Fermionic quantum theory and superselection rules for operational probabilistic theories

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Joint work with
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Outline

1. Are fermions systems of the usual quantum theory?
2. Fermions as “bricks” of a new operational probabilistic theory
3. Informational features:
   - tomography in fermionic quantum theory
   - fermionic entanglement
4. A definition of superselection for a general probabilistic theory:
   - fermionic and real QT as special cases
5. Future perspectives
1. Are fermions systems of the usual quantum theory?

Simulating Physics with Computers

Richard P. Feynman

Department of Physics, California Institute of Technology, Pasadena, California 91107

Received May 7, 1981

The question is, if we wrote a Hamiltonian which involved only these operators, locally coupled to corresponding operators on the other space-time points, could we imitate every quantum mechanical system which is discrete and has a finite number of degrees of freedom? I know, almost certainly, that we could do that for any quantum mechanical system which involves Bose particles. I’m not sure whether Fermi particles could be described by such a system. So I leave that open. Well, that’s an example of what I meant by a general quantum mechanical simulator. I’m not sure that it’s sufficient, because I’m not sure that it takes care of Fermi particles.
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Particle physics

Quantum theory
Relativity: fermionic field

Fermions are anti-commuting systems

\[ \{ \varphi_i, \varphi_j \} = 0 \quad \{ \varphi_i, \varphi_i^\dagger \} = \delta_{ij} I \]
Particle physics

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\[ |\psi\rangle = |\text{even particles}\rangle + |\text{odd particles}\rangle \]

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No a priori \textit{superselection}

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$\{\psi\} = \left\langle \text{odd particles} \right| + \left\langle \text{even particles} \right|$

$\left\langle \text{even particles} \right| + \left\langle \text{odd particles} \right|$
Usual approach to the problem

**Local fermionic mode**: system

\[ |0\rangle \quad \text{empty} \]

\[ \varphi^\dagger |0\rangle \quad 1 \text{ fermion} \]
Usual approach to the problem

Local fermionic mode: system

\[ \varphi^\dagger |0\rangle \rightarrow 1 \text{ fermion} \]

\[ |0\rangle \rightarrow \text{empty} \]

\( N \) Local Fermionic modes (LFM) \( \cong \) \( N \) qubits

Jordan-Wigner isomorphism
Usual approach to the problem

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Price to pay for anti-commutation

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isomorphism

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Price to pay for anti-commutation

\[ N \text{ Local Fermionic modes (LFM)} \]

\[ \cdots \]

\[ \cdots \]

\[ N \text{ qubits} \]

\[ \cdots \]

\[ \cdots \]

2 Warnings: Where does parity superselection come from? What do I map via the Jordan-Wigner map?
2. Fermions as “bricks” of a new operational probabilistic theory

Elementary systems:

*local fermionic modes*
2. Fermions as “bricks” of a new operational probabilistic theory

**Construction of the theory:**

States and maps in terms of the fields $\varphi_i, \varphi_i^\dagger$
2. Fermions as "bricks" of a new operational probabilistic theory

Elementary systems: local fermionic modes

Construction of the theory:

- States and maps in terms of the fields $\varphi_i^$, $\varphi_i^{\dagger}$
- Kraus operator $T(\rho) = \sum_i s_i K_i \rho K_i^{\dagger}$
- States $\rho := \sum_j K_j |\Omega\rangle \langle \Omega| K_j^{\dagger}$

2. Fermions as “bricks” of a new operational probabilistic theory

Construction of the theory:

\[ \mathcal{T}(\rho) = \sum_i s_i K_i \rho K_i^\dagger \]

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Operational assumptions:

The field \( \varphi \) must be “physical”: Maps with single Kraus \( \alpha \varphi + \beta \varphi^\dagger \) are maps of the theory.
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**Construction of the theory:**

Notion of *local operations*: A map made of fields on some modes \( \Rightarrow \) Local on that modes

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**Formalism:**

\( T \subseteq \mathcal{L}(\mathcal{F}) \)

\[ \chi = \{ T(\rho) \} \]

**Proposition:** Fermionic states satisfy the **parity superselection rule**

Fock space: \( \mathcal{F} = \mathcal{F}_e \oplus \mathcal{F}_o \)

Any state is of the form:

\[
\rho = \begin{pmatrix}
    p \rho_e & 0 \\
    0 & (1 - p) \rho_o
\end{pmatrix}
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Fock space: \( \mathcal{F} = \mathcal{F}_e \oplus \mathcal{F}_o \)

- **even sector**
- **odd sector**

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2-LFM:

\[\alpha |00\rangle + \beta |11\rangle\]

\[\alpha |00\rangle + \beta |10\rangle\] (crossed out)

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1-LFM:

\[\alpha|0\rangle + \beta|1\rangle\]

NOTICE: 1-LFM = classical bit
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NOTICE: no conserved quantity
3. Informational features: tomography for fermionic quantum theory
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Local tomography:

