Probabilistic general relativity with agency in an operational framework [1]

Lucien Hardy

Perimeter Institute, Waterloo, Ontario, Canada

Thanks for invitation to GRPL2015

[1] Work in progress

Objective: To develop an operational formulation of General Relativity that accommodates ignorance probabilities and agency: PAGeR. Homage to the Blackberry pager

Motivation: As a step in developing a theory of Quantum Gravity (QGPL2016?)

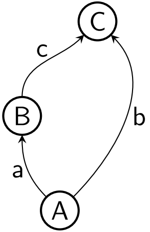
Prelude: composition in physics

The generalized state *is a mathematical object,* A*, associated with an object,* A*, which can be used to calculate the value of those properties we are interested for this object.*

Typically in physics a state pertains to a given time and is used to make predictions for later times. The generalized state is a more general notion than this since we may be making predictions of a more general type. A key question is how do we calculate the generalized state for a composite object? We propose the following principle.

THE COMPOSITION PRINCIPLE: *The generalized state for a composite object can be calculated from the generalized states for the components by means of a calculation having the same structure as the description of the composition of that object.*

Prelude: example



$$\Leftrightarrow \quad \mathsf{A}^{\mathsf{a}_1\mathsf{b}_2}\mathsf{B}^{\mathsf{c}_3}_{\mathsf{a}_1}\mathsf{C}_{\mathsf{b}_2\mathsf{c}_3}$$

We can associate generalized states

$$\begin{aligned}
\mathsf{A}^{\mathsf{a}_1\mathsf{b}_2} &\longrightarrow A^{a_1 b_2} \\
\mathsf{B}^{\mathsf{c}_3}_{\mathsf{a}_1} &\longrightarrow B^{c_3}_{a_1} \\
\mathsf{C}_{\mathsf{b}_2\mathsf{c}_3} &\longrightarrow C_{b_2 c_3}
\end{aligned}$$

with

$$\mathsf{A}^{\mathsf{a}_1\mathsf{b}_2}\mathsf{B}^{\mathsf{c}_3}_{\mathsf{a}_1} \longrightarrow A^{a_1 b_2} B^{c_3}_{a_1}$$

this is in accord with the composition principle.

Earlier work

This is part of an ongoing project (papers on the arXiv).

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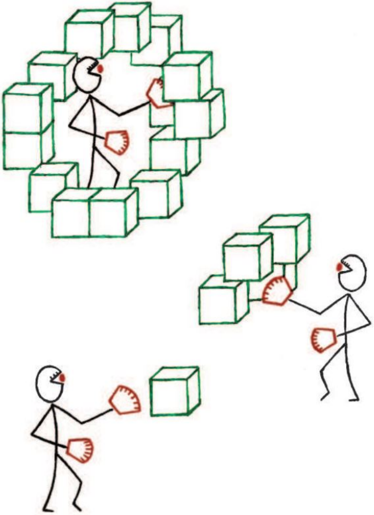
- 2011 The Operator Tensor formulation of Quantum Theory: A manifestly covariant way to formulate QT (for circuits but applicable to space-time).
- 2013 Theory of composition in physics: A general framework for thinking about composition in physics.

Related work

- Chris Fuchs, Rüdiger Schack, - Emphasized agent centric approach to Quantum Foundations (QBism)
- Samson Abramsky and Bob Coecke's categorical (pictorial) approach to quantum theory. This emphasizes compositionality.
- Generalized probability theories (going back to Mackey) - much recent work.
- Some space-time approaches to QT:
  - Quantum causal histories - Markopoulou; Dual point of view - Blute, Ivanov and Panangaden;
  - Aharonov and co-workers - multitime states.
  - General Boundary formulation - Oeckl;
  - Quantum combs - Chiribella, D'Ariano, and Perinotti; Oeckl's positive formulation.
  - Various axiomatic approaches to QT (LH, Dakic and Brukner, Masanes Müeller, CDP, ...) (in particular the tomographic locality axiom).
  - Leifer and Spekkens Quantum Bayesian Inference, more recent work by Henson, Lal, and Pusey.
  - Indefinite causal structure: Brukner, Oreshkov, Costa, Cerf

Fuchs Picture for Quantum Cosmology



General Relativity: fields

In GR have a set of fields

$$\mathbf{\Phi} = (g_{\mu\nu}, \text{matter fields})$$

where the matter fields can be things like

- $J^\mu[a]$ - the current of fluid of type a,
- $F^{\mu\nu}$ - the electromagnetic field
- etc.

