Probabilistic general relativity with agency in an operational framework \(^1\)

Lucien Hardy

Perimeter Institute, Waterloo, Ontario, Canada

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\(^1\)Work in progress
**Objective:** To develop an operational formulation of General Relativity that accommodates ignorance probabilities and agency: PAGeR. Homage to the Blackberry pager

**Motivation:** As a step in developing a theory of Quantum Gravity (QGPL2016?)
The generalized state is a mathematical object, $A$, associated with an object, $A$, which can be used to calculate the value of those properties we are interested for this object.

Typically in physics a state pertains to a given time and is used to make predictions for later times. The generalized state is a more general notion than this since we may be making predictions of a more general type. A key question is how do we calculate the generalized state for a composite object? We propose the following principle.

**THE COMPOSITION PRINCIPLE:** The generalized state for a composite object can be calculated from the generalized states for the components by means of a calculation having the same structure as the description of the composition of that object.
Prelude: example

We can associate generalized states

\[ A^{a_1b_2} \rightarrow A^{a_1b_2} \]
\[ B^{c_3} \rightarrow B^{c_3} \]
\[ C_{a_1} \rightarrow C_{b_2c_3} \]

with

\[ A^{a_1b_2} B^{c_3} \rightarrow A^{a_1b_2} B^{c_3} \]

this is in accord with the composition principle.
Earlier work

This is part of an ongoing project (papers on the arXiv).

- 2005 The causaloid framework: A framework for probabilistic theories with indefinite causal structure
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Related work

- Chris Fuchs, Rüdiger Schack, - Emphasized agent centric approach to Quantum Foundations (QBism)
- Samson Abramsky and Bob Coecke’s categorical (pictorial) approach to quantum theory. This emphasizes compositionality.
- Generalized probability theories (going back to Mackey) - much recent work.
- Some space-time approaches to QT:
  - Quantum causal histories - Markopoulou; Dual point of view - Blute, Ivanov and Panangaden;
  - Aharonov and co-workers - multitime states.
  - General Boundary formulation - Oeckl;
  - Quantum combs - Chiribella, D’Ariano, and Perinotti; Oeckl’s positive formulation.
  - Various axiomatic approaches to QT (LH, Dakic and Brukner, Masanes Müller, CDP, . . . ) (in particular the tomographic locality axiom).
  - Leifer and Spekkens Quantum Bayesian Inference, more recent work by Henson, Lal, and Pusey.
  - Indefinite causal structure: Brukner, Oreshkov, Costa, Cerf
Fuchs Picture for Quantum Cosmology
General Relativity: fields

In GR have a set of fields

$$\Phi = (g_{\mu\nu}, \text{matter fields})$$

where the matter fields can be things like

- $J^{\mu[a]}$ - the current of fluid of type $a$,
- $F^{\mu\nu}$ - the electromagnetic field
- etc.
General Relativity: solutions

We have a set of coupled partial differential equations (including Einstein’s field equations). Solve and find a solution

$$\Psi = \{(p, \Phi) : \forall p \in \mathcal{M}\}$$

Note that, if $\Psi$ is a solution, then so is

$$\varphi^* \Psi = \{(p, \varphi^* \Phi) : \forall p \in \mathcal{M}\}$$

(and it has the same physical content). Here $\varphi$ is a diffeomorphism and it “moves” the fields on $\mathcal{M}$. 
Introducing agency: the agency field

E.g. two fluids *ship* and *wind*. Let

\[ G^\mu[\text{ship}] = \nabla_\nu T^{\mu\nu}[\text{ship}] = \chi^{\mu\alpha} U^\beta[\text{ship}] T_{\mu\nu}[\text{wind}] \]

Can think of \( \chi^{\mu\nu} \) as depending on sail settings.
Introducing agency: the time direction field

Need also a time direction field, $\tau^\mu(p)$ (points into forward light cone).

