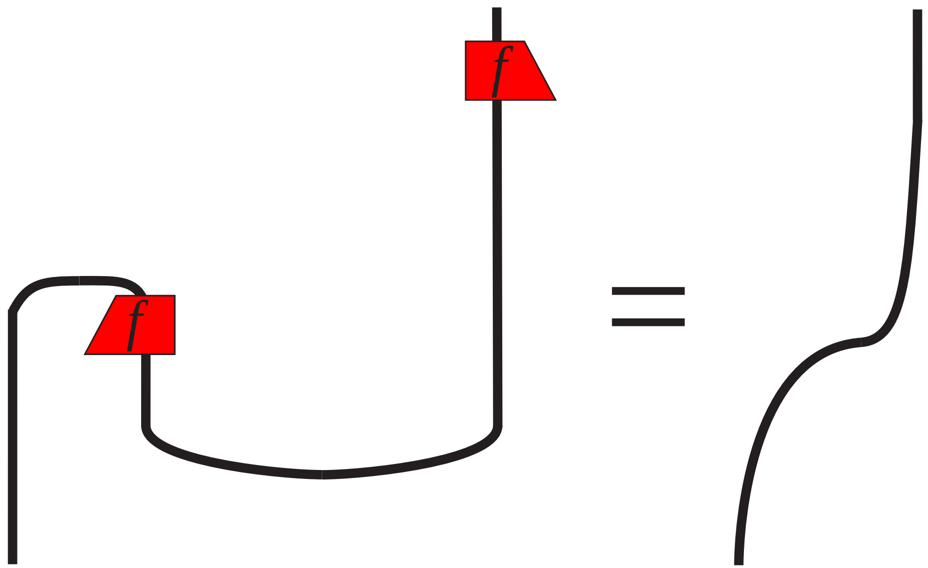
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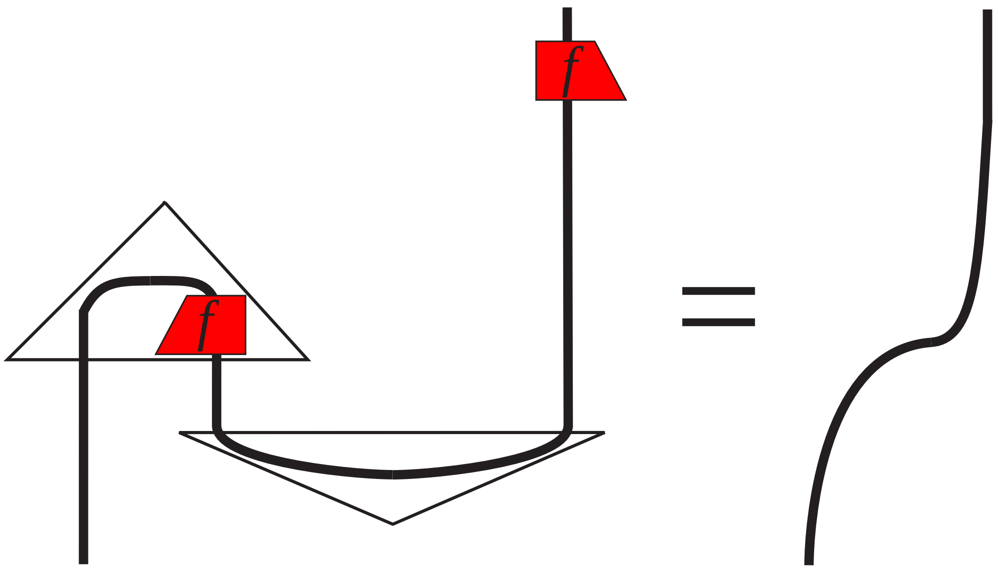
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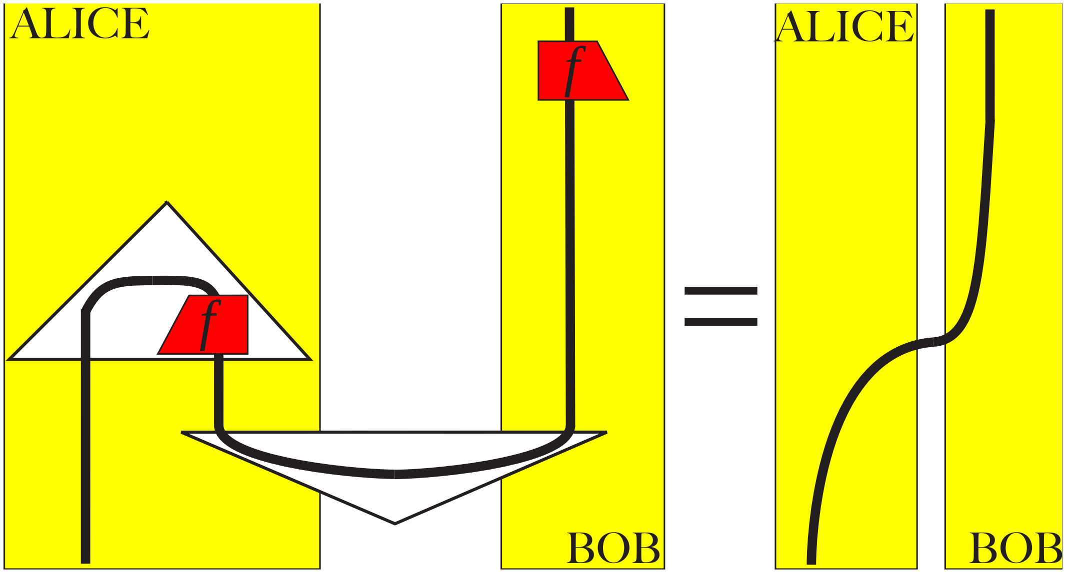
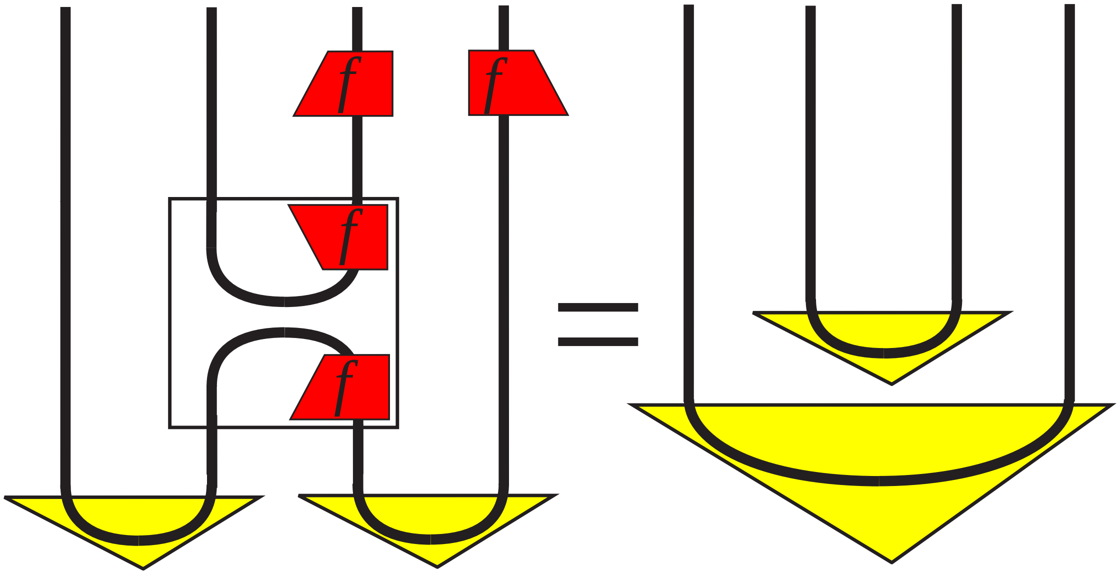


⇒ quantum teleportation



⇒ Entanglement swapping

— *dagger compact categories* —

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Thm. [Kelly-Laplaza '80; Selinger '05] *An equational statement between expressions in dagger compact categorical language holds if and only if it is derivable in the graphical notation via homotopy*.

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Thm. [Selinger '08] *An equational statement between expressions in dagger compact categorical language holds if and only if it is derivable in the category of finite dimensional Hilbert spaces, linear maps, tensor product, and adjoints*.

— *dagger compact categories* —

In words: *Any equation involving:*

- *states, operations, effects*
- *unitarity, adjoints (e.g. self-adjoint), projections*
- *Bell-states/effects, transpose, conjugation*
- *inner-product, trace, Hilbert-Schmidt norm*
- *positivity, completely positive maps*, ...

