

# Quantum Information Processing with Superconducting Circuits

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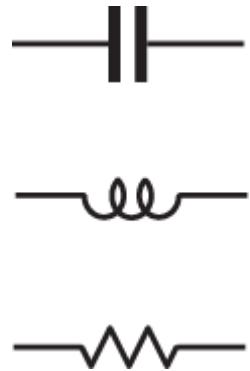


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Swiss Federal Institute of Technology Zurich

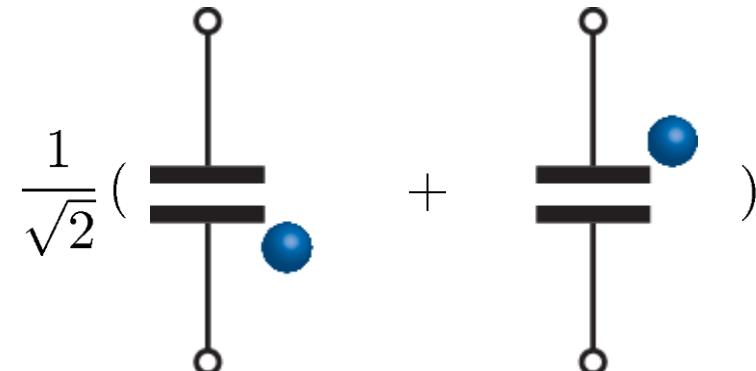


# Classical and Quantum Electronic Circuit Elements

basic circuit elements:



charge on a capacitor:



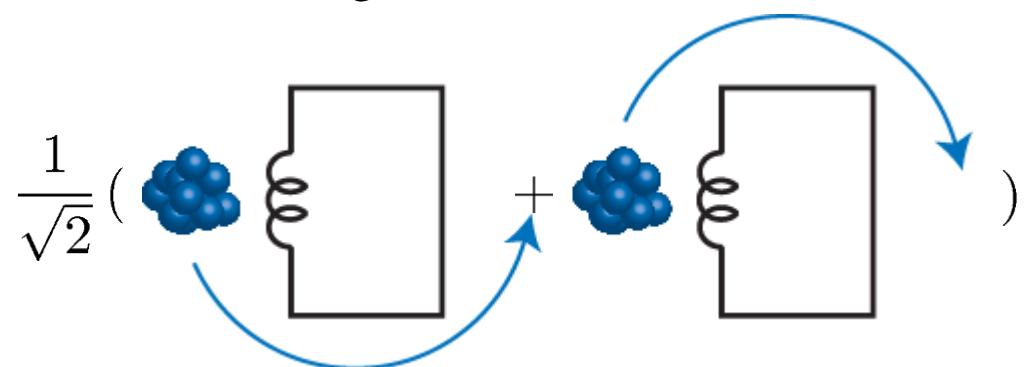
quantum superposition states:

- charge  $q$
- flux  $\phi$

commutation relation (c.f.  $x, p$ ):

$$[\hat{\phi}, \hat{q}] = i\hbar$$

current or magnetic flux in an inductor:



# Constructing Linear Quantum Electronic Circuits

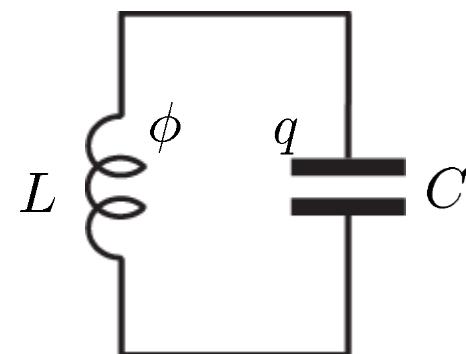
basic circuit elements:



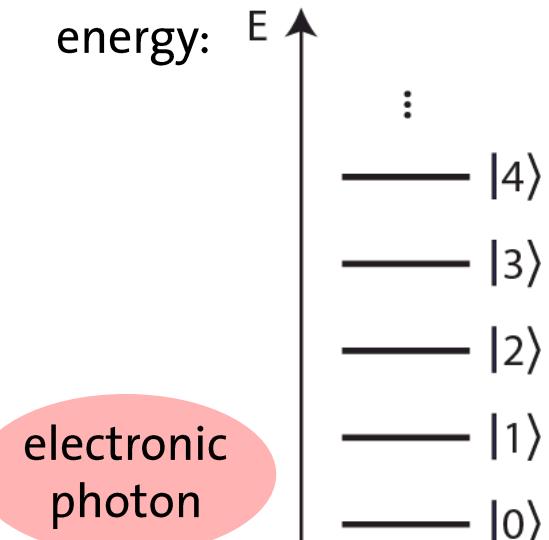
Josephson junction:

a non-dissipative nonlinear element (inductor)

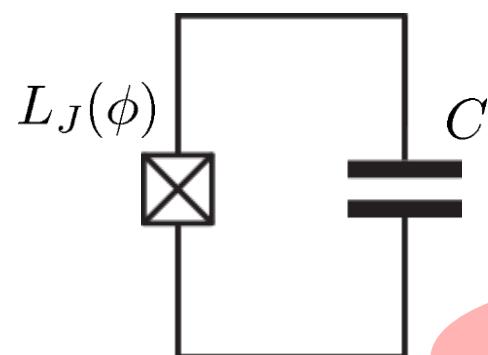
harmonic LC oscillator:



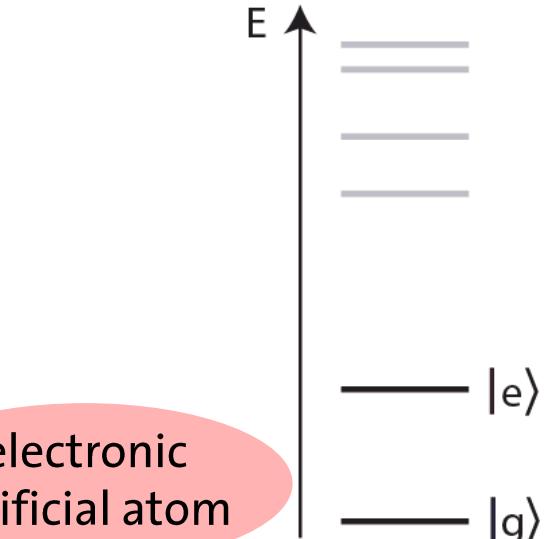
energy:



anharmonic oscillator:



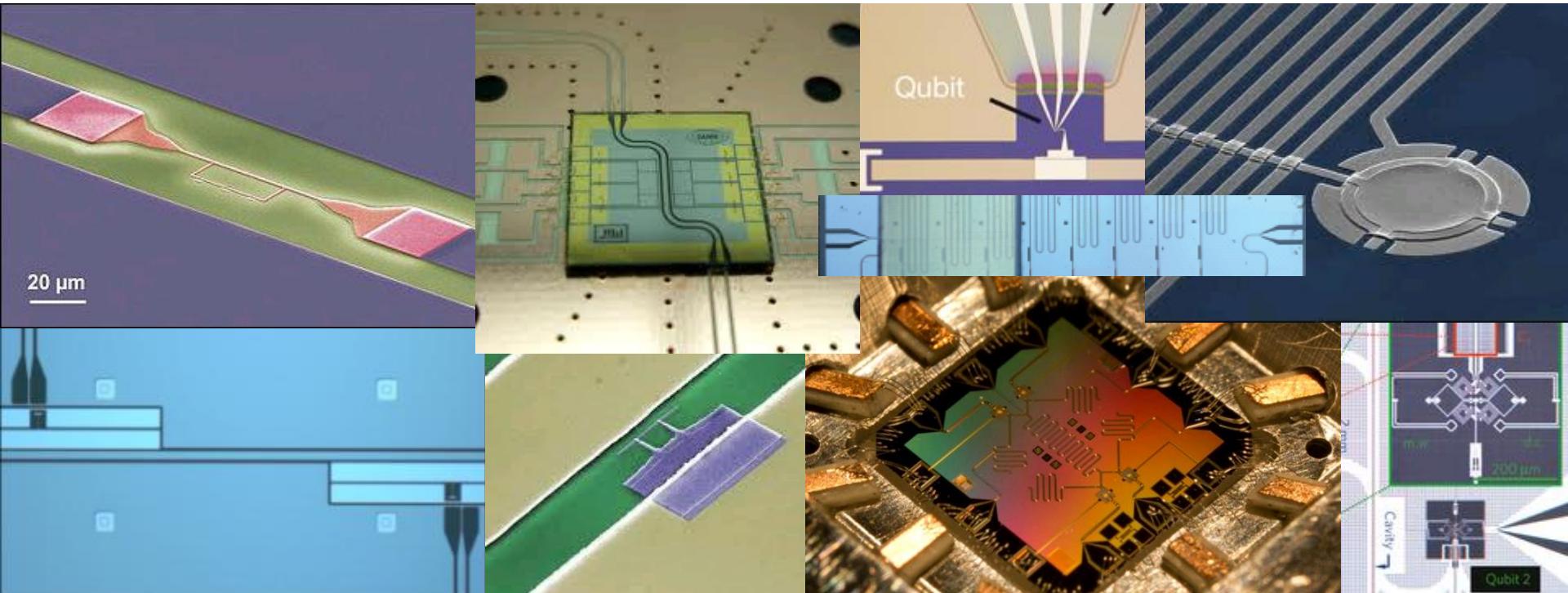
electronic artificial atom



# Superconducting Quantum Electronic Circuits

single or multiple superconducting qubits coupled to harmonic oscillators

- investigated in a few dozen labs around the world
- for basic science and applications

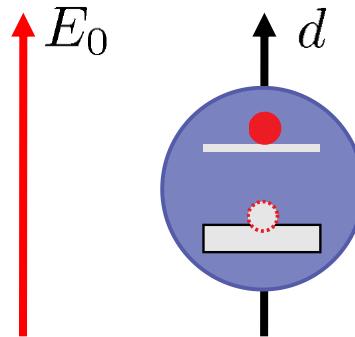


reviews:

- R. J. Schoelkopf, S. M. Girvin, *Nature* **451**, 664 (2008)  
J. Clarke and F. Wilhelm, *Nature* **453**, 1031 (2008)  
J. Q. You and F. Nori, *Nature* **474**, 589 (2011)

# Controlling the Interaction of Photons and Qubits

challenging on the level of single particles

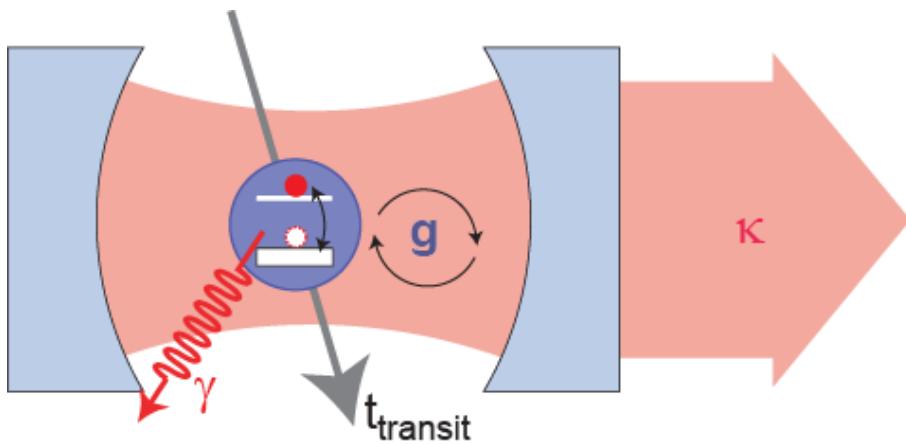


- dipole moment  $d$  in microscopic systems (usually small  $\sim ea_o$ )
- single photon fields  $E_0$  (small in 3D)
- photon/qubit interaction  $\hbar g \sim dE_0$  (usually small)

What to do?