Alice and Bob determine the state by local measurements

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Local tomography:

Alice and Bob determine the state by local measurements

$$D_{AB} = D_A D_B$$

Non-local tomography: local measurements are not enough

$$D_{AB} > D_A D_B$$
Quantum theory

Local tomography

$$D_{AB} = D_A D_B$$

Fermionic quantum theory

$$? = \rho$$
Quantum theory

\[ D_{AB} = D_A D_B \]

Fermionic quantum theory

\[ ? = \rho \]
Quantum theory

Local tomography

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Fermionic quantum theory

\[ \rho = \rho + \rho \]
Quantum theory

Local tomography

\[ D_{AB} = D_A D_B \]

Fermionic quantum theory

\[ D_{AB} > D_A D_B \]

\[ D_{ABC} \leq f(D_A, D_B, D_C, D_{AB}, D_{AC}, D_{BC}) \]

\[ \text{NOTICE: this bound is saturated} \]

3. Informational features: fermionic entanglement

1) Fix a notion of entanglement: **non-separability**
2) Quantify amount of entanglement in operational terms: we choose **Entanglement of formation**
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Quantum entanglement of formation  
Fermionic entanglement of formation
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Quantum entanglement of formation

\[ |\Psi\rangle_{res} \otimes N \xrightarrow{\text{LOCC}} \rho \otimes M \]

$N$ resource states

$M$ copies of $\rho$

Fermionic entanglement of formation

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Proposition:

$$E_F(\rho) \geq p_e E(\rho_e) + p_o E(\rho_o)$$

$$p_e \rho_e + p_o \rho_o$$

Mixed states with maximal entanglement of formation

\[
|\Psi_e\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)
\]

\[
|\Psi_o\rangle = \frac{1}{\sqrt{2}} (|01\rangle + |10\rangle)
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\[
\rho_\star = \frac{1}{2} |\Psi_e\rangle \langle \Psi_e| + \frac{1}{2} |\Psi_o\rangle \langle \Psi_o|
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As qubits state it has no entanglement

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\[ \rho_\ast = \frac{1}{2} |\Psi_e \rangle \langle \Psi_e | + \frac{1}{2} |\Psi_o \rangle \langle \Psi_o | \]

\[ \rho_e \]

\[ \rho_o \]

\[ E(\rho_\ast) = 0 \]

\[ \rho_\ast = \frac{1}{2} |+\rangle \langle +| \otimes 2 + \frac{1}{2} |-\rangle \langle -| \otimes 2 \]
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As Fermionic state has max entanglement

\[ \rho_* = \frac{1}{2} |\Psi_e\rangle \langle \Psi_e | + \frac{1}{2} |\Psi_o\rangle \langle \Psi_o | \]

\[ E(\rho_*) = 0 \]

\[ \rho_* = \frac{1}{2} |+\rangle \langle + | \otimes^2 + \frac{1}{2} |-\rangle \langle - | \otimes^2 \]

\[ p_e \rho_e + p_o \rho_o \]

\[ E_F(\rho) \geq p_e E(\rho_e) + p_o E(\rho_o) \]

\[ E_F(\rho_*) \geq \frac{1}{2} E(\rho_e) + \frac{1}{2} E(\rho_o) \]

Fermionic entanglement is not monogamous

Quantum entanglement is monogamous

3-qubits: $|\Psi\rangle_{ABC}$

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Quantum entanglement is monogamous

3-qubits: $|\Psi\rangle_{ABC} = |\Psi\rangle_{AB} \otimes |\Psi\rangle_{C}$

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Quantum entanglement is *monogamous*

3-qubits:  \[ |\Psi\rangle_{ABC} = |\Psi\rangle_{AB} \otimes |\Psi\rangle_{C} \]

\[ E(\rho_{AB}) + E(\rho_{AC}) \leq 1 \]

1 0


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\[ \begin{array}{cc}
1 & 0
\end{array} \]


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3-LFMs:

\[ |\Psi\rangle_{ABC} = \frac{1}{2} (|001\rangle + |010\rangle + |100\rangle + |111\rangle) \]

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3-qubits: \( |\Psi\rangle_{ABC} = |\Psi\rangle_{AB} \otimes |\Psi\rangle_C \)

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\begin{array}{c}
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3-LFMs:

\( |\Psi\rangle_{ABC} = \frac{1}{2}(|001\rangle + |010\rangle + |100\rangle + |111\rangle) \)

\( \rho_{AB} = \rho_{AC} = \rho_{BC} = \rho_* \) \quad \( E_F(\rho_*) = 1 \)

4. Definition of superselection for probabilistic theory

How to define $\sigma$?
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4. Definition of superselection for probabilistic theory

How to define $\sigma$?

