A quantum advantage for causal inference

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Causal explanations are a singularly useful way of organizing one's knowledge about the world. This has long been recognized by philosophers and, in recent decades, motivated the development of a rigorous mathematical framework, which proved a powerful tool for epidemiology, machine learning and many other fields [8, 10]. The challenge that remains is to infer the correct causal explanation for a phenomenon, given that one can only observe statistical correlations.

The simplest case of causal inference involves just two variables, such as treatment and recovery in a drug trial. Statistical correlations between the two can arise because the treatment causally influences recovery, but they may also be due to a hidden common cause. For example, if men are both more likely to take the drug and more likely to recover spontaneously, then gender constitutes a common cause that induces correlations between treatment and recovery.

Statisticians untangle the two possibilities by randomizing the treatment. Since the *assigned* treatment is statistically independent of any potential common causes, any remaining correlations with recovery herald a causal influence. Similarly, if one records the patients' *intent* to treat and tests how this second variable correlates with recovery, one can determine the common-cause relation between treatment and recovery.

We consider the quantum analogue of a randomized trial, which yields a complete characterization of the causal relation between two quantum systems – the *causal map*. Our characterization scheme encompasses the conventional tomography of processes (which connect cause and effect) and bipartite states (which encode correlations due to a common cause) as special cases, but also captures more general scenarios that do not fit into the conventional categories. Objects that are analogous to our causal maps have been studied under a variety of names: quantum combs [2], operator tensors [3], process matrices [7], quantum conditional states [5, 4] and others [1, 6]. In a sense, these objects encode what one can infer about one quantum system after learning about another; however, they are not an exact quantum analogue of a conventional conditional probability distribution, such as $P(recovery|treatment)$. Instead, they make reference to two 'copies' of the early variable. We give meaning to this 'splitting' by drawing a parallel with the classical analysis of randomized trials, where one must distinguish between "intent to treat" and "assigned treatment".

Our main result, however, concerns the case wherein one does not have the power to intervene, for instance when one is observing distant astronomical objects. We begin by clarifying how this restriction

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limits one's ability to infer causal relations. In a randomized drug trial, one obtains statistically independent information about the variables "intent to treat" and "assigned treatment" and this independence allows one to distinguish common-cause from cause-effect relations. If, however, what one learns about the actual treatment coincides with the intent to treat for all subjects, then the analysis becomes inconclusive. We term this condition *informational symmetry*, and one can show that, in the case of two classical variables, this constraint makes causal inference impossible (unless strong additional assumptions are warranted).

Surprisingly, in an analogous scenario with two quantum variables, we find that one can sometimes determine the causal relation despite the informational symmetry constraint.

The key property that allows one to distinguish common-cause from cause-effect relations between two quantum systems is that bipartite states, which represent common-cause relations, are positive operators, whereas the analogous operators representing quantum channels, viz. cause-effect relations, must be positive under partial transposition (PPT) on the input system [5]. If the observed correlations are only compatible with an operator that is positive, but not PPT, then one can rule out a cause-effect relation, and analogously for common-cause.

A simple example illustrates how this formal difference is reflected in the pattern of correlations one might observe in an experiment. The same Pauli observable σ_s is measured either (a) on two qubits in the singlet state, or (b) on the same qubit, before and after it is subjected to the identity channel; ranging over the settings $s = x, y, z$ over the course of many runs. In scenario (a), one finds perfect anti-correlations between the outcomes for all three measurement settings, $s = x, y, z$. Notably, this pattern of correlations could not arise from a quantum channel, because it would suggest that one is probing the universal NOT gate, which is not a completely positive map. One can therefore rule out a cause-effect relation. Conversely, scenario (b) yields perfect correlations for $s = x, y, z$. This is not compatible with a common-cause explanation, because it would require the 'anti-singlet' state, which is not a positive operator.

To complement the theoretical results, we perform a linear optics experiment wherein we control the causal relation between two qubits, realizing either purely common-cause, purely cause-effect or a probabilistic mixture. Using the quantum analogue of a randomized trial, we characterize the causal relation between the qubits, achieving above 97% fidelity with the expected causal map. We also test our technique for causal inference under the informational symmetry constraint. We implement a probabilistic mixture of causal relations with probability of common-cause p_{exp}, record the correlations between measurement outcomes and analyse the data to determine the best-fitting probability p_{fit}. Our technique yields reconstructed p_{fit} that are extremely close to the implemented p_{exp}, with a root-mean-square deviation of only 0.023.

Our results show that entanglement and coherence provide a distinctly quantum advantage for causal inference. The approach of identifying concrete tasks that are easier in the quantum world is not new, and similar considerations in the context of information processing and cryptography proved extremely fruitful, both in terms of technological applications and by providing a new perspective on foundational questions. More specifically, the advantage for causal inference depends on whether the operators in question are positive under partial transposition; a fact that may provide operational significance to a class of operators that was so far mostly of mathematical interest. On a more practical level, we note that causal inference is closely related to the study of open system quantum dynamics, where our tools may prove useful; in particular as tests of Markovianity.

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