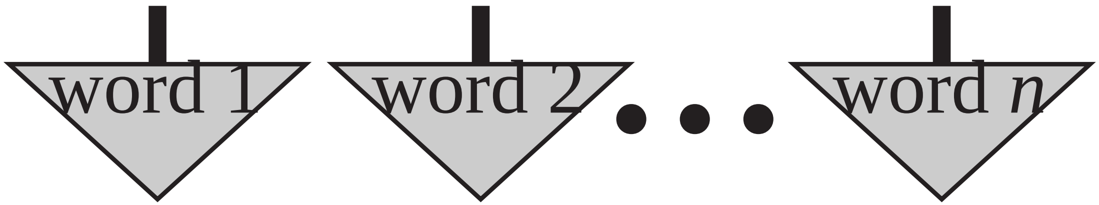
holds in quantum theory *if and only if* *it can be derived in the graphical language via homotopy.*

A SLIGHTLY DIFFERENT LANGUAGE FOR NATURAL LANGUAGE MEANING

Coecke, Sadrzadeh & Clark (2010) *Mathematical Foundations for a Compositional Distributional Model of Meaning*. `arXiv:1003.4394`

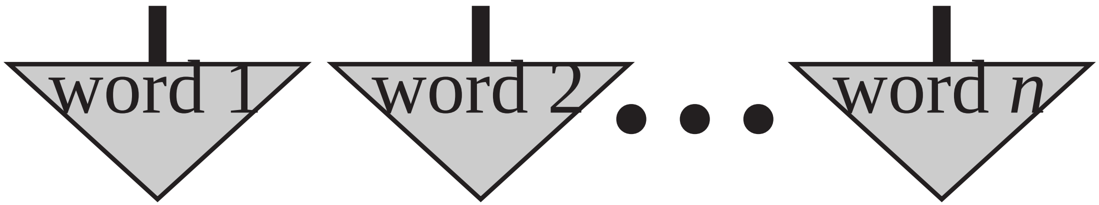
— *the from-words-to-a-sentence process* —

Consider meanings of **words**, e.g. as vectors (cf. Google):



— *the from-words-to-a-sentence process* —

What is the meaning the **sentence** made up of these?



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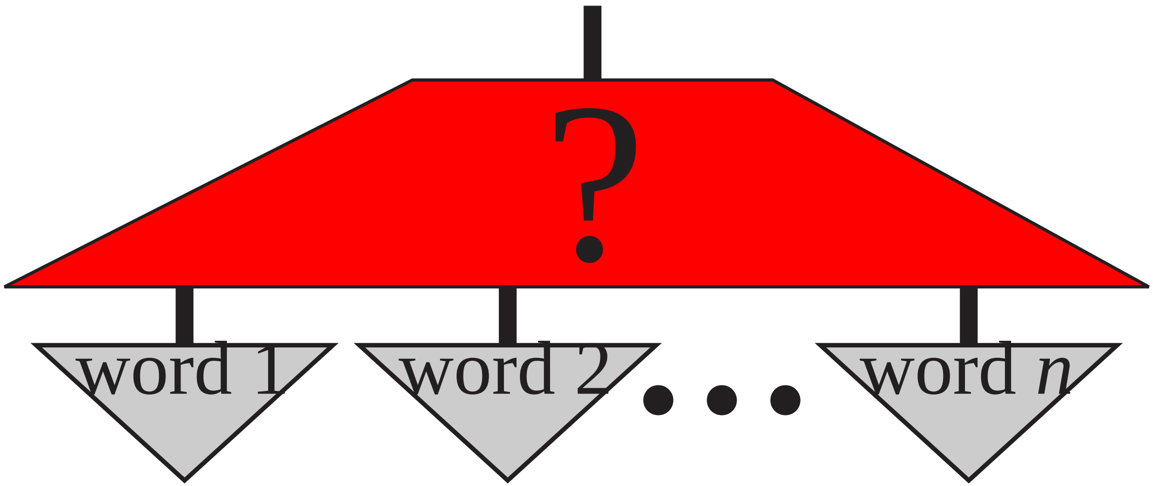
I.e. how do we/machines produce meanings of **sentences**?



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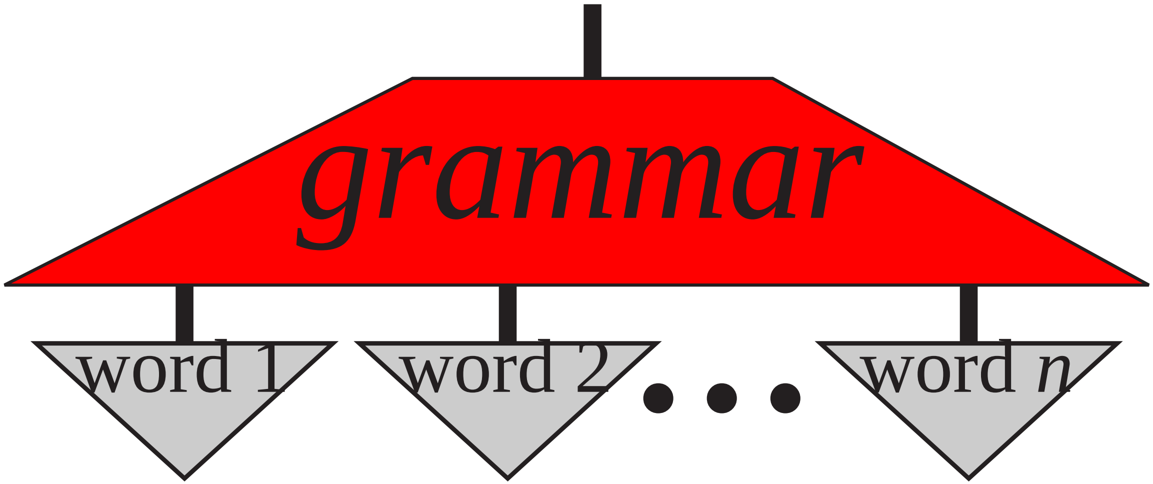
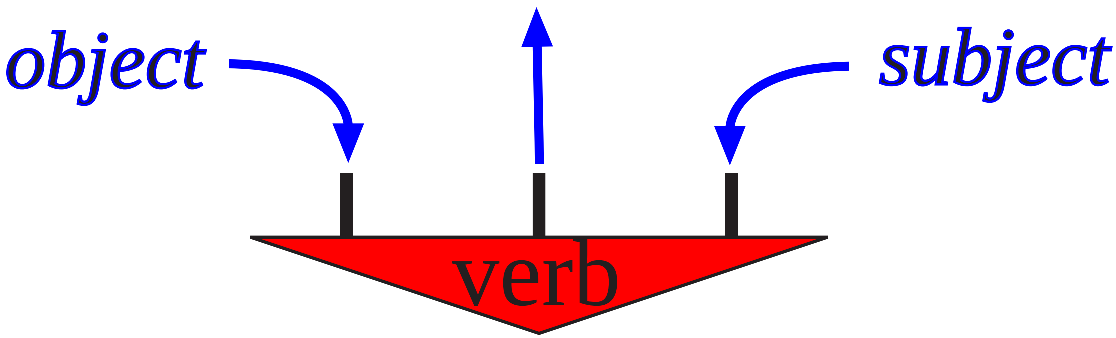


Gerald Gazdar (1996) *Paradigm merger in natural language processing*. In: Computing tomorrow: future research directions in computer science, eds., I. Wand and R. Milner, Cambridge University Press.