General Relativity: solutions

We have a set of coupled partial differential equations (including Einstein's field equations). Solve and find a solution

$$\Psi = \left\{ (p, \mathbf{\Phi}) : \forall p \in \mathscr{M} \right\}$$

Note that, if Ψ is a solution, then so is

$$\varphi^* \Psi = \left\{ (p, \varphi^* \mathbf{\Phi}) : \forall p \in \mathscr{M} \right\}$$

(and it has the same physical content). Here φ is a diffeomorphism and it "moves" the fields on $\mathscr{M}$.

Introducing agency: the agency field

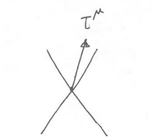
E.g. two fluids *ship* and *wind*. Let

$$G^{\mu}[\text{ship}] = \nabla_{\nu} T^{\mu\nu}[\text{ship}] = \chi^{\mu\alpha} U^{\beta}[\text{ship}] T_{\mu\nu}[\text{wind}]$$

Can think of $\chi^{\mu\nu}$ as depending on sail settings.

Introducing agency: the time direction field

Need also a time direction field, $\tau^\mu(p)$ (points into forward light cone).



Gauge freedom $\boldsymbol{\tau}' = \sigma\boldsymbol{\tau}$ where σ is a time orientation preserving Lorentz boost from point of view of local inertial frame at p.

Introducing agency: solutions

Now have

$$\Psi = \left\{ (p, \boldsymbol{\Phi}, \boldsymbol{\chi}, \boldsymbol{\tau}) : \forall p \in \mathscr{M} \right\}$$

Gauge group is now

$$\theta \in G^{+}$$

where $\theta = \sigma\varphi$ (with σ acting only on $\boldsymbol{\tau}$).

Beables in General Relativity

(Usually called observables in GR community)
A beable is given by any function having the property

$$B(\Psi) = B(\theta^{*}\Psi) \ \ \forall \theta \in G^{+}$$

Locality is an issue here. No function that depends on fields only in some $\mathscr{A} \subset \mathscr{M}$ can be a beable.

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The coefficient of friction between reality and the manifold is zero.

Observables and WS-space

Assertion: Observables correspond to scalars having specified values in coincidence.

Example of scalars.

$$S[ab] = g_{\mu\nu} J^{\mu}[a] J^{\nu}[b]$$

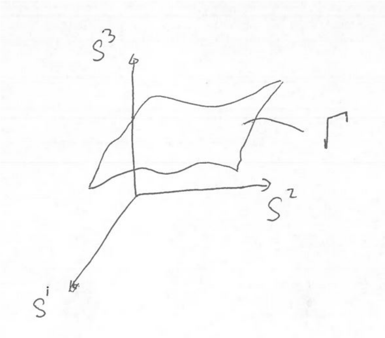
We *nominate* a set of scalars generated by $\mathbf{\Phi}$

$$\mathbf{S} = (S^1, S^2, \ldots S^K)$$

These should be rich enough to capture our experience.

The Westman-Sonego space

WS-space



The surface, Γ, of points $\mathbf{S}$ corresponding to a solution Ψ

- Is invariant under $\theta \in G^+$.
- Has intrinsic dimension less than, or equal to $D = \dim(\mathscr{M})$.

Agency Again

Can only set what we can observe. Thus, settings should be described by scalars. For example,

$$Q^{kl} = \frac{\partial S^k}{\partial x^\mu}\frac{\partial S^l}{\partial x^\mu}\chi^{\mu\nu}$$

In general,

$$\mathbf{Q} = (Q^1, Q^2, \ldots Q^L)$$

This is the setting.
An *Agency strategy* is a choice

$$\mathbf{Q}(\mathbf{S}) \quad \forall S \in \text{WS-space}$$

Specify what would do at every point, $\mathbf{S}$.