- confine qubit and photon in a cavity (cavity QED)
- engineer qubit/light interaction in solid state circuits

# Cavity QED with Superconducting Circuits



coherent interaction of photons with  
quantum two-level systems ...

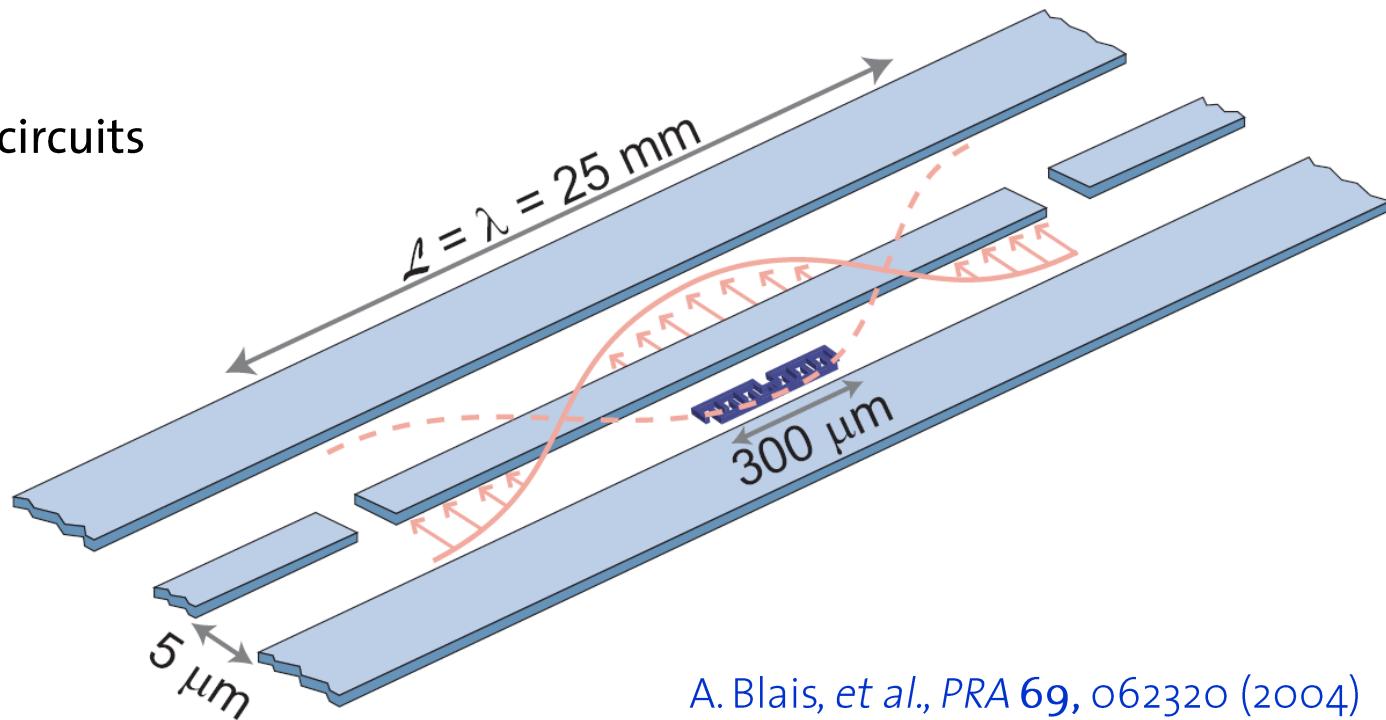
J. M. Raimond et al., *Rev. Mod. Phys.* **73**, 565 (2001)

S. Haroche & J. Raimond, *OUP Oxford* (2006)

J. Ye., H. J. Kimble, H. Katori, *Science* **320**, 1734 (2008)

... in superconducting circuits

circuit quantum  
electrodynamics



A. Blais, et al., *PRA* **69**, 062320 (2004)

A. Wallraff et al., *Nature (London)* **431**, 162 (2004)

R. J. Schoelkopf, S. M. Girvin, *Nature (London)* **451**, 664 (2008)

# Resonant Vacuum Rabi Mode Splitting ...

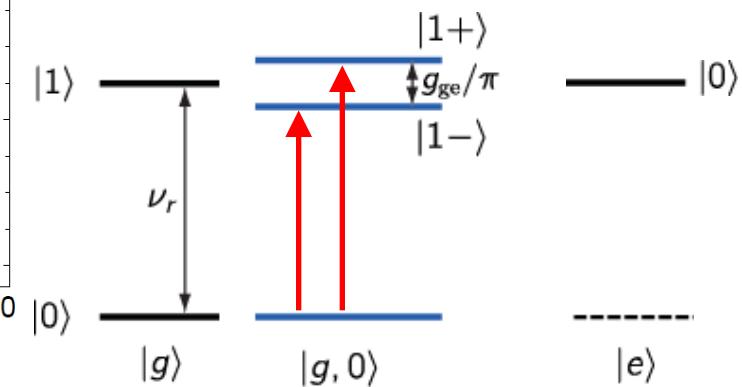
... with one photon ( $n = 1$ ):

very strong coupling:

$$g_{ge}/\pi = 308 \text{ MHz}$$

$$\kappa, \gamma < 1 \text{ MHz}$$

$$g_{ge} \gg \kappa, \gamma$$



forming a 'molecule' of a qubit and a photon

first demonstration in a solid: A. Wallraff et al., *Nature (London)* 431, 162 (2004)

this data: J. Fink et al., *Nature (London)* 454, 315 (2008)

R. J. Schoelkopf, S. M. Girvin, *Nature (London)* 451, 664 (2008)

# Quantum Physics with Circuit QED ... some examples

## Vacuum Rabi Mode Splitting

A. Wallraff *et al.*, *Nature* **431**, 162 (2004)

## Coherent Flux-Qubit / SQUID Coupling

I. Chiorescu *et al.*, *Nature* **431**, 159 (2004)

## Quantum AC-Stark Shift

D. Schuster *et al.*, *Nature* **445**, 515 (2007)

## Lamb Shift

A. Fragner *et al.*, *Science* **322**, 1357 (2008)

## Fock and Arbitrary Photon States

M. Hofheinz *et al.*, *Nature* **454**, 310 (2008)

M. Hofheinz *et al.*, *Nature* **459**, 546 (2009)

## Root n Nonlinearity

J. Fink *et al.*, *Nature* **454**, 315 (2008)

## Two Photon Nonlinearities

F. Deppe *et al.*, *Nat. Phys.* **4**, 686 (2008)

## Parametric Amplification

Castellanos-Beltran *et al.*, *Nat. Phys.* **4**, 928 (2008)

