Defunctionalized Interpreters for Call-by-Need Programming Languages

— a functional pearl with hygiene —

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The contributions

- A hygienic standard call-by-need reduction for the $\lambda$-calculus.
- The notion of explicit evaluation contexts.
- Towards an abstract machine and a natural semantics for call by need through refocusing, refunctionalization, etc.
The starting point

The standard call-by-need reduction of

- Ariola and Felleisen, 1997
  JFP 7(3):265-301

- Maraist, Odersky and Wadler, 1998
  JFP 8(3):275-317
The starting point

The standard call-by-need reduction of

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The goal: to extract a computational content.
Syntax

\[ T ::= x \mid \lambda x. T \mid T \cdot T \mid \text{let } x \text{ be } T \text{ in } T \]

\[ A ::= \lambda x. T \mid \text{let } x \text{ be } T \text{ in } A \]

\[ E ::= [] \mid E \cdot T \mid \text{let } x \text{ be } T \text{ in } E \]

\[ \text{let } x \text{ be } E \text{ in } E[x] \]
Axioms

\((\lambda x. T) \ T_1 \ \to \ \text{let } x \ \text{be } T_1 \ \text{in } T\)

\(\text{let } x \ \text{be } \lambda x. T \ \text{in } E[x] \ \to \ \text{let } x \ \text{be } \lambda x. T \ \text{in } E[\lambda x. T]\)

\((\text{let } x \ \text{be } T_1 \ \text{in } A) \ T_2 \ \to \ \text{let } x \ \text{be } T_1 \ \text{in } A \ T_2\)

\(\text{let } x_2 \ \text{be let } x_1 \ \text{be } T \ \to \ \text{let } x_1 \ \text{be } T \ \text{in } A \ \text{in let } x_2 \ \text{be } A\)

\(\text{in } E[x_2] \ \ \text{in } E[x_2]\)
In practice

...too hard to test!
La même chose, with integers

Syntax:

$$T ::= \llbracket n \rrbracket \mid \text{succ } T \mid x \mid \ldots$$

$$A ::= \llbracket n \rrbracket \mid \lambda x. T \mid \text{let } x \text{ be } T \text{ in } A$$

$$E ::= [ ] \mid \text{succ } E \mid E \ T \mid \ldots$$
La même chose, with integers

Three extra axioms:

\[
\text{succ } \langle n \rangle \rightarrow \langle n' \rangle
\]

where \( n' = n + 1 \)

\[
\text{let } x \text{ be } \langle n \rangle \text{ in } E[x] \rightarrow \text{let } x \text{ be } \langle n \rangle \text{ in } E[\langle n \rangle]
\]

\[
\text{succ (let } x \text{ be } T \text{ in } A) \rightarrow \text{let } x \text{ be } T \text{ in } \text{succ } A
\]
Some exegesis

1. The potential redexes
2. Barendregt’s variable convention
3. The evaluation contexts
1. The potential redexes

A helpful grammar:

\[ R ::= \text{succ } A \mid A \ T \mid \text{let } x \text{ be } A \text{ in } E[x] \]

where

\[ A ::= \overline{n} \mid \lambda x. T \mid \text{let } x \text{ be } T \text{ in } A \]
2. Barendregt’s variable convention (1/3)

It is assumed, e.g., in

\[(\text{let } x \text{ be } T_1 \text{ in } A) \ T_2 \rightarrow \text{let } x \text{ be } T_1 \text{ in } A \ T_2\]

\[\text{let } x \text{ be } \lambda x. T \text{ in } E[x] \rightarrow \text{let } x \text{ be } \lambda x. T \text{ in } E[\lambda x. T]\]
2. Barendregt’s variable convention (2/3)

One axiom, however, yields terms that do not satisfy the convention:

\[
\text{let } x \text{ be } \lambda x. T \text{ in } E[x] \rightarrow \text{let } x \text{ be } \lambda x. T \text{ in } E[\lambda x. T]
\]
2. Barendregt’s variable convention (3/3)

Simple fix:

\[
\text{let } x \text{ be } \lambda x. T \rightarrow \text{let } x \text{ be } \lambda x. T
\]
\[
\text{in } E[x] \quad \text{ in } E[\lambda x'. T']
\]
\[
\text{where } \lambda x'. T' = \text{freshen}\_\text{up}(\lambda x. T)
\]
3. The evaluation contexts

The grammar of contexts is unusual because it includes identifiers within (delimited) contexts.
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The grammar of contexts is unusual because it includes identifiers within (delimited) contexts.