Information flow within a verb:



— the from-words-to-a-sentence process —

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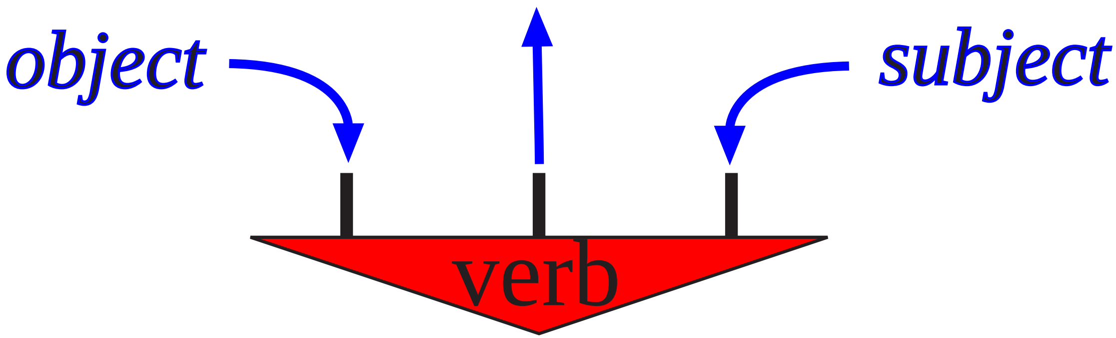
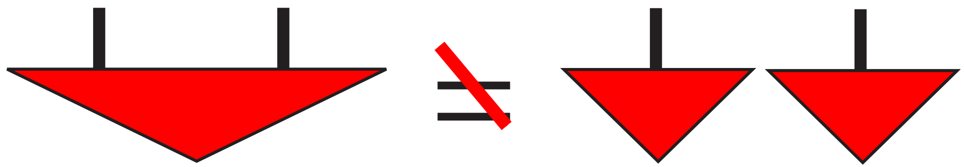
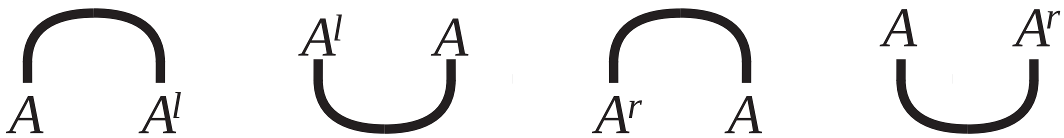
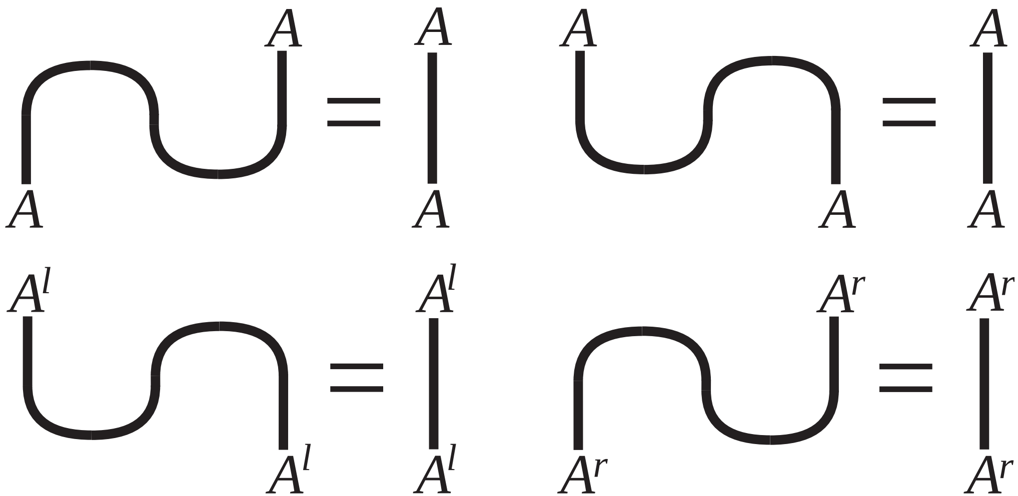


Again we have:

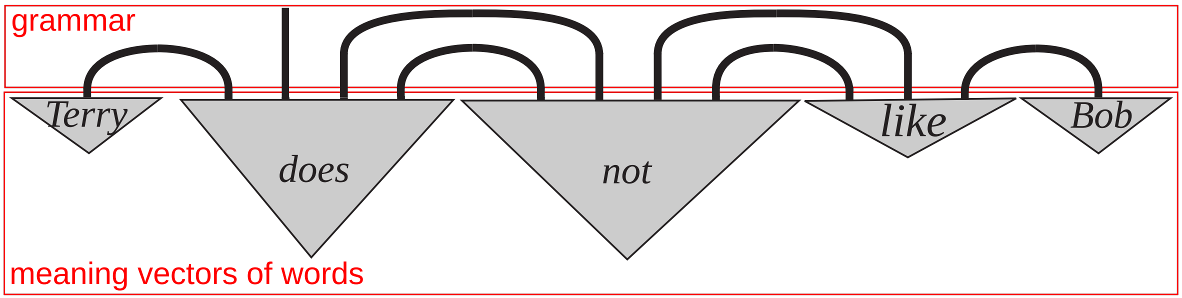


— *going non-symmetric* —

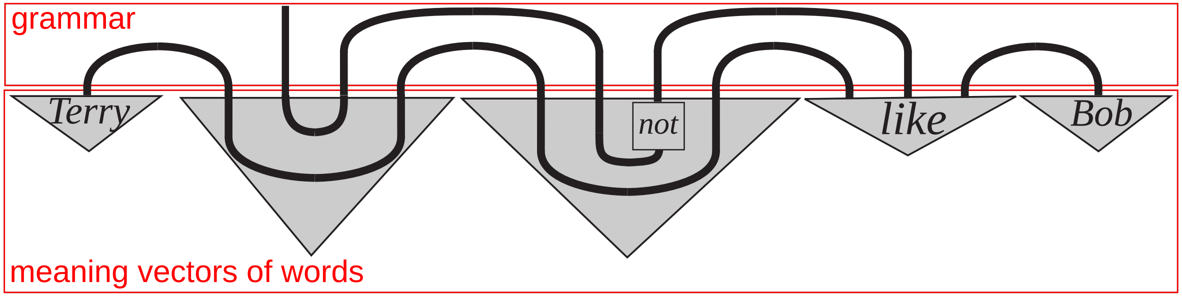




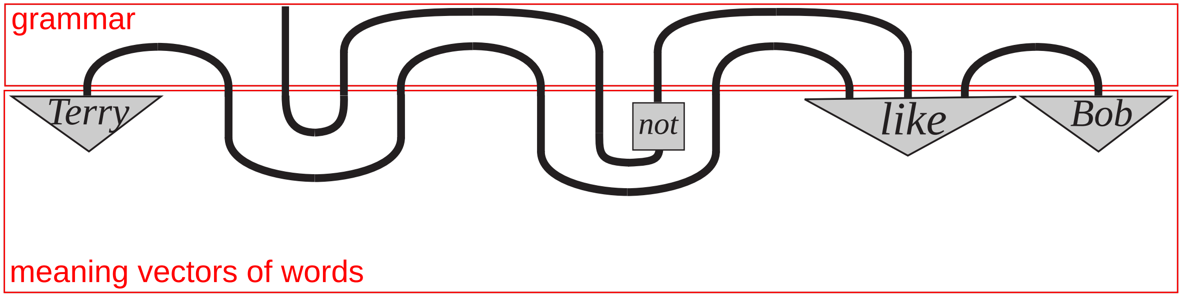
$$— \overrightarrow{Alice} \otimes \overrightarrow{does} \otimes \overrightarrow{not} \otimes \overrightarrow{like} \otimes \overrightarrow{Bob} —$$



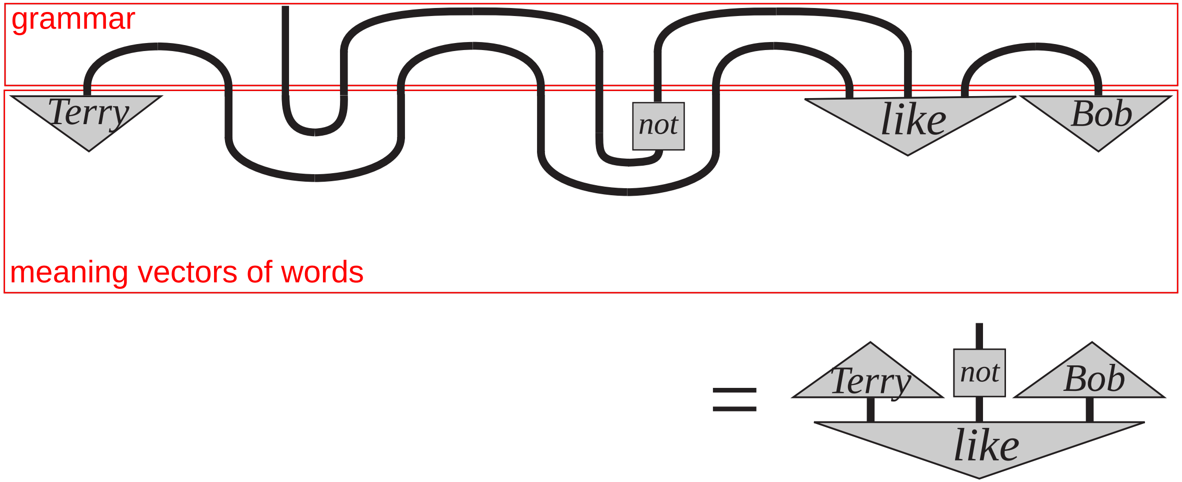
$$— \overrightarrow{Alice} \otimes \overrightarrow{does} \otimes \overrightarrow{not} \otimes \overrightarrow{like} \otimes \overrightarrow{Bob} —$$



$$— \overrightarrow{Alice} \otimes \overrightarrow{does} \otimes \overrightarrow{not} \otimes \overrightarrow{like} \otimes \overrightarrow{Bob} —$$



$$— \overrightarrow{Alice} \otimes \overrightarrow{does} \otimes \overrightarrow{not} \otimes \overrightarrow{like} \otimes \overrightarrow{Bob} —$$



— *experiment: word disambiguation* —

E.g. what is "saw"" in: "Alice saw Bob with a saw".