## Super Splitting and Root n Nonlinearity

L. Bishop *et al.*, *Nat. Phys.* **5**, 105 (2009)

## Ultrastrong Coupling

T. Niemczyk *et al.*, *Nat. Phys.* **6**, 772 (2010)

## Single Photon Source

A. Houck *et al.*, *Nature* **449**, 328 (2007)

## Single Qubit MASER

O. Astafiev *et al.*, *Nature* **449**, 588 (2007)

## Single Qubit Resonance Fluorescence

O. Astafiev *et al.*, *Science* **327**, 840 (2010)

## QND Measurement of Single Photon

B. Johnson *et al.*, *Nat. Phys.* **6**, 663 (2010)

## Correlation Function Measurements

D. Bozyigit *et al.*, *Nat. Phys.* **7**, 154 (2011)

## Cooling and Amplification

M. Grajcar *et al.*, *Nat. Phys.* **4**, 612 (2008)

## Quantum Algorithms & Entangled States

L. DiCarlo *et al.*, *Nature* **460**, 240 (2009)

L. DiCarlo *et al.*, *Nature* **467**, 574 (2010)

A. Fedorov *et al.*, *Nature* **481**, 170 (2012)

M. Reed *et al.*, *Nature* **481**, 382 (2012)

## Quantum Bus

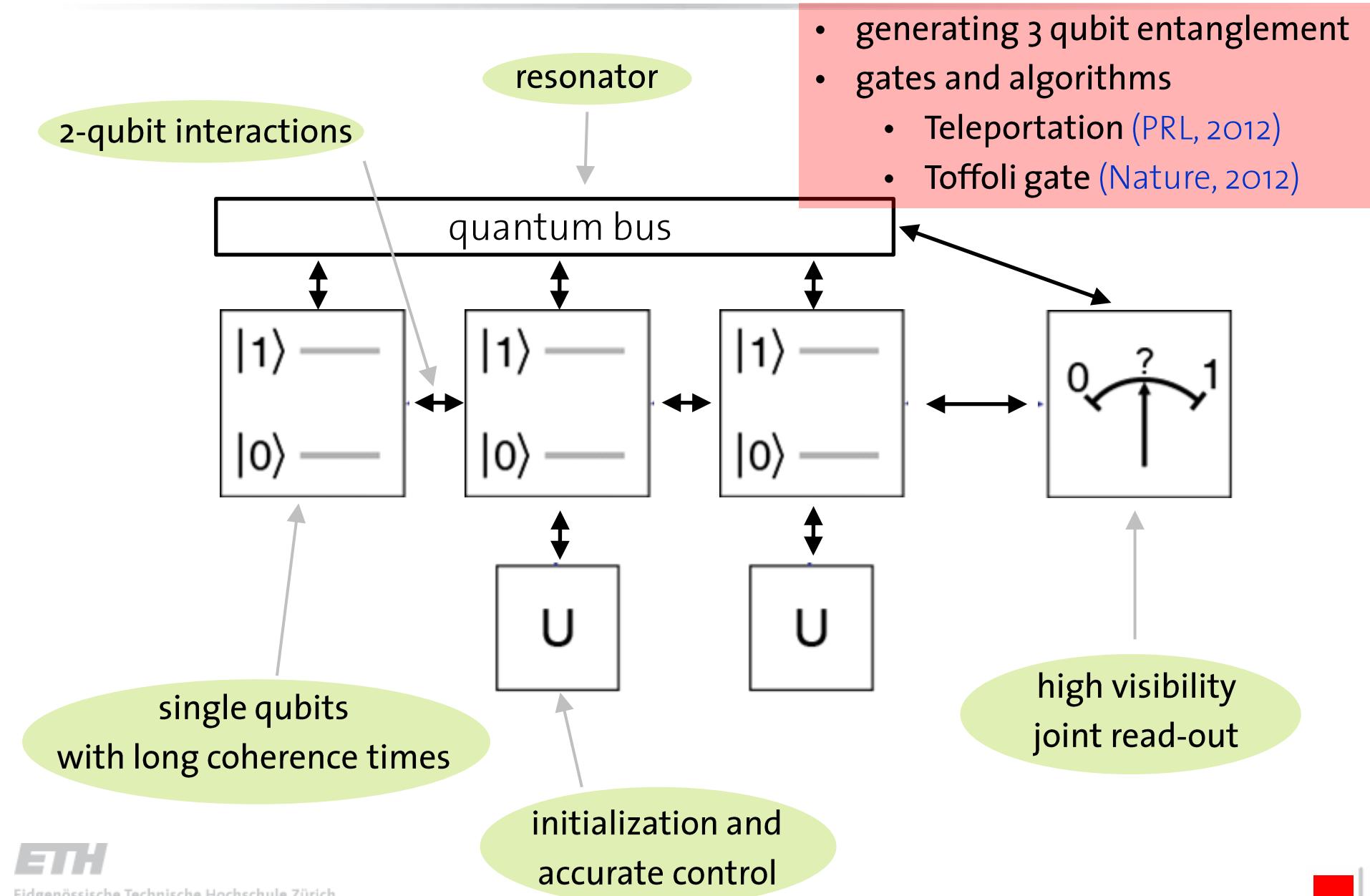
M. Sillanpaa *et al.*, *Nature* **449**, 438 (2007)

H. Majer *et al.*, *Nature* **449**, 443 (2007)

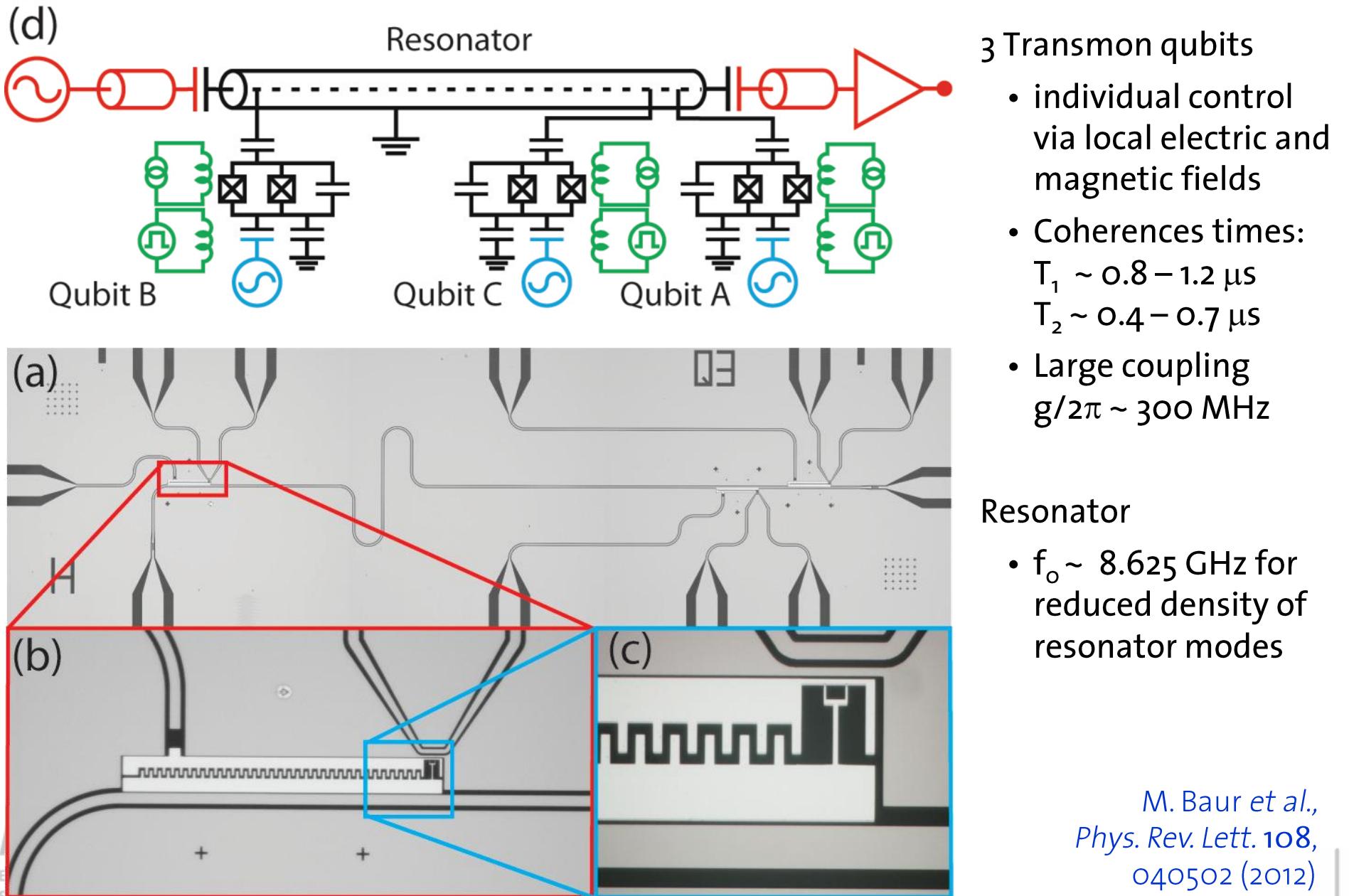
M. Mariantoni *et al.*, *Nat. Phys.* **7**, 287 (2011)

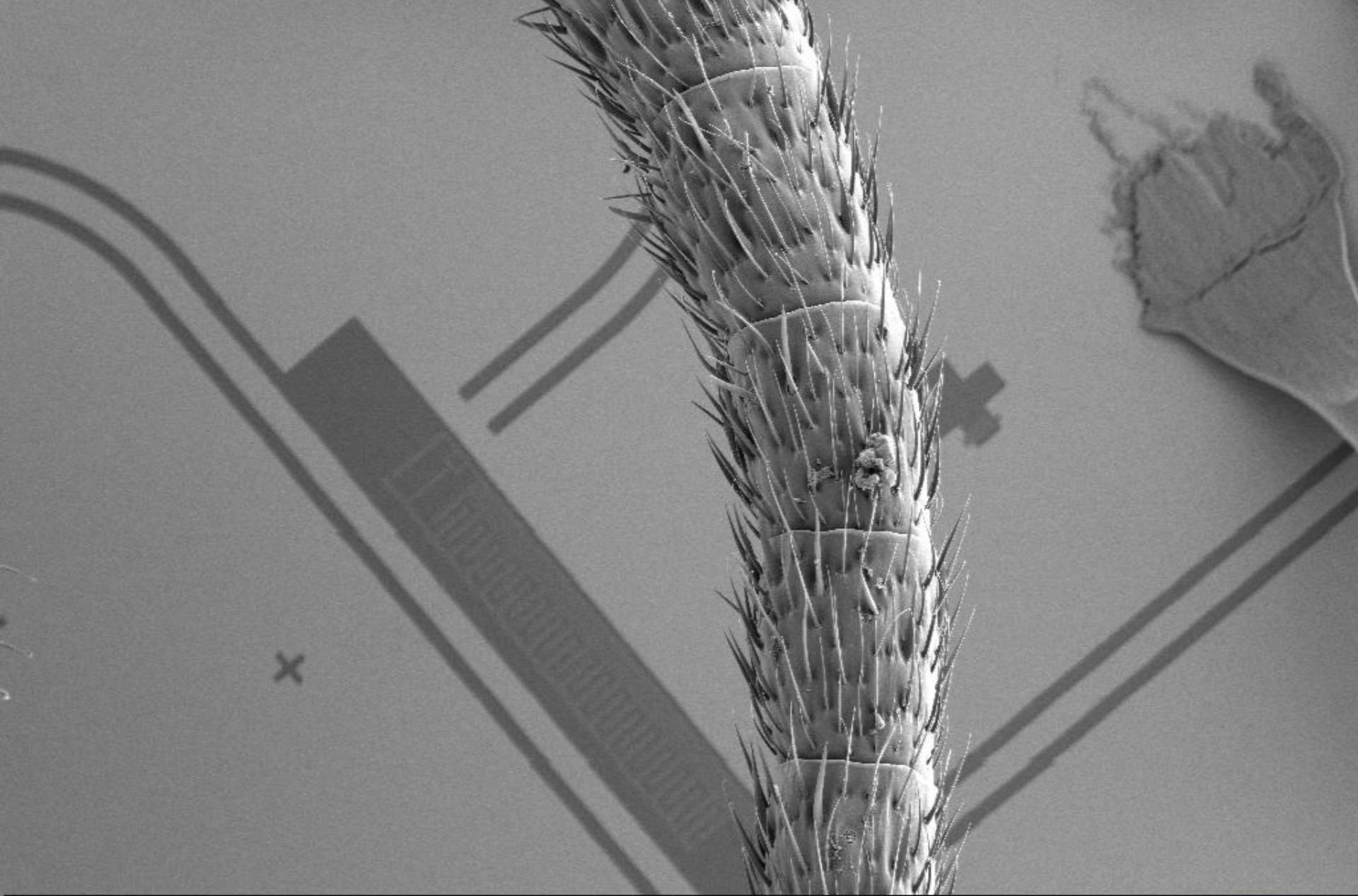
M. Mariantoni *et al.*, *Science* **334**, 61 (2011)

