Complete positivity and natural representation of quantum computations

QPL'15

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Outline

Types for quantum computation

How to build representations of completely positive maps

Application: Quantum domain theory

Concluding remarks

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Where we are, sofar

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- Void: $[\![()]\!] = \mathbb{C}$
- Natural numbers: $[\![\text{nat}]\!] = \bigoplus_{n \in \mathbb{N}} \mathbb{C}$

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  - unital: preserves the unit.
  - positive: preserves observables
    - positive element: $a = x^*x$ for some x.
    - observables are determined by positive elements.

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- Our contribution: a method to consider complete positive maps as natural families of positive maps.

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What is a representation?

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- Representation of $\mathbf{C}$ in $\mathbf{R}$
  - full and faithful functor

$$F : \mathbf{C} \to \mathbf{R}$$

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  - Natural isomorphism

$$\mathbf{C}(A, B) \cong \mathbf{R}(F(A), F(B))$$

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- Biggest advantage: it gives more structure to types without altering the interpretation of programs.

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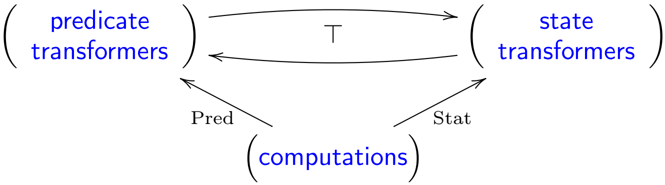

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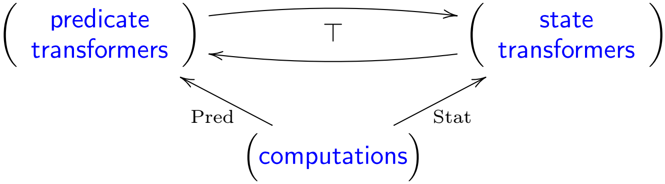

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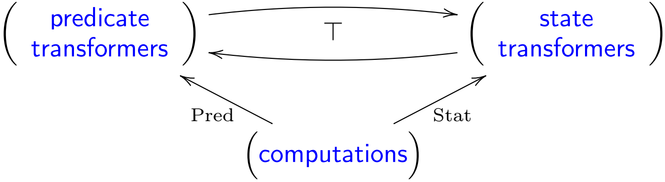

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- **Goal: Make this view compositional for quantum computation**

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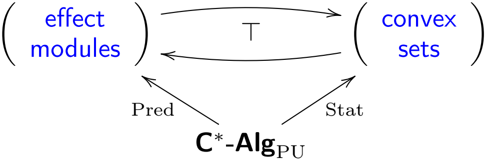

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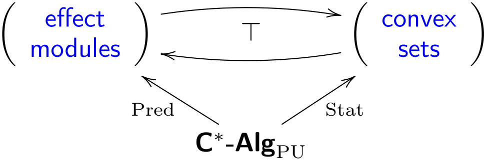

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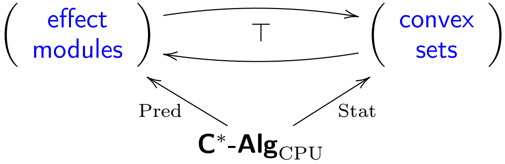

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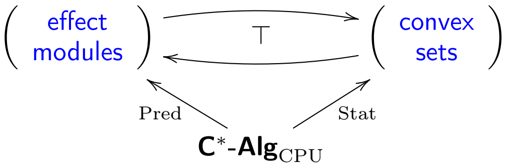

- Pred and Stat are **NOT** representations.

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- Functor $M : \mathbf{C^*\text{-}Alg}_{\mathrm{CPU}} \to [\mathbb{N}_{\mathrm{Isom}}, \mathbf{C^*\text{-}Alg}_{\mathrm{PU}}]$
  - (C*-algebra) $\mapsto$ (indexed family of C*-algebras)
    $$M(A) = \{M_n(A)\}_n$$
  - (CPU-map) $\mapsto$ (natural family of PU-maps)
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- You get a representation of $\mathbf{C^*\text{-}Alg}_{\mathrm{CPU}}$ in $[\mathbb{N}_{\mathrm{Isom}}, \mathbf{R}]$!
- Crucial point: Representing a completely positive map as a natural family of maps rather than as a unique map.

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- Example: unit interval of the reals.

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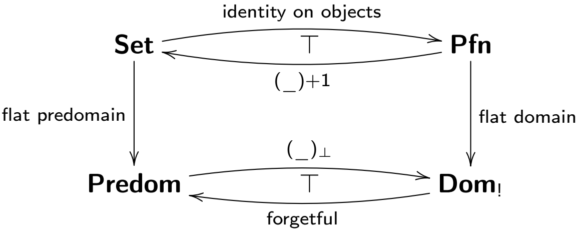

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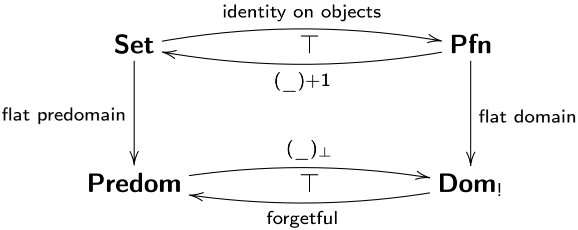

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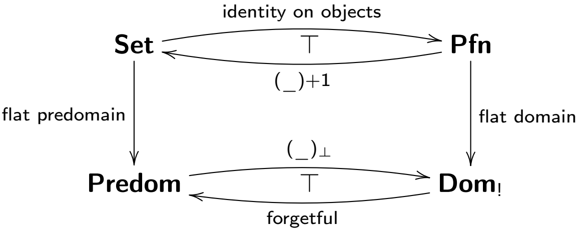

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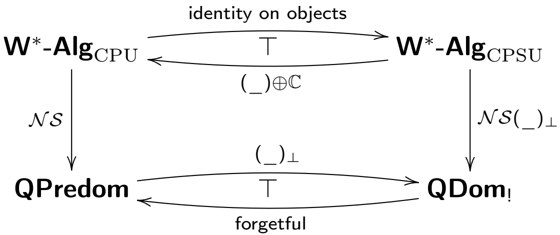

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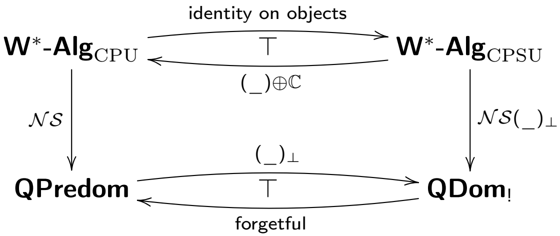

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cf. S. Staton. POPL'15.

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- Algebraic compactness and quantum (pre)domains (to appear).

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The functor $M : \mathbf{C^*\text{-}Alg}_{\mathrm{CPU}} \to [\mathbb{N}_{\mathrm{CPU}}, \mathbf{C^*\text{-}Alg}_{\mathrm{PU}}]$ *yields a representation of* $\mathbf{C^*\text{-}Alg}_{\mathrm{CPU}}$ *in* $[\mathbb{N}_{\mathrm{CPU}}, \mathbf{C^*\text{-}Alg}_{\mathrm{PU}}]$.

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- Trick for quantum domain theory: replacing Scott-continuous maps by natural families of Scott-continuous maps.