-system $A \in \Theta$ fully specified by its set of states $\text{St}(A)$
-system $\bar{A} \in \bar{\Theta}$ $\text{St}(\bar{A})$ linear section of $\text{St}(A)$

$\sigma : \Theta \rightarrow \bar{\Theta}$ $(A, \text{St}(A)) \mapsto (\bar{A}, \text{St}(\bar{A}))$

$\text{St}(\bar{A}) := \{ \rho \in \text{St}(A) \mid (s_i^\sigma | \rho) = 0, \quad i = 1, \ldots, V_A^\sigma \}$

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linear constraint

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system $\bar{A} \in \bar{\Theta}$ $St(\bar{A})$ linear section of $St(A)$

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$$St(\bar{A}) := \{ \rho \in St(A) \mid (s_i^\sigma | \rho) = 0, \quad i = 1, \ldots, V_A^\sigma \}$$
Superselection-holism tradeoff

Open question: how does the tomography of the theory change after superselection?

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Question:
Θ local tomographic $\sigma$ tomography of $\Theta$ ?

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$\Theta$ local tomographic $\xrightarrow{\sigma}$ tomography of $\Theta$ ?

The intuition

less local measures

tradeoff

less states

Superselection-holism tradeoff

Open question: how does the tomography of the theory change after superselection?

Question:
\[ \Theta \rightarrow \sigma \rightarrow \text{tomography of } \Theta \]

Lemma: number of constraints on composite systems
\[ L(\sigma, A, B) \leq V_{AB}^\sigma \leq U(\sigma, A, B) \]

The intuition

Superselection-holism tradeoff

**Open question:** how does the tomography of the theory change after superselection?

**Question:**
\[ \Theta \text{ local tomographic } \xrightarrow{\sigma} \text{ tomography of } \bar{\Theta} ? \]

**Lemma:** number of constraints on composite systems
\[ L(\sigma, A, B) \leq V_{AB}^{\sigma} \leq U(\sigma, A, B) \]

**Proposition:** If \( \Theta \) satisfies *local tomography* then

i) **minimal SSR** \[ V_{AB}^{\sigma} = L(\sigma, A, B) \quad \Rightarrow \quad \bar{\Theta} \text{ bilocal tomographic} \]

ii) **maximal SSR** \[ V_{AB}^{\sigma} = U(\sigma, A, B) \quad \Rightarrow \quad \bar{\Theta} \text{ local tomographic} \]

\[ D'\text{Ariano, G.M., Manessi, F., Perinotti, P., Tosini, A. EPL 107(2), 20,009 (2014)} \]
Fermionic and Real quantum theory

Qubit

Rebit

bit
Fermionic and Real quantum theory

1 linear constraint

\[ \text{Tr}[\rho \sigma_y] = 0 \]
Fermionic and Real quantum theory

- **Qubit**: 1 linear constraint
  \[ \text{Tr}[\rho \sigma_y] = 0 \]

- **Rebit**: 2 linear constraints
  \[ \text{Tr}[\rho \sigma_y] = 0 \]
  \[ \text{Tr}[\rho \sigma_x] = 0 \]
Fermionic and Real quantum theory

Qubit

1 linear constraint
\[ \text{Tr}[\rho \sigma_y] = 0 \]

Extended minimally to composite systems

Real quantum theory

Rebit

2 linear constraints
\[ \text{Tr}[\rho \sigma_y] = 0 \]
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Extended minimally to composite systems

Fermionic quantum theory

bit
Fermionic and Real quantum theory

Real and Fermionic quantum theory are
(the only two)
minimal SSR of QT

### Real quantum theory

1 linear constraint
\[ \text{Tr}[\rho \sigma_y] = 0 \]

Extend minimally
to composite systems

### Fermionic quantum theory

2 linear constraints
\[ \text{Tr}[\rho \sigma_y] = 0 \]
\[ \text{Tr}[\rho \sigma_x] = 0 \]

Extend minimally
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Fermionic and Real quantum theory

Real and Fermionic quantum theory are (the only two) minimal SSR of QT


minimal SSR => bilocal tomography
5. Future perspectives
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Quantum Theory has been proved to be an operational theory of information processing


Axioms regarding how information can or cannot be manipulated
5. Future perspectives

Quantum Theory has been proved to be an operational theory of information processing.


**Question**: informational derivation of Fermionic quantum theory?

What is missing?
5. Future perspectives

Quantum Theory has been proved to be an operational theory of **information processing**


**Question:** informational derivation of Fermionic quantum theory?

What is missing?

- Spacetime?
- Dynamical quantities?
- Equation of their evolution?
- “mechanical” side?
5. Future perspectives

Quantum Theory has been proved to be an operational theory of information processing


Question: informational derivation of Fermionic quantum theory?

What is missing?

Spacetime?
Dynamical quantities?
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Question: extend the operational informational framework to Quantum Field Theory

Alternative to Algebraic Quantum Field Theory