# Quantum Information Processing



# Quantum Processor Platform with 3 Qubits





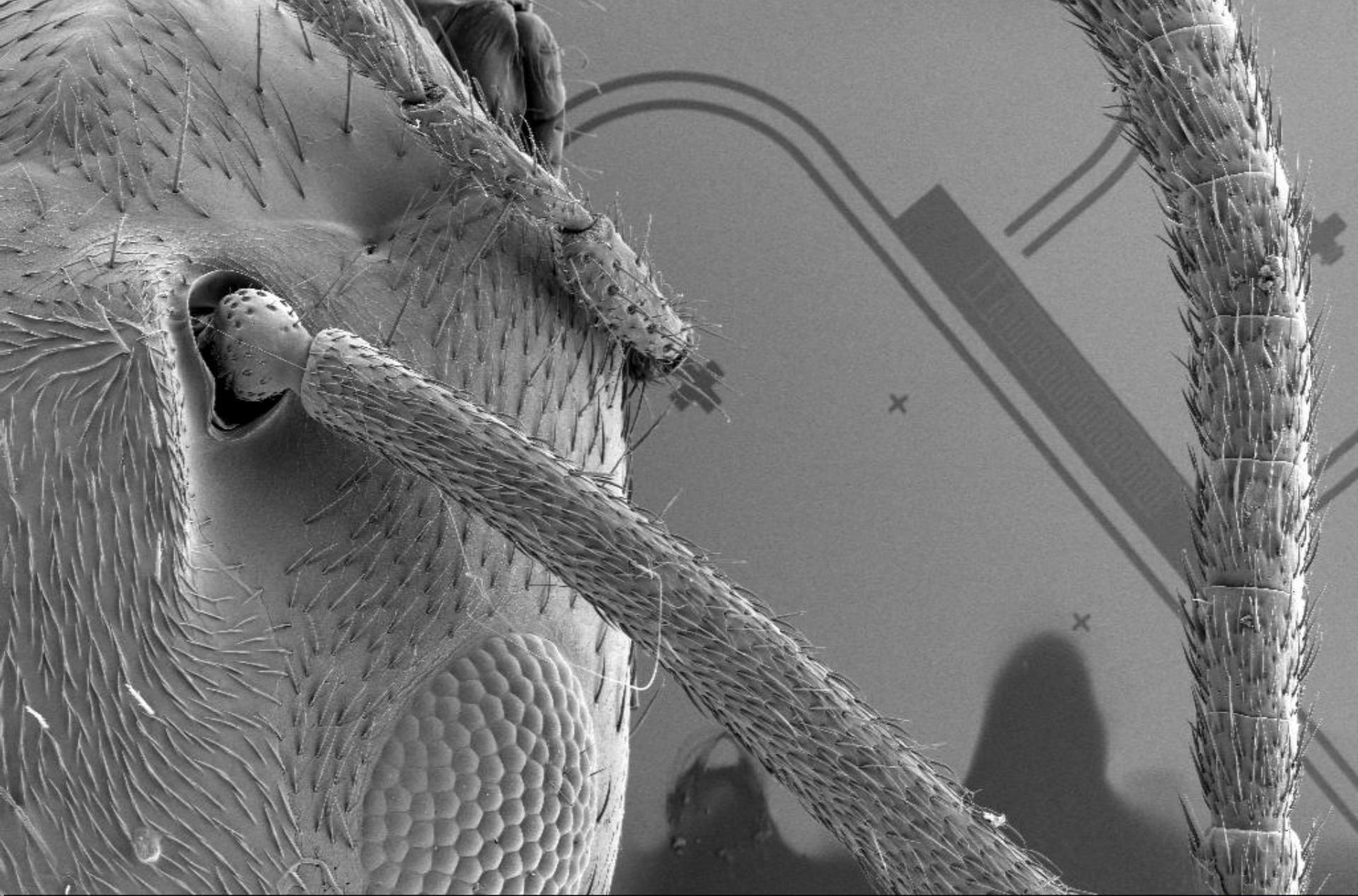
20 µm

EHT = 4.00 kV  
WD = 6.5 mm

Signal A = SE2  
Photo No. = 8311

Date :27 Mar 2012  
Time :17:54:36





100 µm



EHT = 4.00 kV

WD = 6.5 mm

Signal A = SE2

Photo No. = 8347

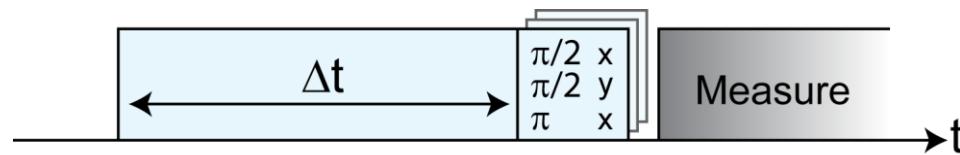
Date :27 Mar 2012

Time :18:38:14

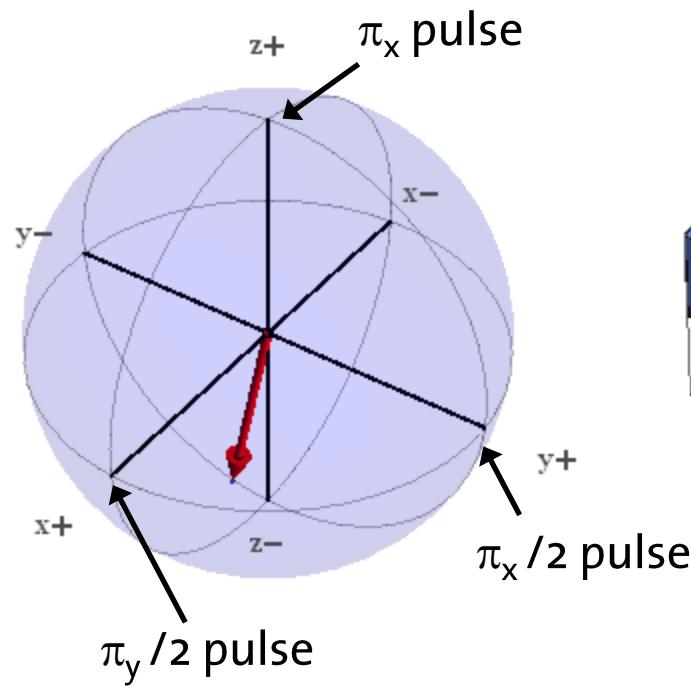


# Single Qubit Operations and Measurement

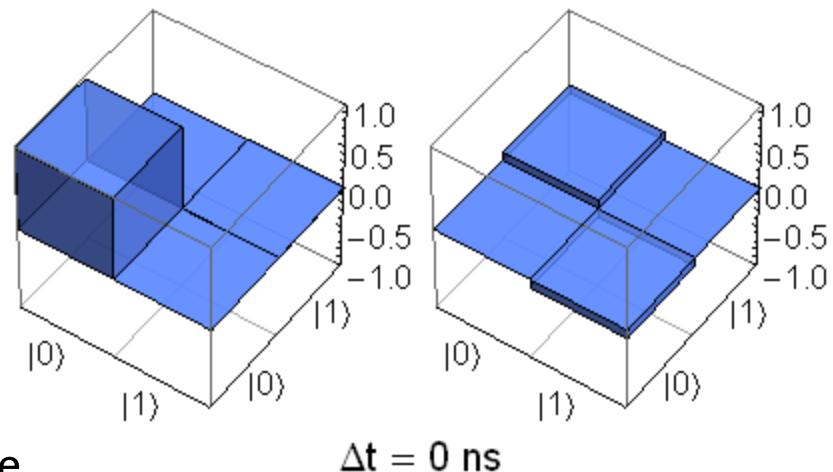
Pulse sequence for qubit rotation and readout