Model	High	Low	ρ
Baseline	0.47	0.44	0.16
Add	0.90	0.90	0.05
Multiply	0.67	0.59	0.17
Categorical (1)	**0.73**	**0.72**	**0.21**
Categorical (2)	**0.34**	**0.26**	**0.28**
UpperBound	4.80	2.49	0.62

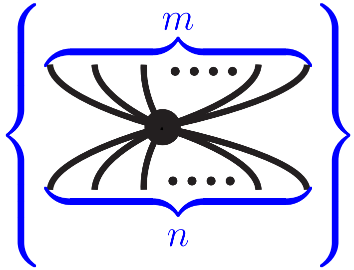
Edward Grefenstette & Mehrnoosh Sadrzadeh (2011) *Experimental support for a categorical compositional distributional model of meaning*. Accepted for: Empirical Methods in Natural Language Processing (EMNLP'11).

Mehrnoosh Sadrzadeh Edward Grefenstette

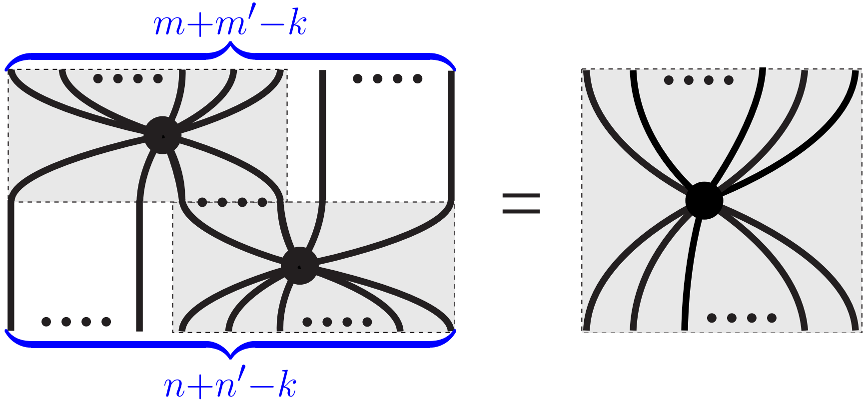
AN EXTENDED LANGUAGE:
CLASSICALITY & OBSERVABLES

— *observables* —

'spiders' =

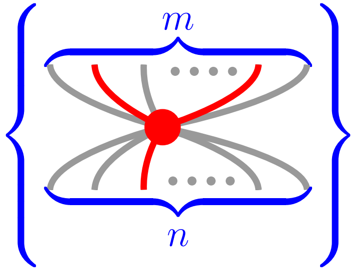


such that, for $k > 0$:

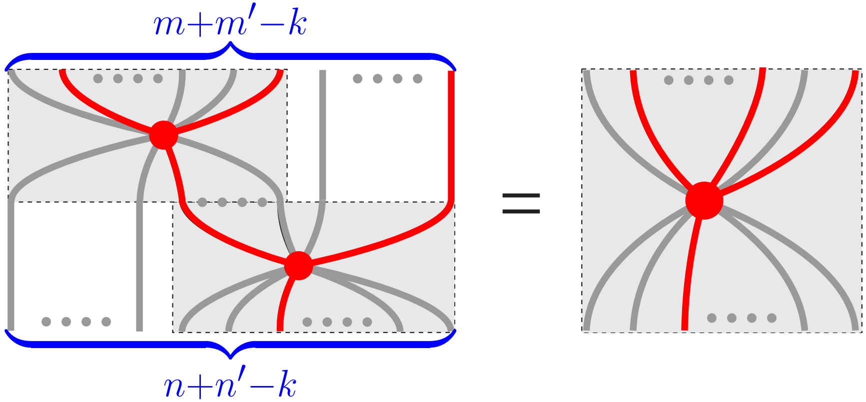


— *observables* —

'spiders' =

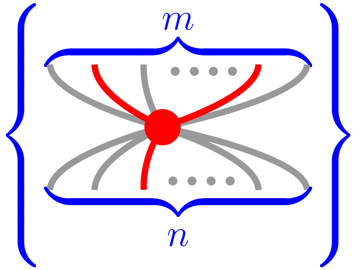


such that, for $k > 0$:

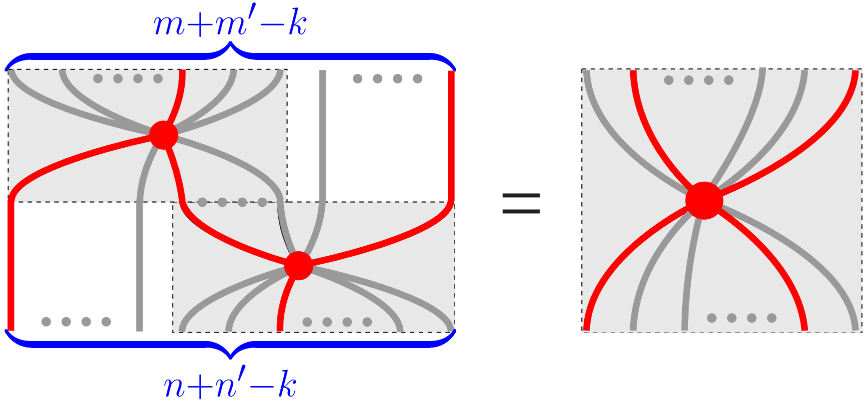


— *observables* —

'spiders' =



such that, for $k > 0$:



— *observables* —

Theorem 1. ('folklore' - Kock's TQFT '03; Lack '04) In any dagger symmetric monoidal category such families of spiders and dagger special commutative Frobenius algebras are in canonical bijective correspondence.

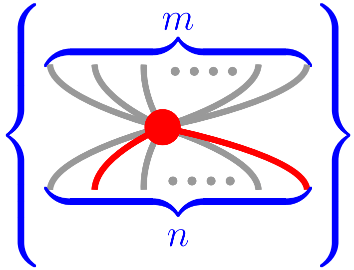
Theorem 2. (Coecke-Pavlovic-Vicary) In **FdHilb** dagger (special) commutative Frobenius algebra are exactly ortho(normal) bases, nl. those of copyable elts.

Coecke & Pavlovic (2007) *Quantum measurement without sums*. In: Mathematics of Quantum Computing and Technology. `quant-ph/0608035`

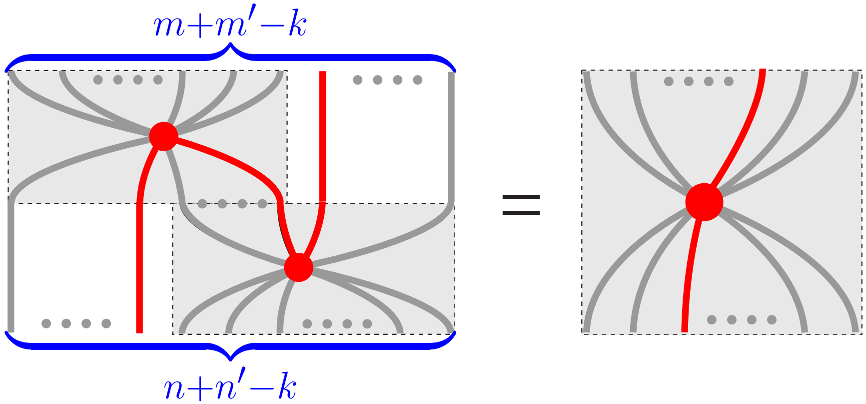
Coecke, Pavlovic & Vicary (2008) *A new description of orthogonal bases*. Mathematical Structures in Computer Science. `0810.0812`