- These contexts are constructed outside in.
- All the others are constructed inside out.
Towards explicit evaluation contexts

Analogy with explicit substitutions:
delay the actual substitution.

Here: delay the recomposition, i.e.,
keep $E$ instead of having $\lambda x. E[x]$.

Joint work with Kristoffer Rose
Contexts as lists of frames

\[ F ::= \text{successor} \quad \square \quad | \]
\[ \quad \square \ T \quad | \]
\[ \quad \text{let } x \text{ be } \square \text{ in } C_{oi}[x] \quad | \]
\[ \quad \text{let } x \text{ be } T \text{ in } \square \]

\[ C_{oi} ::= \bullet \quad | \quad F \circ C_{oi} \]

\[ C_{io} ::= \bullet \quad | \quad F \circ C_{io} \]
Recomposition of outside-in contexts

\[ \langle \bullet, T \rangle_{oi} \uparrow_{rec} T \]

\[ \langle C_{oi}, T \rangle_{oi} \uparrow_{rec} T_0 \]

\[ \langle (\Box T_1) \circ C_{oi}, T \rangle_{oi} \uparrow_{rec} T_0 T_1 \]

\[ \ldots \]
Recomposition of inside-out contexts

\[
\langle \bullet, T \rangle_{io} \uparrow_{rec} T
\]

\[
\langle (\Box T_1) \circ C_{io}, T \rangle_{io} \uparrow_{rec} \langle C_{io}, T T_1 \rangle_{io}
\]

\[\ldots\]
Decomposition

A convenient format: as a transition system.

Accepting states: $\langle T, C_{io} \rangle_{term}$

$\langle C_{io}, A \rangle_{context}$

$\langle C_{io}, (C_{oi}, \chi) \rangle_{reroot}$

Final states: $\langle A \rangle_{answer}$

$\langle R, C_{io} \rangle_{decomposition}$
One-step reduction

$$T \overset{\text{let}}{\longrightarrow} T' \text{ if } \begin{cases} \langle T, \bullet \rangle_{\text{term}} \downarrow_{\text{dec}}^{*} \langle R, C_{io} \rangle_{\text{decomposition}} \\
(R, R') \in \text{...the axioms...} \\
\langle C_{io}, R' \rangle_{io} \uparrow_{\text{rec}}^{*} T' \end{cases}$$
Reduction-based evaluation

\[ T \xrightarrow{\text{let}}^* A \]
Good news

The rest is (essentially) mechanical.

Reference: Defunctionalized Interpreters for Programming Languages, ICFP’08.
The syntactic correspondence

- **Refocusing**: from reduction semantics to small-step abstract machine

- **Lightweight fusion**: from small-step abstract machine to big-step abstract machine

- **Transition compression**: from big-step abstract machine to big-step abstract machine
The functional correspondence

- **Refunctionalization**: from abstract machine to continuation-passing interpreter
- **Back to direct style**: from continuation-passing interpreter to first-order natural semantics
- **Refunctionalization**: from first-order natural semantics to higher-order natural semantics
Main results

- A readable, hygienic abstract machine.
- A readable, hygienic natural semantics.
Orthogonal issues

- Adding a garbage-collection rule
- Introducing a heap
- Introducing a store
Variants

Ensuring hygiene.
Latest news

More aggressive transition compression (using a global invariant) makes outside-in contexts unnecessary.

Good news for Simon’s head: a continuation-free account of lazy evaluation.

Work in progress.
Conclusion

- The standard call-by-need reduction of the lambda-calculus, plus hygiene, can be uniformly mirrored into an abstract machine and a natural semantics that make sense.
- Further transition compression leads to a continuation-free account of call by need.

Thank you.