experimental Bloch vector

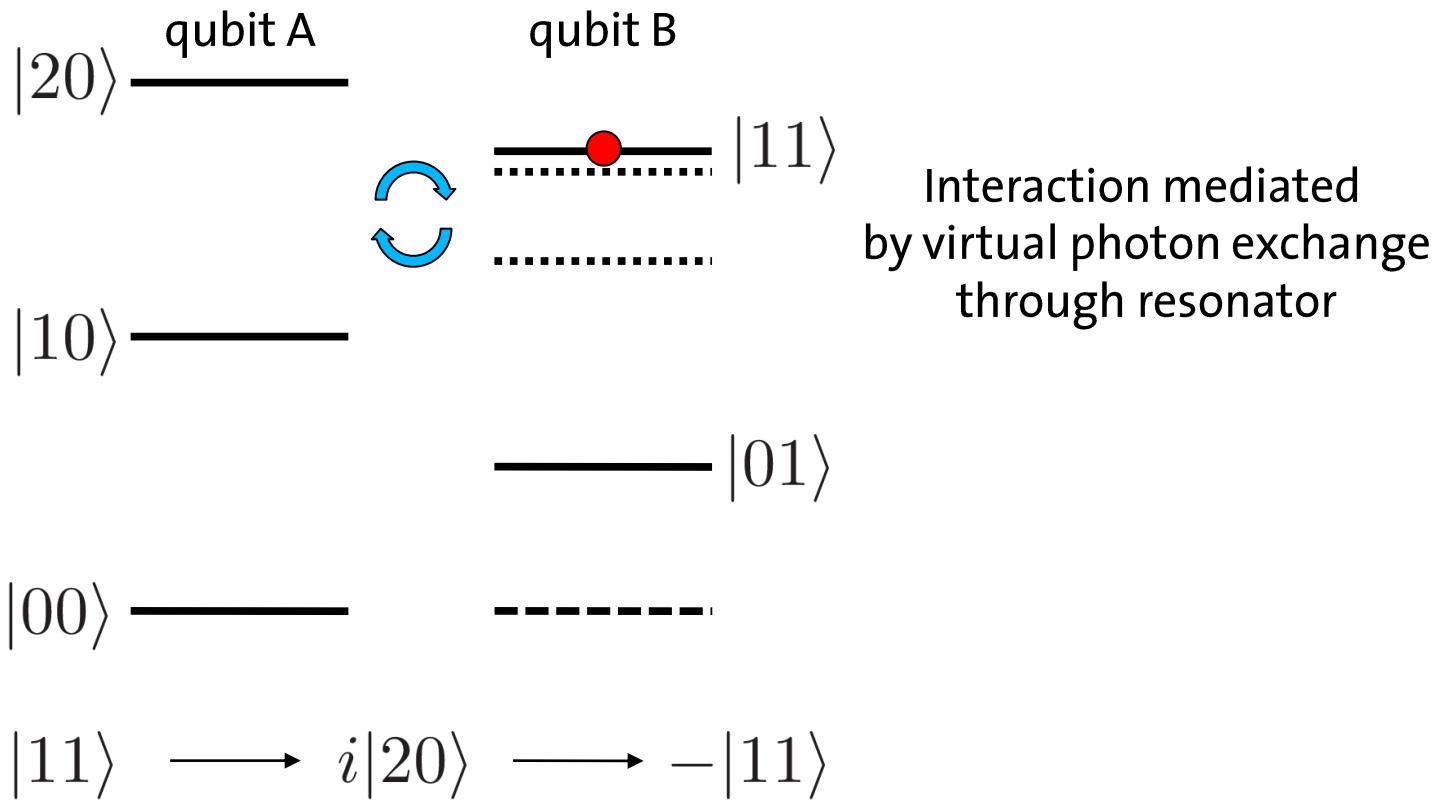


experimental density matrix



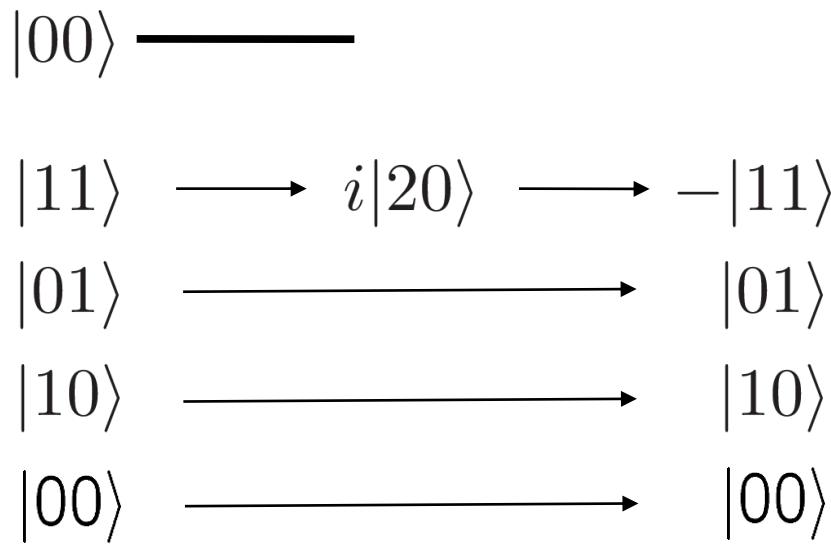
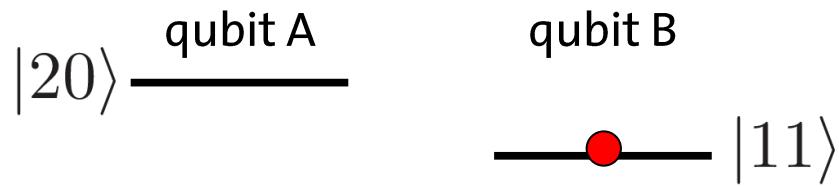
# Universal Two-Qubit Gate

Tune levels into resonance using magnetic field



proposal: F. W. Strauch, *Phys. Rev. Lett.* **91**, 167005 (2003).  
first implementation: L. DiCarlo, *Nature* **460**, 240 (2010).

# Controlled Phase Gate



Universal two-qubit gate. Used together with single-qubit gates to create any quantum operation.

C-Phase gate:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

proposal: F. W. Strauch, *Phys. Rev. Lett.* **91**, 167005 (2003).  
first implementation: L. DiCarlo, *Nature* **460**, 240 (2010).

# Process Tomography: C-Phase Gate

arbitrary  
quantum  
process

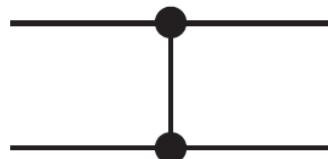
$$\rho' = \mathcal{E}(\rho)$$

decomposed into

$$\mathcal{E}(\rho) = \sum_{mn} \tilde{E}_m \rho \tilde{E}_n^\dagger \chi_{mn}$$

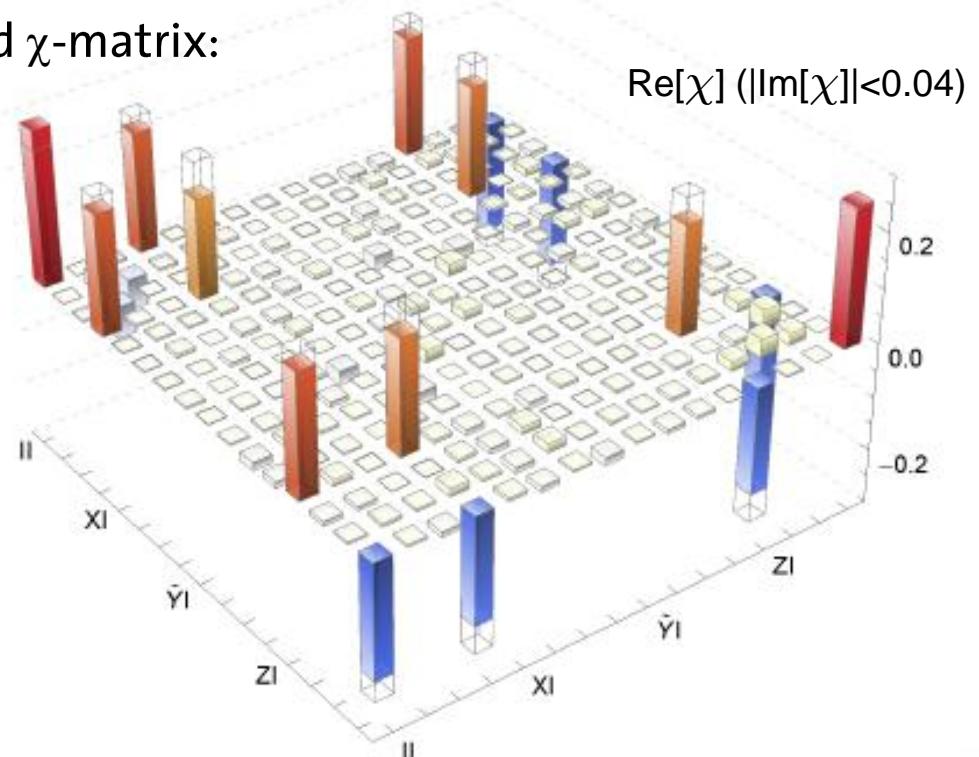
$\{\tilde{E}_k\}$  is an operator basis  
 $\chi$  is a positive semi definite Hermitian matrix  
characteristic for the process

Controlled phase gate



$$cZ_{00} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Measured  $\chi$ -matrix:



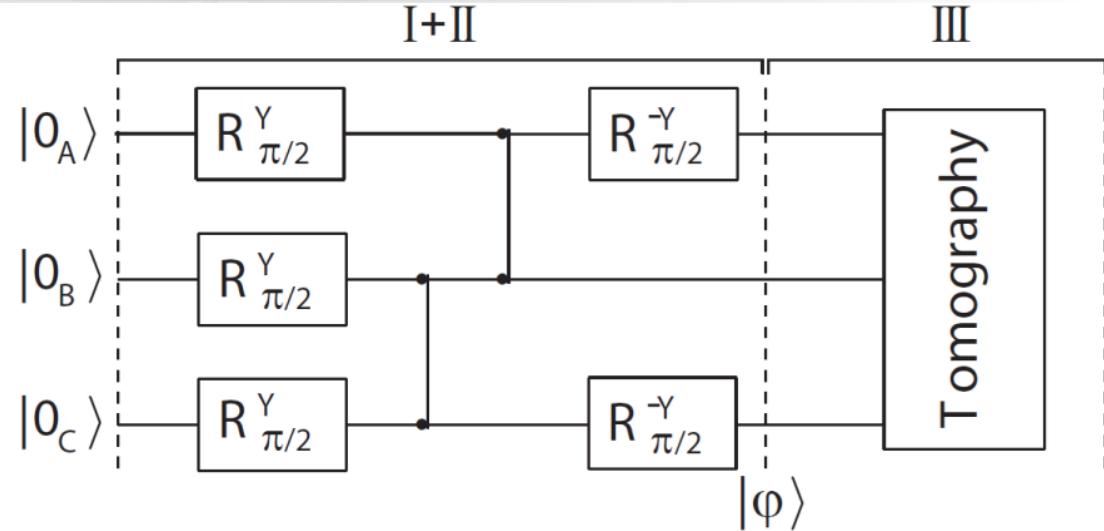
$$F = \text{Tr}[\chi_{\text{meas}} \chi_{\text{ideal}}] = 0.86$$

# Maximally Entangled Three Qubit States

Generation of GHZ class, e.g.