Turning a solution inside out

Put

$$\left(\boldsymbol{\Phi}(p), \boldsymbol{\chi}(p)\right) \leftrightarrow \left(\mathbf{S}(p), \mathbf{Q}(p), \boldsymbol{\omega}(p)\right)$$

Now put

$$\boldsymbol{\lambda}(\mathbf{S}) = \{(p, \boldsymbol{\omega}(p), \boldsymbol{\tau}(p)) : \forall p \in \mathscr{M}_{\mathbf{S}}\}$$

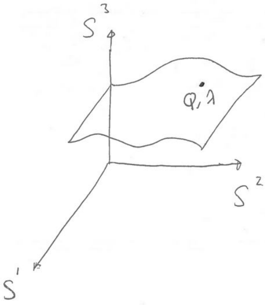
where $\mathscr{M}_{\mathbf{S}}$ is all the points in $\mathscr{M}$ having given value of $\mathbf{S}$.

Write

$$\Psi = \{(\mathbf{S}, \mathbf{Q}, \boldsymbol{\lambda}) : \forall \mathbf{S} \in \Gamma\}$$

This is the *inside out form*.
Can convert back to

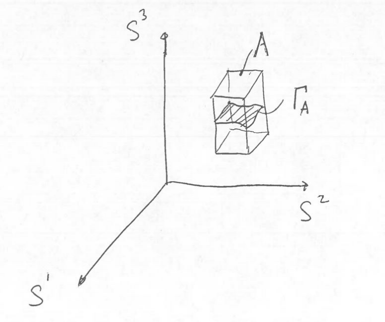
$$\Psi = \left\{(p, \boldsymbol{\Phi}(p), \boldsymbol{\chi}(p), \boldsymbol{\tau}(p)) : \forall p \in \mathscr{M}\right\}$$



Parts of a solution in WS-space

Consider region, $\mathtt{A}$, of WS-space.
Have

$$\Gamma_{\mathtt{A}} = \Gamma \cap \mathtt{A}$$



In this region, the solution is

$$\Psi_{\mathtt{A}} = \{(\mathbf{S}, \mathbf{Q}, \boldsymbol{\lambda}) : \forall \mathbf{S} \in \Gamma_{\mathtt{A}}\}$$

In region, $\mathtt{A}$, the agent strategy can be written

$$\mathbf{Q}_{\mathtt{A}} = \{(\mathbf{S}, \mathbf{Q}(\mathbf{S})) : \forall \mathbf{S} \in \mathtt{A}\}$$

Propositions for $\mathtt{A}$

Operational Propositions These are the propositions we can directly verify.

- *Basic:* $\mathsf{Prop}_{\mathtt{A}}[\Gamma_{\mathtt{A}}]$
- *Course-grained:* $\mathsf{Prop}[\{\Gamma_{\mathtt{A}}^{\alpha} : \alpha \in O_{\mathtt{A}}\}]$

Ontic Propositions
Define

$$\tilde{\Psi}_{\mathtt{A}} = \left\{\theta^* \Psi_{\mathtt{A}} : \forall \theta \in G^+\right\}$$

(Heavy handed way to give gauge invariant presentation of solution.)

- *Basic:* $\mathsf{Prop}[\tilde{\Psi}_{\mathtt{A}}]$
- *Course-grained:* $\mathsf{Prop}[\{\tilde{\Psi}_{\mathtt{A}}^{r} : r \in R_{\mathtt{A}}\}]$

Operational propositions can be written as course-grained ontic propositions.

Composition

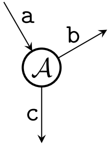
Principle of general compositionality: the laws of physics should be written in such a way that they apply to any compositional description of any object and in terms of a calculation having the same compositional structure as this description.

Compare with *Principle of general covariance* - the laws of physics should be written in such a way that they take the same form in any coordinate system.

Encapsulated propositions, $\mathcal{A}$, $\mathcal{B}$, ...

An encapsulated proposition

$$\mathcal{A}_{\mathtt{a}}^{\mathtt{bc}} = \Big(\mathsf{prop}(\mathcal{A}), \mathbf{Q}_{\mathcal{A}}, (\mathtt{bc}, \mathtt{a})\Big) =$$



Here $\mathtt{a}$, $\mathtt{b}$, ... are directed bounding surfaces.