— *observables* —

'spiders' =

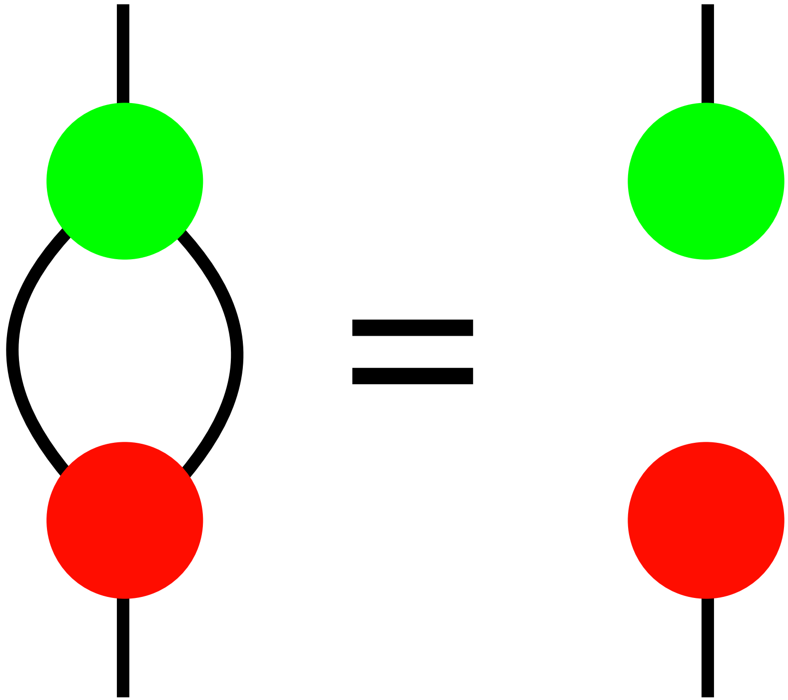


such that, for $k > 0$:

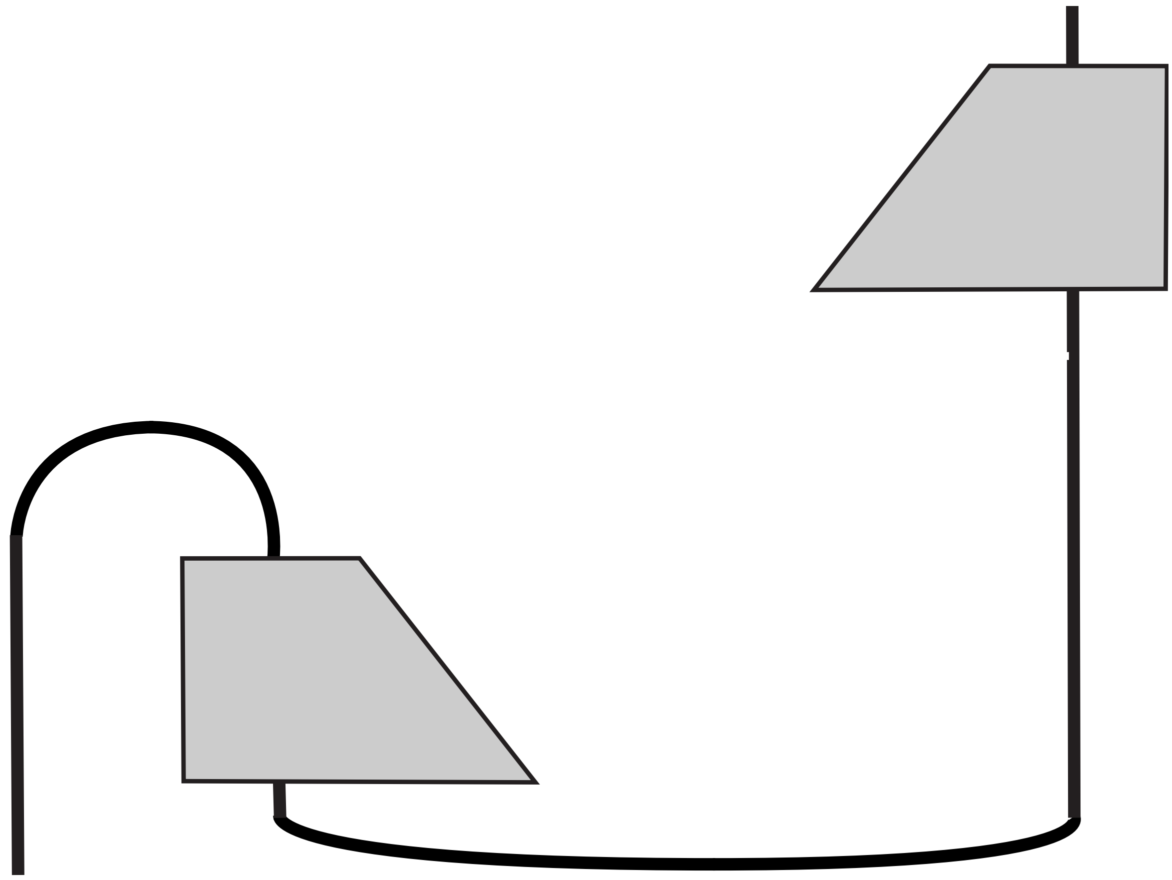


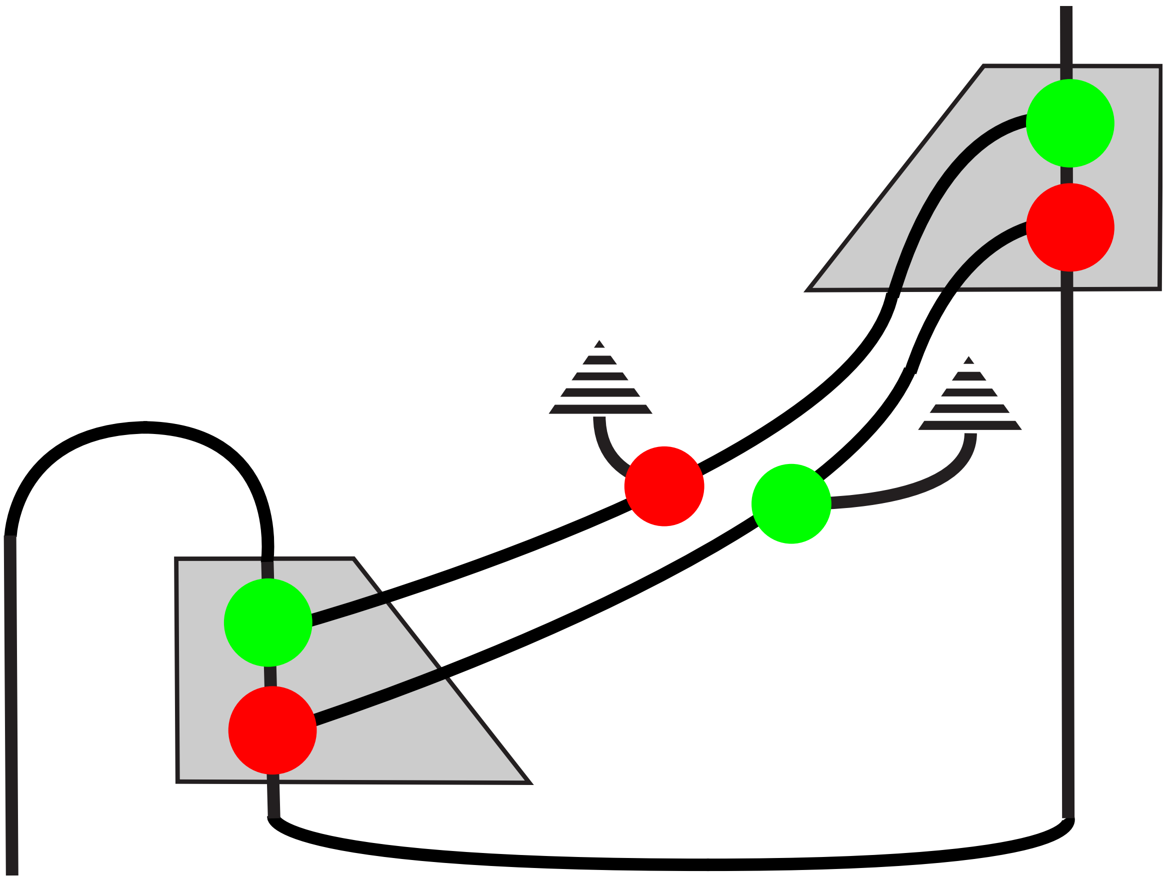
— *complementary observables* —

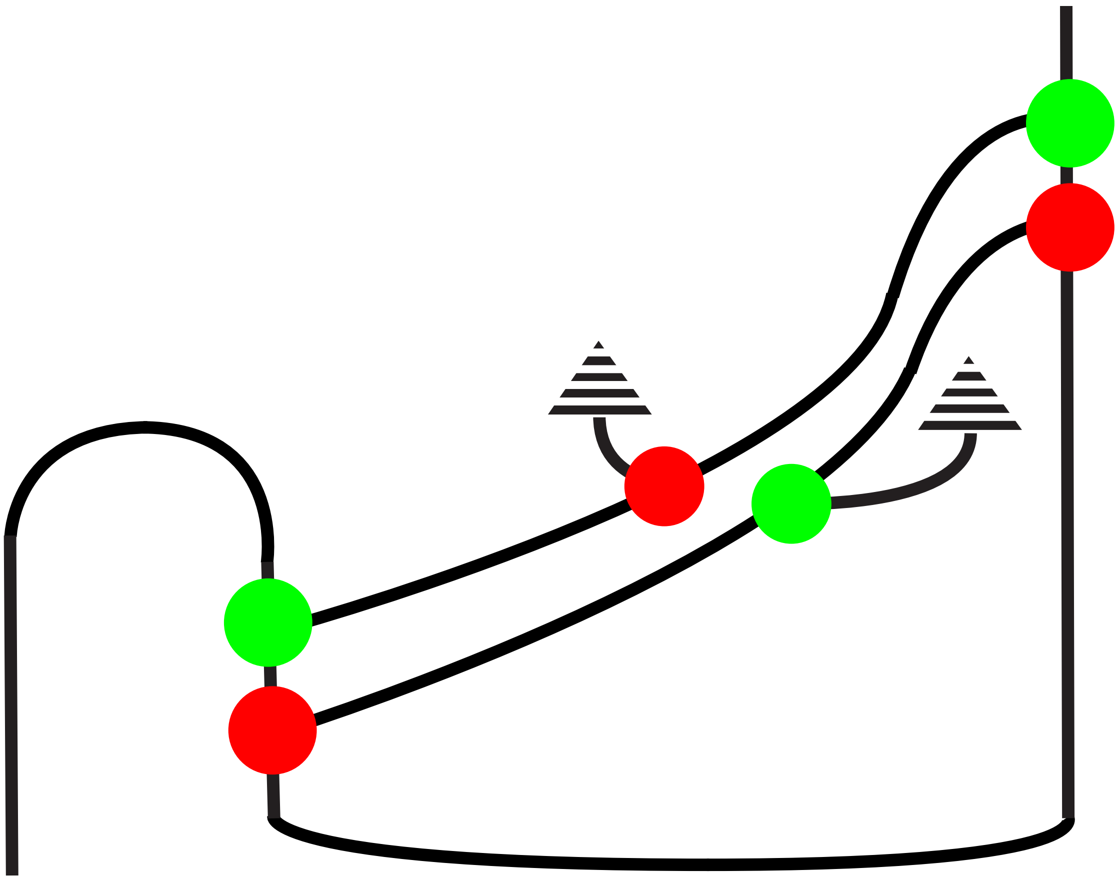
— complementary observables —

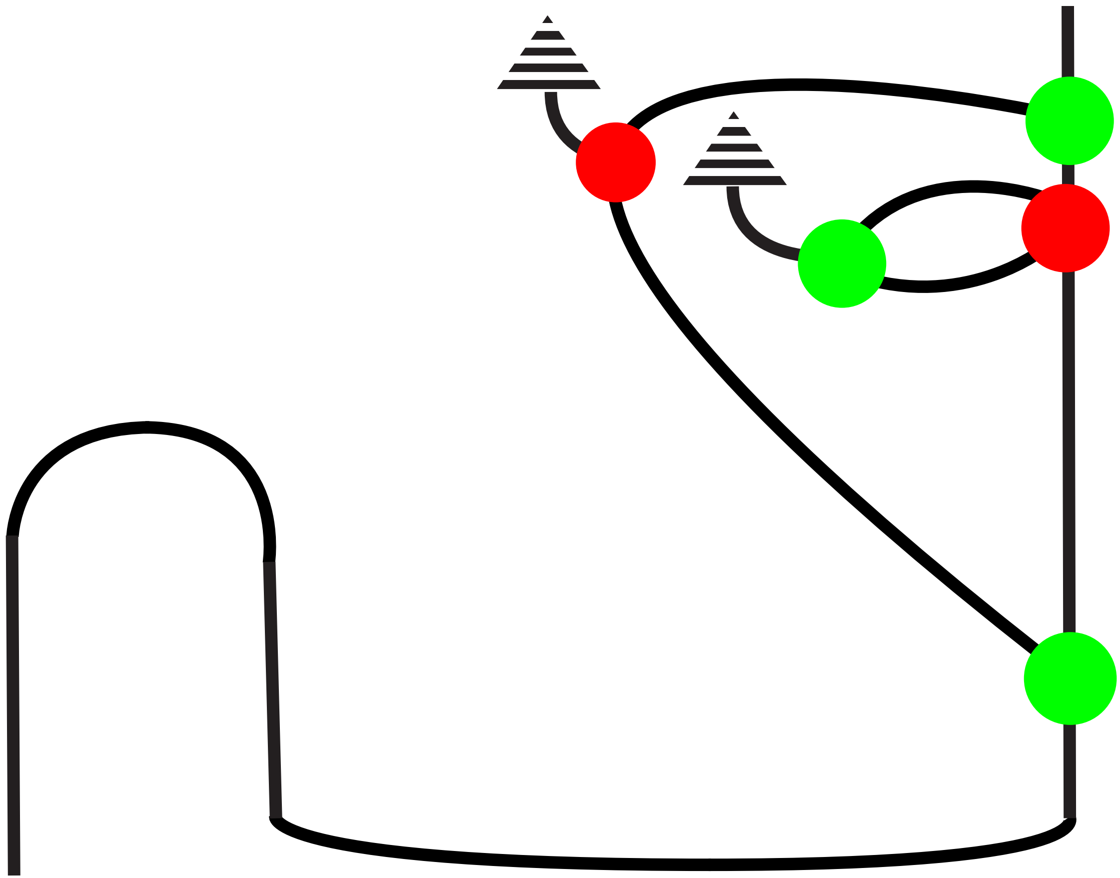


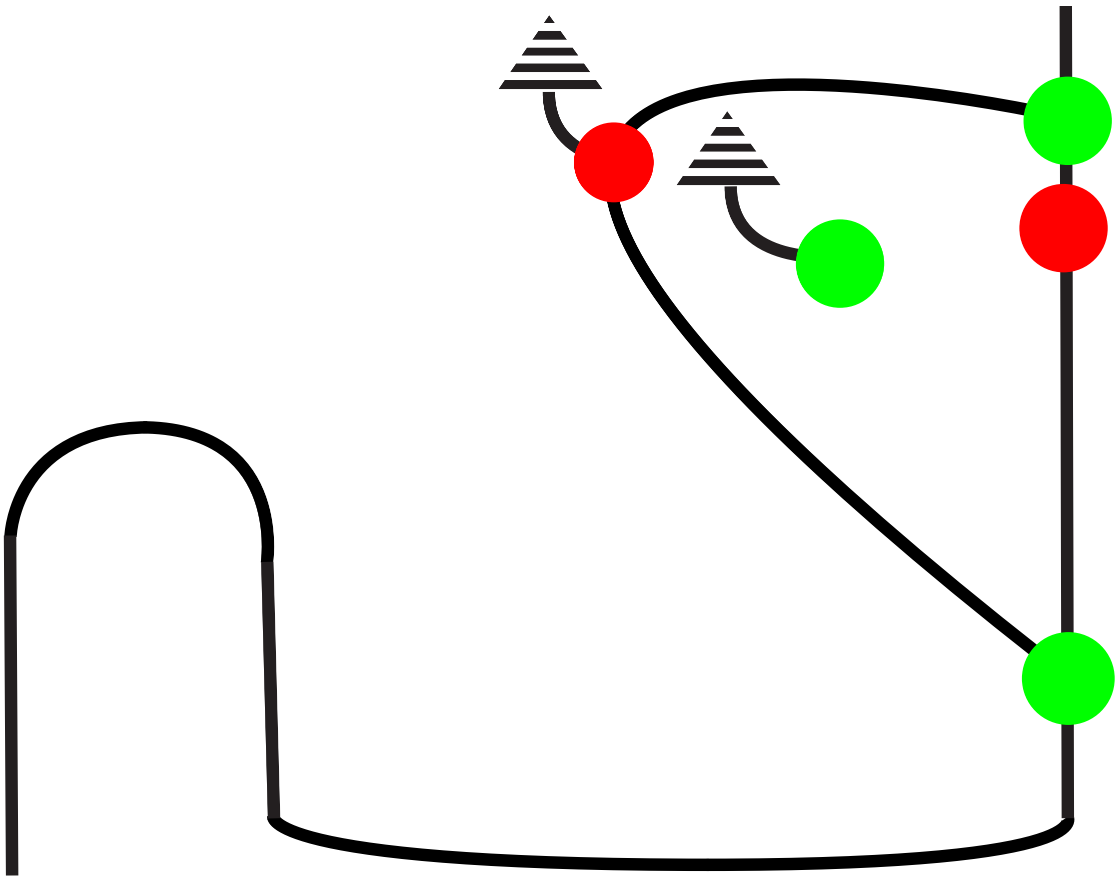
Coecke & Ross Duncan (2008) *Interacting quantum observables*. In: ICALP'08.
`arXiv:0906.4725`