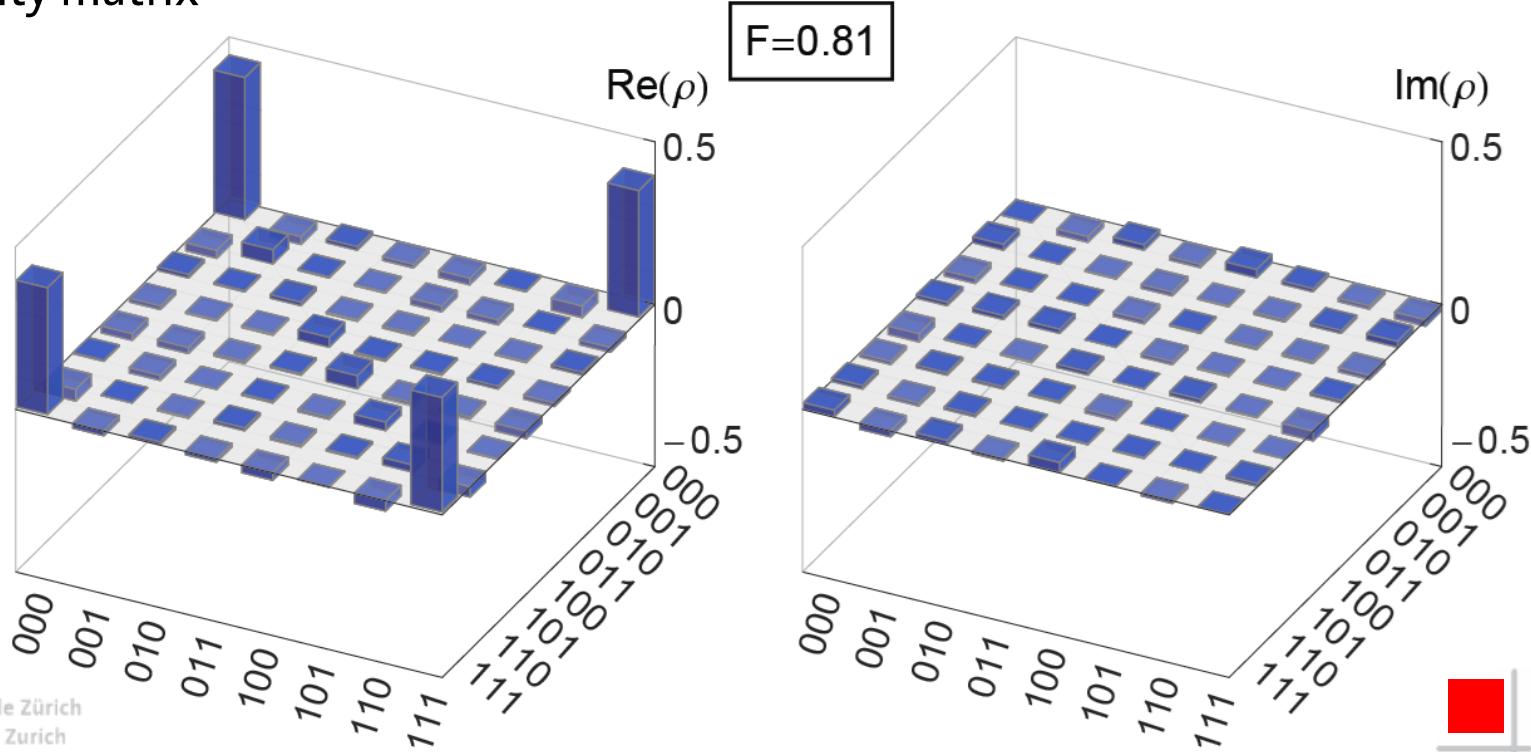
$|000\rangle + |111\rangle$ , states:

- single qubit gates
- C-PHASE gates



Measured density matrix

- high fidelity



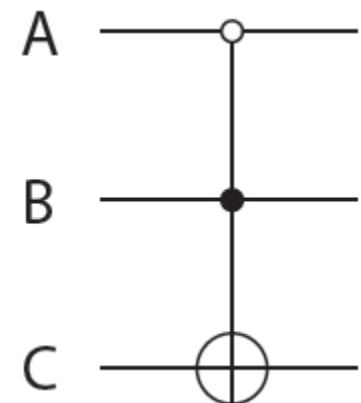
# The Toffoli Gate

proposed by *Tommaso Toffoli* in 1980

- any reversible computation can be performed with only the Toffoli gate

function:

- inverts qubit C only if qubits A and B are in selected basis states



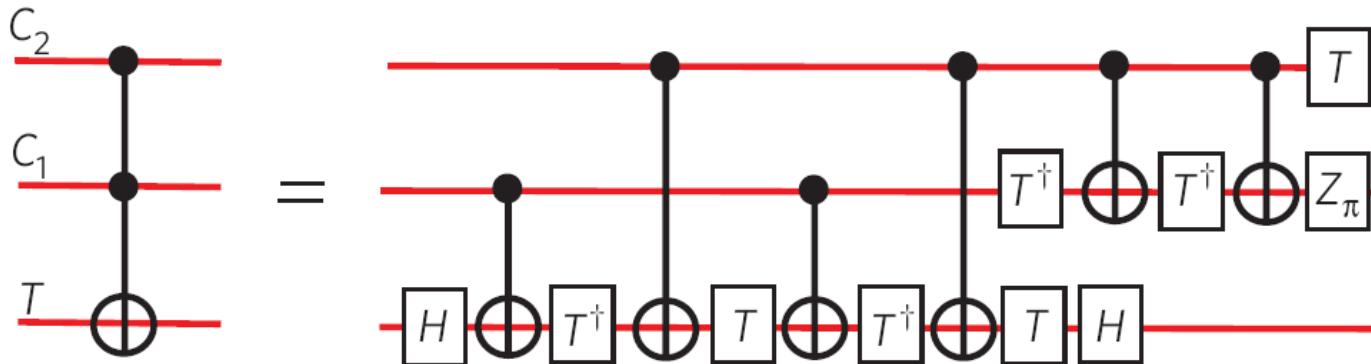
applications:

- for universal reversible classical computation
- for simplification of complex quantum circuits
- used in quantum error-correction schemes  
(essential for any practical quantum processor)

# Implementation of a Toffoli Gate

with only single and two-qubit gates requires:

- 6 CNOT gates
- 10 single qubit gates



Inefficient decomposition not feasible with any solid state system yet!

Idea by T. C. Ralph *et. al.*, PRA 75, 022313 (2007):

- use higher levels (qutrits) for efficient decomposition

# Circuit Diagram

New approach: use qubit-qutrit gates for the *more efficient* decomposition!

CC-PHASE – inverts the sign for only one basis state

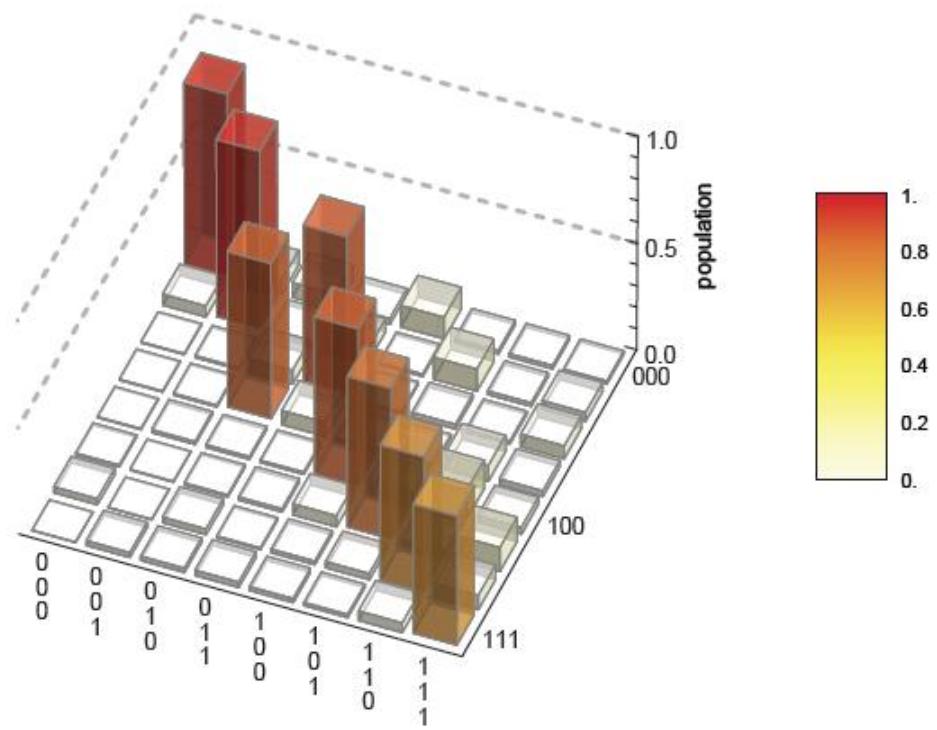
Equivalent to Toffoli up to single qubit rotations

same amount of resources, more efficient

# Truth Table

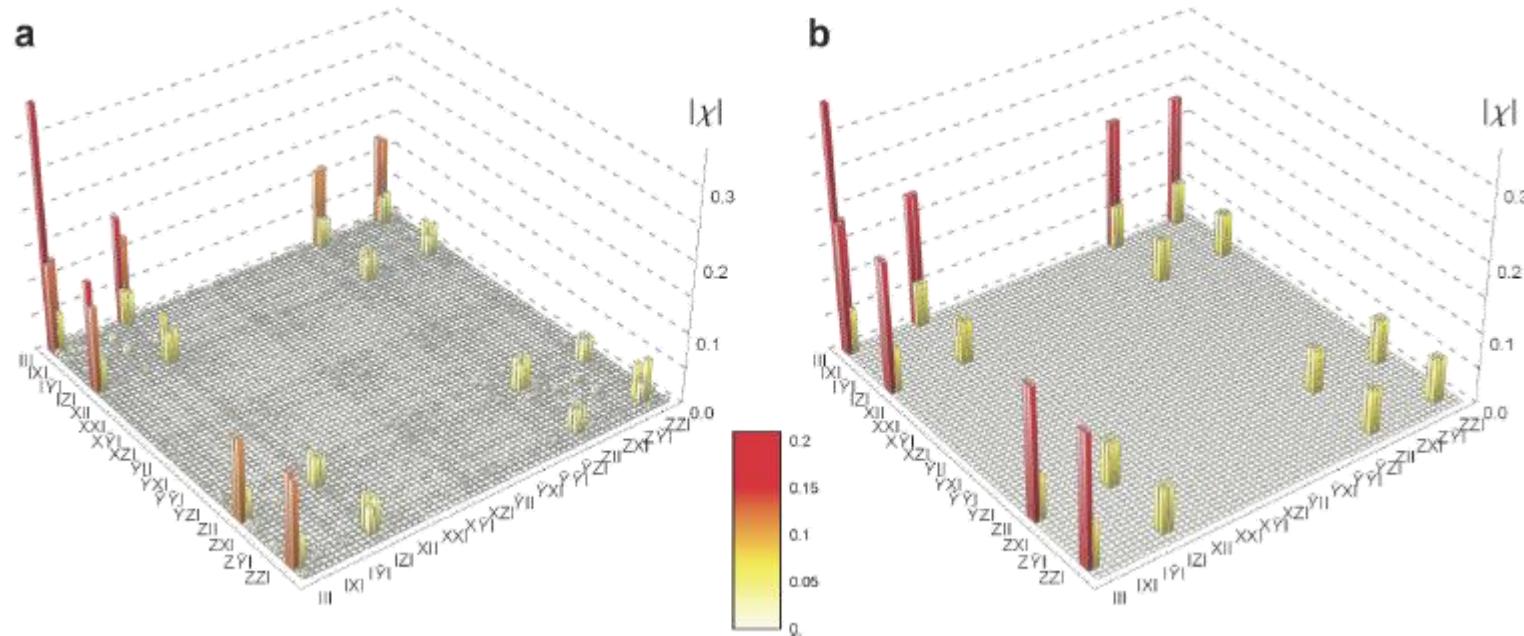
- characterizes the action of the Toffoli gate on the basis input states
- Fidelity

$$F = \frac{1}{8} \text{Tr} [U_{\text{exp}}^T U_{\text{ideal}}] = 76\%$$



# Process Tomography

- Fully characterizes the process by evaluating  $\chi$ -matrix (ML)