Can simplify an encapsulated proposition using physics. Let

$$\Omega(\mathcal{A}, \mathbf{Q}_{\mathcal{A}})$$

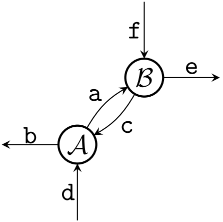
be set of allowed $\tilde{\Psi}_{\mathcal{A}}$. Then can replace

$$\mathsf{prop}(\mathcal{A}) \to \mathsf{prop}_{\Omega}(\mathcal{A}) = \mathsf{Prop}[\{\tilde{\Psi}^{r}_{\mathcal{A}} : \forall r\} \cap \Omega(\mathcal{A}, \mathbf{Q}_{\mathcal{A}})]$$

Composing encapsulated propositions

Can consider two encapsulated propositions joined as follows

$$\mathcal{C}^{\mathtt{ab}}_{\mathtt{cd}}\mathcal{B}^{\mathtt{ec}}_{\mathtt{af}} \quad \Longleftrightarrow$$



Can simplify using physical matching. Prune so as to only keep cases that match. This is subtle because of the G^+ group.
For each element of $\theta_{\mathcal{B}}$ and $\theta_{\mathcal{C}}$ on each side, have a condition

$$\text{cond}[\mathtt{ac}]$$

that must take the same value on each side. For these cases, the joint solution

$$\Psi^q_{\mathcal{A}} \cup \Psi^r_{\mathcal{B}}$$

is a solution for joint region.

Equivalence classes under composition

Let

$$a = \left\{\theta^* \mathsf{cond}[a] : \forall\theta \in G^+\right\}$$

(Heavy handed). If there exists a $\Psi_{\mathcal{A}} \in \tilde{\Psi}_{\mathcal{A}}$ satisfying one of these conditions then there will be one satisfying every one of these conditions. Then

$$\mathcal{A}_{\mathsf{a}}^{\mathsf{bc}} \quad \text{and} \quad \mathcal{A'}_{\mathsf{a}}^{\mathsf{bc}}$$

belong to equivalence class under composition if they have same (bc, a) at boundary.

Probabilities

Assume that (for whatever reason) we have probabilities for different ontic encapsulated propositions associated with

$$\mathcal{A}_{\mathtt{a}}^{\mathtt{bc}}$$

Then we write

$$A_a^{bc} = \text{probability density of matching } (bc, a) \text{ at boundary}$$

Need a measure, da, to talk of probability density.

Can now avail ourselves of the duotensor machinery (with continuous rather than discrete indices).

Principle of general compositionality

$$\mathcal{A}_{\mathtt{a}}^{\mathtt{bc}} \longrightarrow A_a^{bc}$$

$$\mathcal{B}_{\mathtt{be}}^{\mathtt{d}} \longrightarrow B_{be}^{d}$$

$$\mathcal{A}_{\mathtt{a}}^{\mathtt{bc}} \mathcal{B}_{\mathtt{be}}^{\mathtt{d}} \longrightarrow A_a^{bc} B_{be}^{d}$$

Repeated index implies integration (rather than summation) using measure $\mathrm{d}b$.

F-locality

Relative probability

$$\frac{\mathsf{Prob}(\mathcal{A}^{\mathtt{a}}_{\mathtt{bc}})}{\mathsf{Prob}(\mathcal{A}'^{\mathtt{a}}_{\mathtt{bc}})}$$

well conditioned iff generalized states are proportional

$$A^a_{bc} = kA'^a_{bc}$$

Then relative probability is equal to k.
Allows us to do calculations in face of indefinite causal structure.

Causality

Special to General Relativity

$$\left[g_{\mu\nu} = \eta_{\mu\nu}\right] \to \left[g_{\mu\nu} \text{ satisfies } G_{\mu\nu} = 8\pi T_{\mu\nu}\right]$$

Compare with

$$\left[\text{Pavia causality condition}\right] \to \left[???\right]$$

Can demand that a deterministic effect is unique employing $\boldsymbol{\tau}$ field. Does this fully characterize generalized states?

Conclusions

- Have indicated route to an operational probabilistic formulation of GR
- Various challenges remain: measure da, causality condition, constraints on generalized states, ...
- Will introduce fiducials and free encapsulated propositions (not tied to a particular region of WS-space).
- Can sketch route to QG.

Plan of attack on Quantum Gravity

$$\begin{tikzcd}
\text{OpQT} \arrow[r, "\text{GRize}"] & \text{QuAGeR} \\
\text{CProbT} \arrow[u, "\text{quantize}"] \arrow[r, "\text{GRize}"'] & \text{PAGeR} \arrow[u, "\text{quantize}"']
\end{tikzcd}$$

- Quantization: simplex $\Longrightarrow$ curved convex set from a Hilbert space.
- GRization: fixed causal structure $\Longrightarrow$ fuzzy causal structure.