Logical pre- and post-selection paradoxes are proofs of contextuality

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joint work with Matthew S. Leifer
The three box paradox is a proof of contextuality

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Three boxes

Prepare $|1\rangle + |2\rangle + |3\rangle$
Three boxes

Prepare $|1\rangle + |2\rangle + |3\rangle$

Post-select $|1\rangle + |2\rangle - |3\rangle$
Three boxes

Prepare $|1\rangle + |2\rangle + |3\rangle$

Post-select $|1\rangle + |2\rangle - |3\rangle$

Intermediate measurement “Look in box 1”:
\[ \{ |1\rangle \langle 1|, |2\rangle \langle 2| + |3\rangle \langle 3| \} , \]

or “Look in box 2”:
\[ \{ |2\rangle \langle 2|, |1\rangle \langle 1| + |3\rangle \langle 3| \} . \]
Three boxes

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Post-select $|1\rangle + |2\rangle - |3\rangle$

Intermediate measurement “Look in box 1”: 
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or “Look in box 2”: 
\{ |2\rangle \langle 2|, |1\rangle \langle 1| + |3\rangle \langle 3| \}.
Three boxes

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Intermediate measurement “Look in box 1”: 
{$|1\rangle \langle 1| , |2\rangle \langle 2| + |3\rangle \langle 3|}$

or “Look in box 2”: 
{$|2\rangle \langle 2| , |1\rangle \langle 1| + |3\rangle \langle 3|}$. 
Three boxes

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or “Look in box 2”:
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Three boxes

Prepare $|1\rangle + |2\rangle + |3\rangle$

Post-select $|1\rangle + |2\rangle \quad \text{--} \quad |3\rangle$

Intermediate measurement “Look in box 1”:
$\{ |1\rangle \langle 1| , |2\rangle \langle 2| + |3\rangle \langle 3| \}$,

or “Look in box 2”:
$\{ |2\rangle \langle 2| , |1\rangle \langle 1| + |3\rangle \langle 3| \}$.
Kochen-Specker non-contextuality

1. *Outcome determinism for projective measurements*: One outcome of a projective measurement is assigned probability 1, the rest 0.

2. *Measurement non-contextuality for projective measurements*: The assignment to a projector is independent of the other outcomes in the measurement.
Avoiding KS contextuality

quant-ph/0412179
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arXiv:1207.3114
Relation to KS contextuality

quant-ph/0412178
Generalised non-contextuality$^1$

If two procedures are equivalent at the operational level, then they are equivalent at the ontological level.

$^1$quant-ph/0406166
Generalised non-contextuality\textsuperscript{1}

If two procedures are equivalent at the operational level, then they are equivalent at the ontological level.

“Procedures” encompasses preparations, transformations and measurements.

\textsuperscript{1}quant-ph/0406166
Necessity of disturbance
Two equivalent transformations

\[ P \rho P + Q \rho Q \]
Two equivalent transformations

\[ P \rho P + Q \rho Q \]

\[ = \frac{1}{2} (P \rho P + Q \rho Q + P \rho P + Q \rho Q) \]
Two equivalent transformations

\[ P \rho P + Q \rho Q \]

\[ = \frac{1}{2} (P \rho P + Q \rho Q + P \rho P + Q \rho Q) \]

\[ = \frac{1}{2} (P \rho P + Q \rho P + P \rho Q + Q \rho Q - Q \rho P - P \rho Q + Q \rho Q) \]
Two equivalent transformations

\[ P \rho P + Q \rho Q \]

\[ = \frac{1}{2} (P \rho P + Q \rho Q + P \rho P + Q \rho Q) \]

\[ = \frac{1}{2} (P \rho P + Q \rho P + P \rho Q + Q \rho Q - Q \rho P - P \rho Q + Q \rho Q) \]

\[ = \frac{1}{2} ((P + Q) \rho (P + Q) + (P - Q) \rho (P - Q)) \]
Two equivalent transformations

\[ P \rho P + Q \rho Q \]

\[ = \frac{1}{2} (P \rho P + Q \rho Q + P \rho P + Q \rho Q) \]

\[ = \frac{1}{2} (P \rho P + Q \rho P + P \rho Q + Q \rho Q - Q \rho P - P \rho Q + Q \rho Q) \]

\[ = \frac{1}{2} ((P + Q) \rho (P + Q) + (P - Q) \rho (P - Q)) \]

\[ = \frac{1}{2} (\rho + (P - Q) \rho (P - Q)) \]
Two equivalent transformations

\[ P\rho P + Q\rho Q = \frac{1}{2} (\rho + U\rho U^\dagger) \]

where \( U = P - Q \).
Read the paper, arXiv:1506.07850 for...

- All logical pre-and post-selection paradoxes (e.g. “quantum pigeonhole principle”)
- Measurement non-contextuality instead of transformation non-contextuality
- Weak measurement versions
- Importance of 0/1 probabilities, von-Neumann update rule.