- Fidelity  $F = \frac{1}{8} \text{Tr} [\chi_{\text{exp}}^T \chi_{\text{ideal}}] = 69 \pm 3 \%$
- Monte Carlo process certification does not rely on maximum-likelihood procedures [da Silva et al., *PRL* 107, 210404 (2011), Steffen et al., arXiv:1202.5196]

$$F = 68.5 \pm 0.5 \%$$

# Toffoli Gate and Error Correction in Circuits

- Realization and full characterization of 3 qubit Toffoli gate also with efficient process certification  
A. Fedorov et al., *Nature (London)* **481**, 170 (2012)  
L. Steffen et al., arXiv:1202.5196 (2012)
- Realization of Toffoli-class gate with only two qubits (used resonator as 3<sup>rd</sup> qubit) and limited characterization (phase fidelity)  
M. Mariantoni et al., *Science* **334**, 61 (2011)
- First use of Toffoli gate for correcting an induced error  
M. D. Reed et al., *Nature (London)* **482**, 382 (2012)



# Teleportation

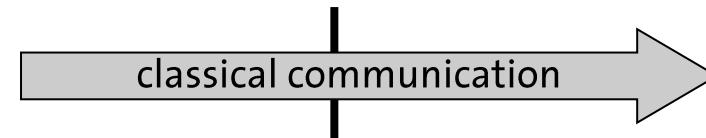
Task:

- transmit state of unknown qubit (A) from Alice to Bob

Resources:

- a pair of entangled qubits (B+C)
- a small quantum computer
- classical communication

Alice



Bob



## Features:

- exploits non-local quantum correlations
- contains the same essential steps as for realizing a quantum computer: important benchmark to pass!

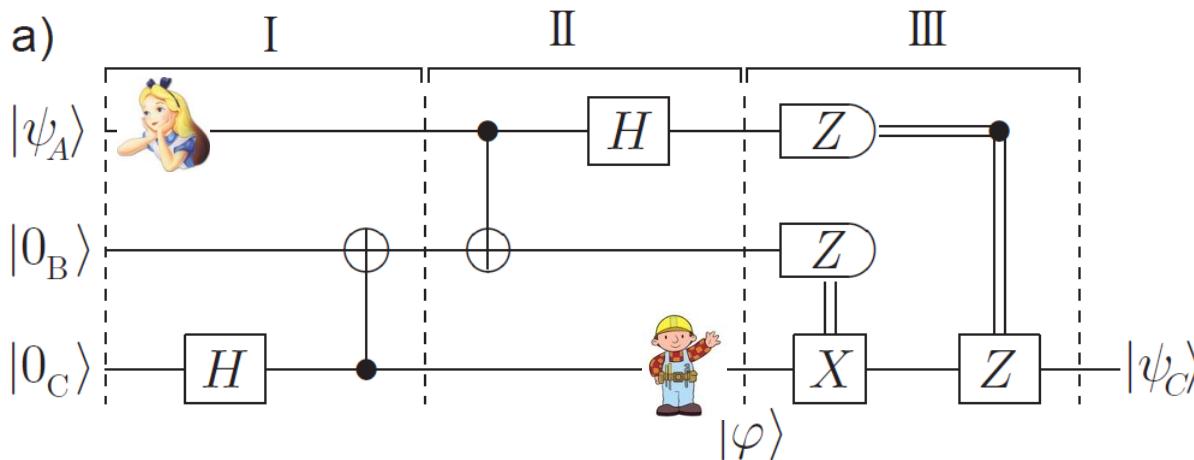
## Applications:

- quantum repeaters
- simplification of quantum circuits
- universal computation

Has been demonstrated for photons and ions, but not yet in any solid state system!

# Quantum Circuit for Teleportation

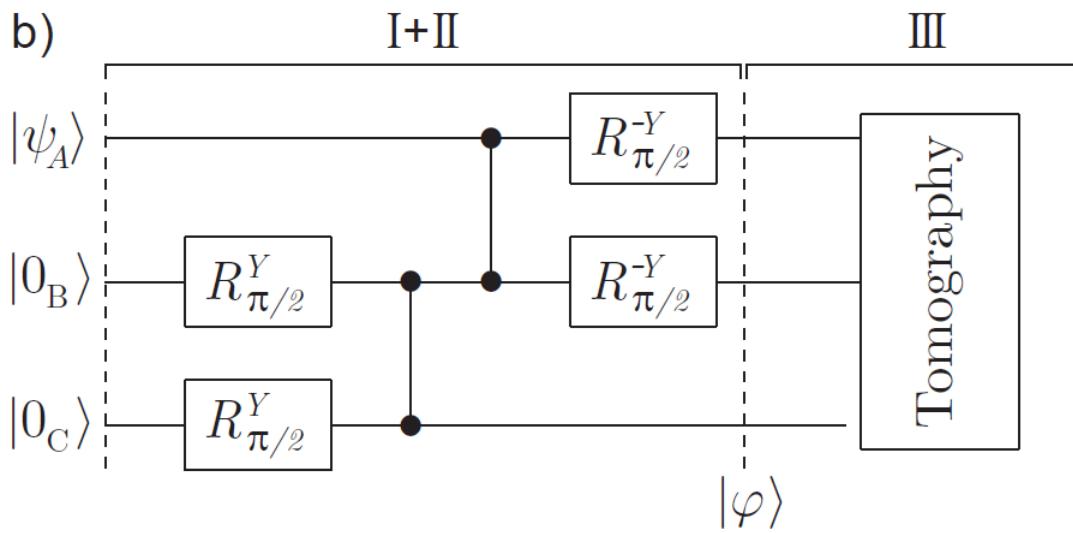
a)



teleportation scheme with single shot measurement and feed-back

here: demonstrate scheme without use of feed-back

b)

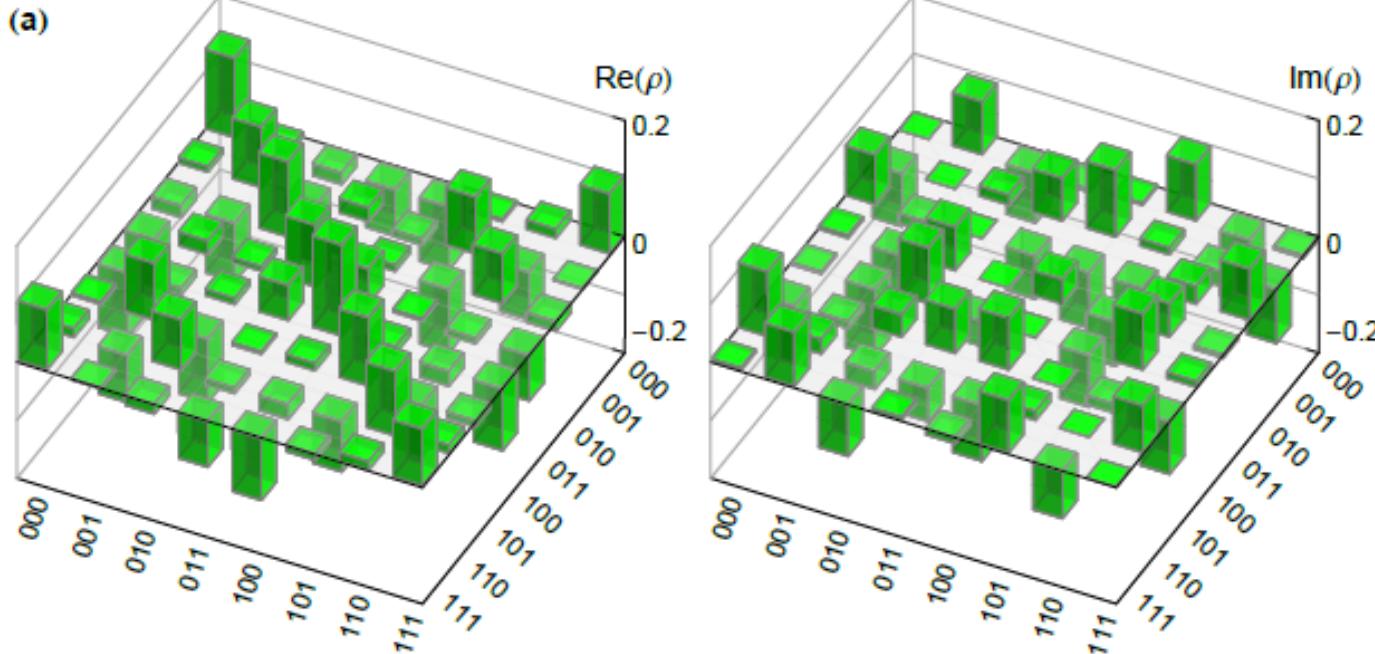


implemented three qubit tomography at step III

projection on A and B to recover density matrix of teleported state

$$|\psi\rangle\langle\psi| = \langle 0_A 0_B | \rho_{A,B,C} | 0_A 0_B \rangle$$

# Characterizing the Full System State



measured 3-qubit entangled state after execution of circuit:

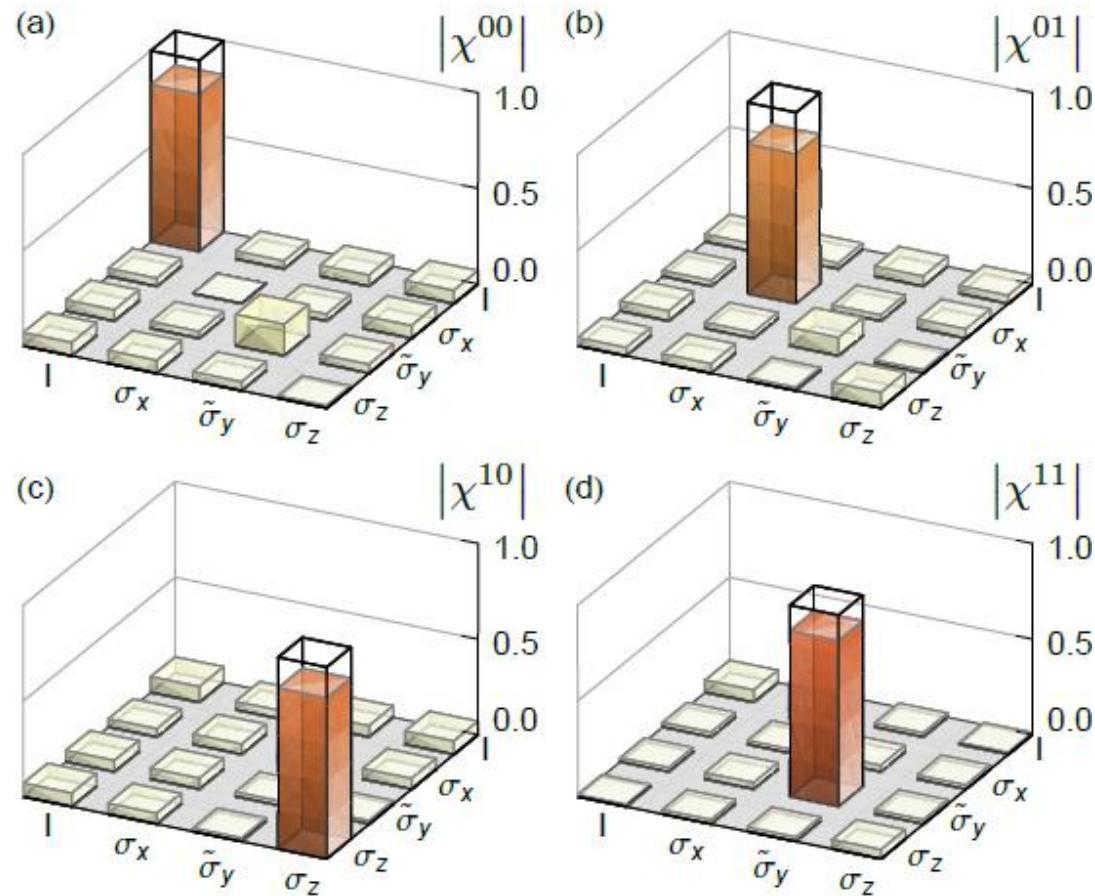
input state:  $|\psi\rangle = (|0\rangle - i|1\rangle)/\sqrt{2}$

density matrix of  
teleported state  
when projecting  
qubits A & B onto  
 $|00\rangle$

# Benchmarking the Teleportation Process

procedure:

- prepare set of input states
- run circuit
- perform tomography on full system state
- project on basis states for qubits A and B
- analyze process acting on qubit C (see a-d)
- average fidelity of teleportation 83%

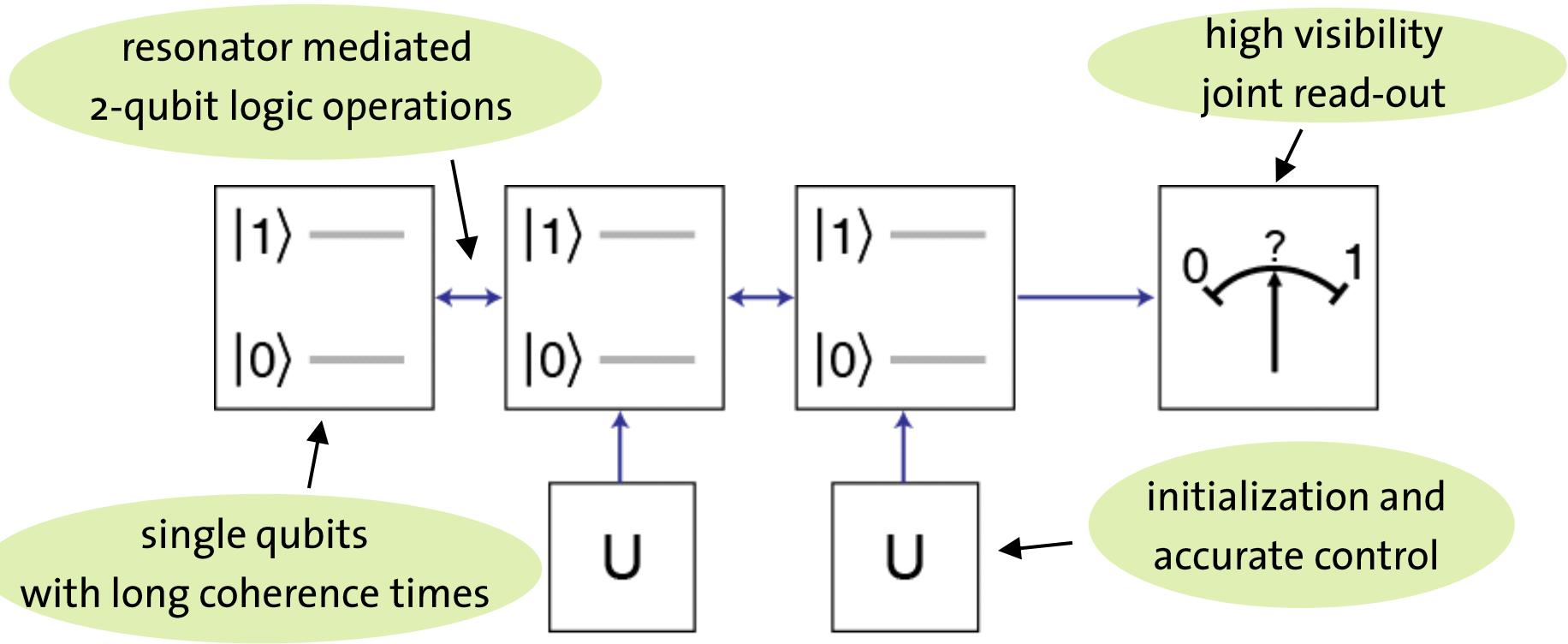


summary:

- verified operation of teleportation circuit & demonstrated 3-qubit quantum processor
- without single-shot projective read-out or feed-back



# Quantum Computing using Circuit QED



## Recently:

- Toffoli gate for error correction  
A. Fedorov et al., *Nature* 481, 170 (2012)
- benchmarking of teleportation circuit  
M. Baur et al., *PRL* 108, 040502 (2012)

## Future challenges:

- improve coherence times
- address scalability questions
- realize feed-back
- solve interesting problems (quantum simulation, error correction)



# The ETH Zurich Quantum Device Lab

Postdoc and PhD positions available



SWISS NATIONAL SCIENCE FOUNDATION



Eidgenössische Technische Hochschule Zürich  
Swiss Federal Institute of Technology Zurich



# Selected Circuit QED Publications

## Circuit QED Proposal:

- Blais et al., *PRA* **69**, 062320 (2004)

## Strong Coupling & Vacuum Rabi Mode Splitting:

- Wallraff et al., *Nature* **431**, 162 (2004)
- Fink et al., *Nature* **454**, 315 (2008)
- Fink et al., *PRL* **105**, 163601 (2010)

## Tavis-Cummings Multi-Atom QED:

- Fink et al., *PRL* **103**, 083601 (2009)

## AC-Stark & Lamb Shift, Autler-Townes and Mollow Transitions

- Schuster et al., *PRL* **94**, 123062 (2005)
- Gambetta et al., *PRA* **74**, 042318 (2006)
- Schuster et al., *Nature* **445**, 515 (2007)
- Fragner et al., *Science* **322**, 1357 (2008)
- Baur et al., *PRL* **102**, 243602 (2009)

## Itinerant Photons, Tomography, Photon Blockade

- da Silva et al., *PRA* **82**, 043804 (2010)
- Bozyigit et al., *Nat. Phys.* **7**, 154 (2011)
- Eichler et al., *PRL* **106**, 220503 (2011)
- Lang et al., *PRL* **106**, 243601 (2011)
- Eichler et al., *PRL* **107**, 113601 (2011)

## One-, Two-, Three-Qubit Gates and Algorithms:

- Wallraff et al., *PRL* **95**, 060501 (2005)
- Blais et al., *PRA* **75**, 032329 (2007)
- Wallraff et al., *PRL* **99**, 050501 (2007)
- Majer et al., *Nature* **449**, 443 (2007)
- Leek et al., *Science* **318**, 1889 (2007)
- Leek et al., *PRB* **79**, 180511(R) (2009)
- Filipp et al., *PRL* **102**, 200402 (2009)
- Leek et al., *PRL* **104**, 100504 (2010)
- Bianchetti et al., *PRL* **105**, 223601 (2010)
- A. Fedorov et al., *Nature* **481**, 170 (2012)
- M. Baur et al., *PRL* **108**, 040502 (2012)

## Hybrid Systems:

- Frey et al., *PRL* **108**, 046807 (2012)
- Hogan et al., *PRL* **108**, 063004 (2012)

## Device Fabrication:

- Frunzio et al., *IEEE Trans. Appl. Sup.* **15**, 860 (2005)
- Goeppel, et al. *J. Appl. Phys.* **104**, 113904 (2008)

## Review (gr.):

- Wallraff, *Physik Journal* **7** (12), 39 (Dez. 2008)

Additional Information: [www.qudev.ethz.ch](http://www.qudev.ethz.ch)