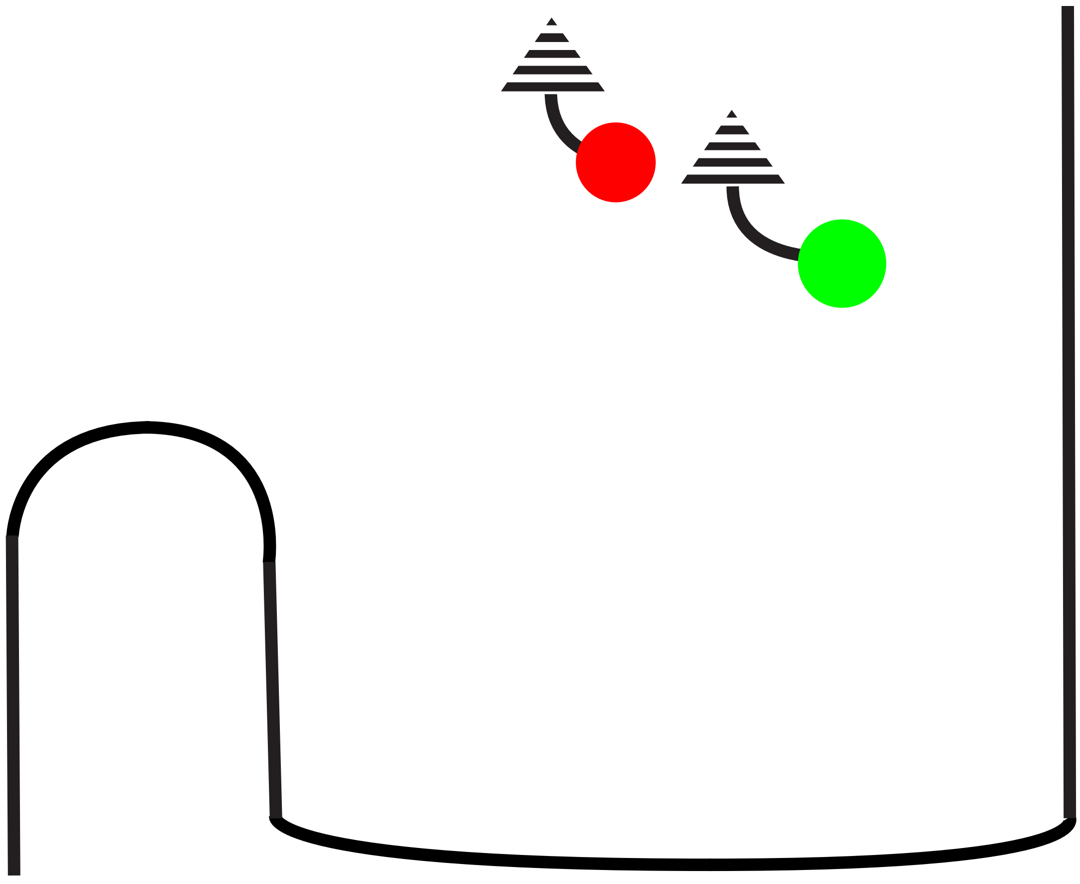


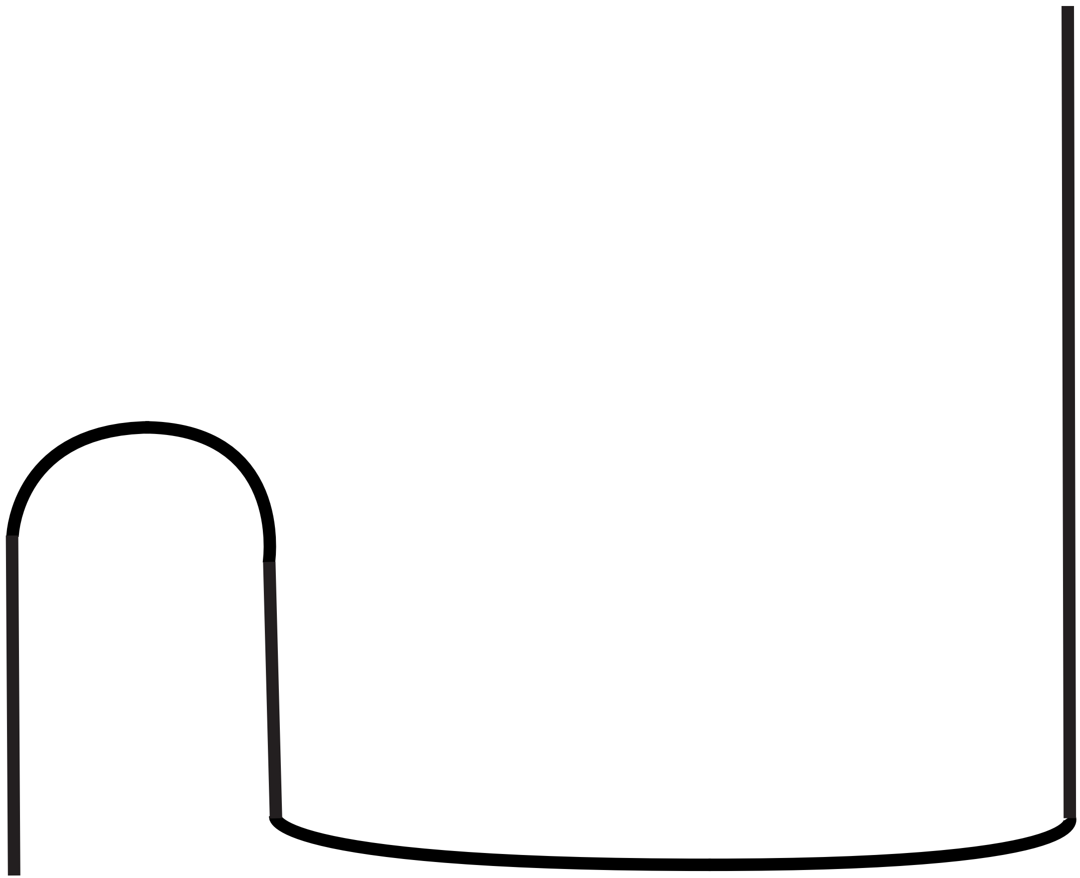












A UNIVERSAL LANGUAGE

THM (Phased Z/X-calc.). Any $f : \mathbb{C}^n \to \mathbb{C}^m$ decomposes in complementary "phased" 3-spiders:

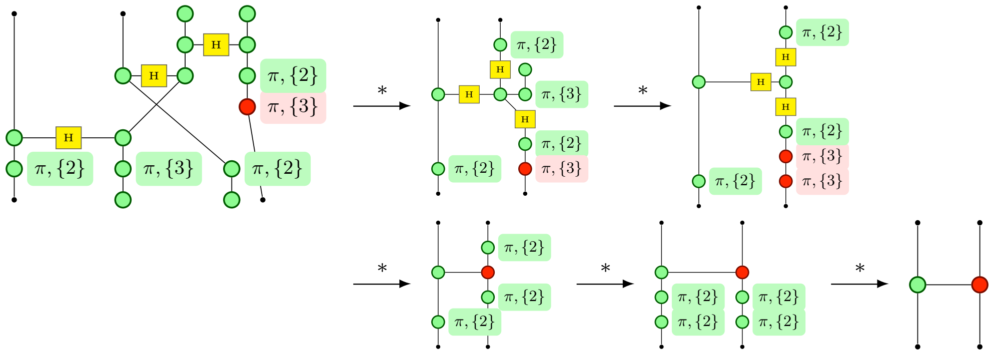
These phases arise as an **Abelian group structure** that comes with the spiders for purely abstract reasons, where inverses are the abstract conjugates.