Gauge freedom $\tau' = \sigma \tau$ where $\sigma$ is a time orientation preserving Lorentz boost from point of view of local inertial frame at $p$. 
Now have

\[ \Psi = \{ (p, \Phi, \chi, \tau) : \forall p \in \mathcal{M} \} \]

Gauge group is now

\[ \theta \in G^+ \]

where \( \theta = \sigma \varphi \) (with \( \sigma \) acting only on \( \tau \)).
Beables in General Relativity

(Usually called observables in GR community)
A beable is given by any function having the property

\[ B(\Psi) = B(\theta^*\Psi) \quad \forall \theta \in G^+ \]

Locality is an issue here. No function that depends on fields only in some \( A \subset M \) can be a beable.
Beables in General Relativity

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*The coefficient of friction between reality and the manifold is zero.*
**Assertion:** Observables correspond to scalars having specified values in coincidence.

Example of scalars.

\[ S[ab] = g_{\mu\nu}J^\mu[a]J^\nu[b] \]

We *nominate* a set of scalars generated by \( \Phi \)

\[ S = (S^1, S^2, \ldots S^K) \]

These should be rich enough to capture our experience.
The Westman-Sonego space

WS-space

The surface, $\Gamma$, of points $S$ corresponding to a solution $\Psi$

- Is invariant under $\theta \in G^+$.
- Has intrinsic dimension less than, or equal to $D = \dim(M)$. 
Can only set what we can observe. Thus, settings should be described by scalars. For example,

\[ Q^{kl} = \frac{\partial S^k}{\partial x^\mu} \frac{\partial S^l}{\partial x^\mu} \chi_{\mu\nu} \]

In general,

\[ Q = (Q^1, Q^2, \ldots Q^L) \]

This is the setting.

An **Agency strategy** is a choice

\[ Q(S) \quad \forall S \in WS\text{-space} \]

Specify what would do at every point, \( S \).
Turning a solution inside out

Put

\[(\Phi(p), \chi(p)) \leftrightarrow (S(p), Q(p), \omega(p))\]

Now put

\[\lambda(S) = \{(p, \omega(p), \tau(p)) : \forall p \in M_S\}\]

where \(M_S\) is all the points in \(M\) having given value of \(S\).

Write

\[\Psi = \{(S, Q, \lambda) : \forall S \in \Gamma\}\]

This is the inside out form.
Can convert back to

\[\Psi = \{(p, \Phi(p), \chi(p), \tau(p)) : \forall p \in M\}\]
Parts of a solution in WS-space

Consider region, \( A \), of WS-space. Have

\[ \Gamma_A = \Gamma \cap A \]

In this region, the solution is

\[ \Psi_A = \{ (S, Q, \lambda) : \forall S \in \Gamma_A \} \]

In region, \( A \), the agent strategy can be written

\[ Q_A = \{ (S, Q(S)) : \forall S \in A \} \]
**Propositions for A**

*Operational Propositions* These are the propositions we can directly verify.

- **Basic:** \( \text{Prop}_A[\Gamma_A] \)
- **Course-grained:** \( \text{Prop}\{\{\Gamma_A^\alpha : \alpha \in O_A\}\} \)

*Ontic Propositions*

Define

\[
\tilde{\Psi}_A = \{ \theta^* \Psi_A : \forall \theta \in G^+ \}
\]

(Heavy handed way to give gauge invariant presentation of solution.)

- **Basic:** \( \text{Prop}[\tilde{\Psi}_A] \)
- **Course-grained:** \( \text{Prop}\{\{\tilde{\Psi}_A^r : r \in R_A\}\} \)

Operational propositions can be written as course-grained ontic propositions.
Composition

*Principle of general compositionality:* the laws of physics should be written in such a way that they apply to any compositional description of any object and in terms of a calculation having the same compositional structure as this description.