These phases 'add' when spiders fuse, which can be described as families of 'group-decorated' spiders.

Coecke & Ross Duncan (2008) *Interacting quantum observables*. In: ICALP'08.
Extended version: `arXiv:0906.4725`

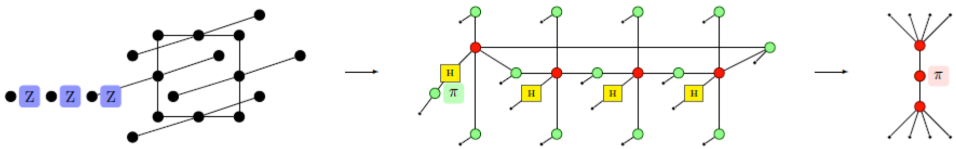
— *applications to QC models* —

Translation to circuits, required resources and determinism in measurement based quantum computations:



Ross Duncan & Simon Perdrix (2010) Rewriting measurement-based quantum computations with generalised flow. ICALP'10.

Similar stuff for TMBQC (Clare Horsman NJP'11):



— *applications to quantum foundations* —

Toy qubits vs. true quantum theory in one language:

$$\frac{\text{Spekkens' qubit QM}}{\text{stabilizer qubit QM}} = \frac{Z_2 \times Z_2}{Z_4} = \frac{\text{local}}{\text{non-local}}$$

Coecke, Bill Edwards & Robert W. Spekkens (2010) Phase groups and the origin of non-locality for qubits. **QPL'10** `arXiv:1003.5005`

— *applications to quantum foundations* —

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Coecke, Bill Edwards & Robert W. Spekkens (2010) Phase groups and the origin of non-locality for qubits. **QPL'10** `arXiv:1003.5005`

Generalized Mermin arg. ⇔ strong complementarity

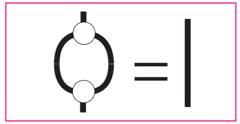
Coecke, Duncan, Kissinger & Quanlong Wang (2012) *Strong complementarity and non-locality in categorical quantum mechanics*. **LiCS'12**. arXiv:1203.4988

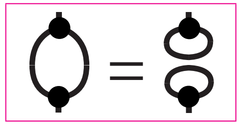
— *multipartite entanglement structure* —

Tripartite SLOCC-classes as comm. Frobenius algs:

$$\frac{GHZ = |000\rangle + |111\rangle}{W = |001\rangle + |010\rangle + |100\rangle} = \frac{\text{‘special’ CFAs}}{\text{‘anti-special’ CFAs}}$$

=



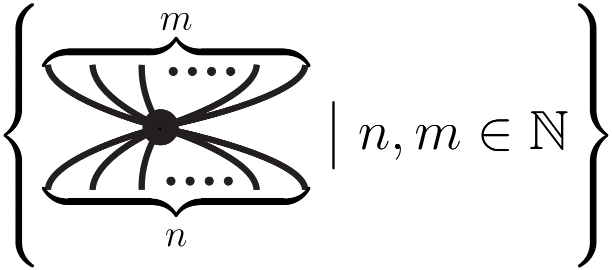


$$= \frac{\times}{+} \quad \Rightarrow \text{distributivity}$$

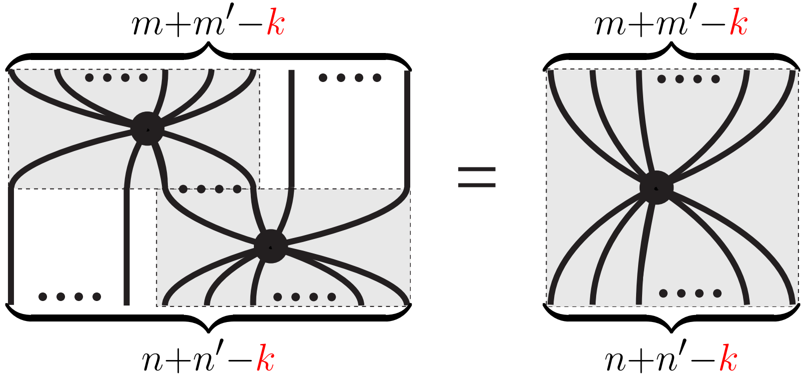
Coecke & Aleks Kissinger (2010) The compositional structure of multipartite quantum entanglement. ICALP'10. `arXiv:1002.2540`

— *GHZ-spiders* —

Data:

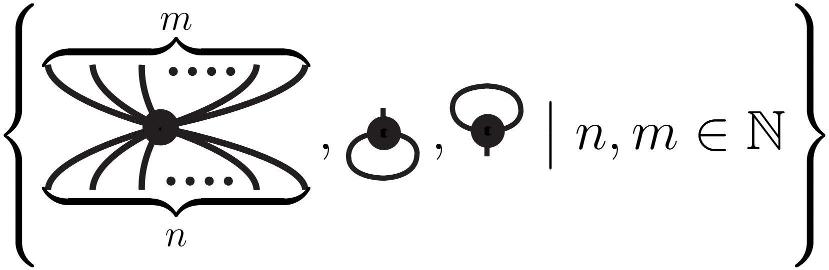


Rules:

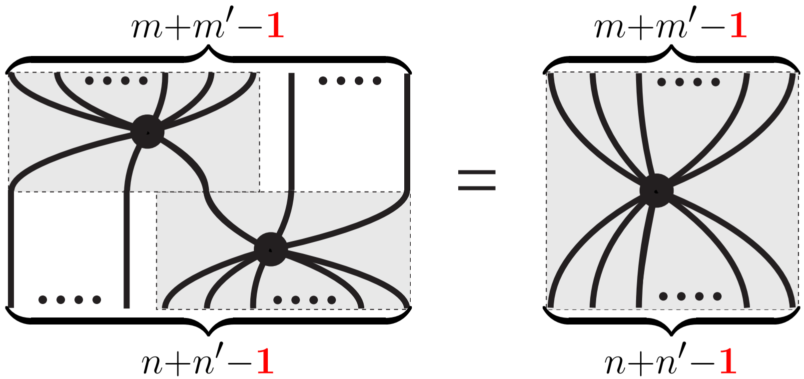


— *W-spiders* —

Data:

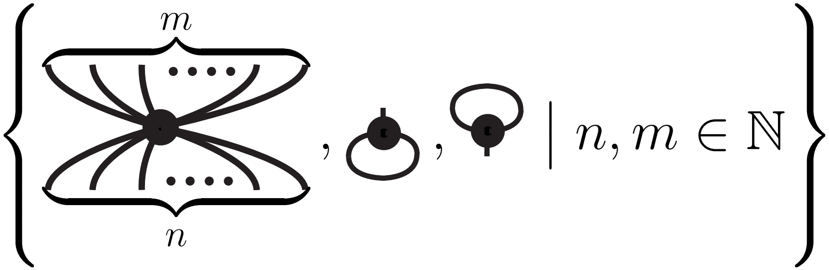


Rules:

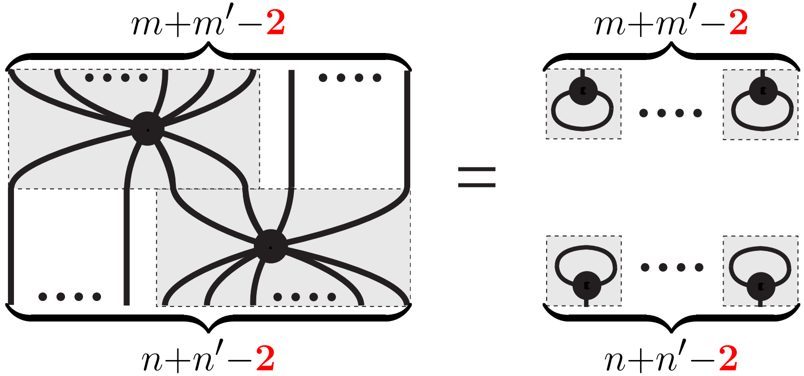


— *W-spiders* —

Data:



Rules:



— *automation* —

Stages:

- Automated reasoning — `quantomatic`
- Automated theory generation — `quantocosy`
- Automated theorem extraction — ???

— *automated quantum reasoning* —

Duncan, Soloviev, Kissinger, Merry, Dixon