Compare with *Principle of general covariance* - the laws of physics should be written in such a way that they take the same form in any coordinate system.
Encapsulated propositions, $\mathcal{A}$, $\mathcal{B}$, $\ldots$

An encapsulated proposition

$$\mathcal{A}^{bc}_a = \left( \text{prop}(\mathcal{A}), \mathcal{Q}_\mathcal{A}, (bc, a) \right) = \mathcal{A}$$

Here $a$, $b$, $\ldots$ are directed bounding surfaces.
Can simplify an encapsulated proposition using physics. Let

$$\Omega(\mathcal{A}, Q_\mathcal{A})$$

be set of allowed $\tilde{\Psi}_\mathcal{A}$. Then can replace

$$\text{prop}(\mathcal{A}) \rightarrow \text{prop}_\Omega(\mathcal{A}) = \text{Prop}[\{\tilde{\Psi}_\mathcal{A}^r : \forall r\} \cap \Omega(\mathcal{A}, Q_\mathcal{A})]$$
Composing encapsulated propositions

Can consider two encapsulated propositions joined as follows

\[ C_{cd} B_{af} \leftrightarrow \]

Can simplify using physical matching. Prune so as to only keep cases that match. This is subtle because of the \( G^+ \) group. For each element of \( \theta_B \) and \( \theta_C \) on each side, have a condition

\[
\text{cond}[ac]
\]

that must take the same value on each side. For these cases, the joint solution

\[
\Psi^q_A \cup \Psi^r_B
\]

is a solution for joint region.
Equivalence classes under composition

Let

$$a = \{ \theta^* \text{cond}[a] : \forall \theta \in G^+ \}$$

(Heavy handed). If there exists a \( \Psi_A \in \tilde{\Psi}_A \) satisfying one of these conditions then there will be one satisfying every one of these conditions. Then

$$\mathcal{A}_a^{bc} \quad \text{and} \quad \mathcal{A}'_a^{bc}$$

belong to equivalence class under composition if they have same \((bc, a)\) at boundary.
Assume that (for whatever reason) we have probabilities for different ontic encapsulated propositions associated with

\[ A_{a}^{bc} \]

Then we write

\[ A_{a}^{bc} = \text{probability density of matching } (bc, a) \text{ at boundary} \]

Need a measure, \( da \), to talk of probability density.

Can now avail ourselves of the duotensor machinery (with continuous rather than discrete indices).
Principle of general compositionality

\[ A^{bc}_a \rightarrow A^{bc}_a \]
\[ B^d_{be} \rightarrow B^d_{be} \]
\[ A^{bc}_a B^d_{be} \rightarrow A^{bc}_a B^d_{be} \]

Repeated index implies integration (rather than summation) using measure \( db \).
Relative probability

\[
\frac{\text{Prob}(A_{bc}^a)}{\text{Prob}(A'_{bc}^a)}
\]

well conditioned iff generalized states are proportional

\[A_{bc}^a = kA'_{bc}^a\]

Then relative probability is equal to \(k\).

Allows us to do calculations in face of indefinite causal structure.
Causality

Special to General Relativity

\[ g_{\mu \nu} = \eta_{\mu \nu} \rightarrow [g_{\mu \nu} \text{ satisfies } G_{\mu \nu} = 8\pi T_{\mu \nu}] \]

Compare with

\[ \text{[Pavia causality condition]} \rightarrow [???] \]

Can demand that a deterministic effect is unique employing \( \tau \) field. Does this fully characterize generalized states?
Conclusions

- Have indicated route to an operational probabilistic formulation of GR
- Various challenges remain: measure $da$, causality condition, constraints on generalized states, ...
- Will introduce fiducials and free encapsulated propositions (not tied to a particular region of WS-space).
- Can sketch route to QG.
Plan of attack on Quantum Gravity

- Quantization: simplex $\implies$ curved convex set from a Hilbert space.
- GRization: fixed causal structure $\implies$ fuzzy causal structure.