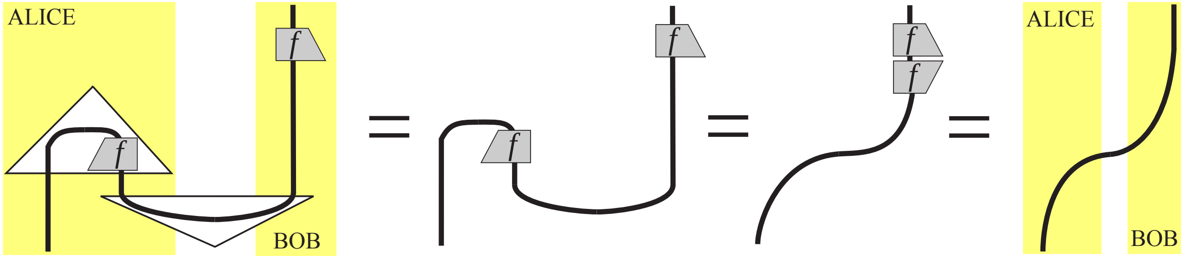
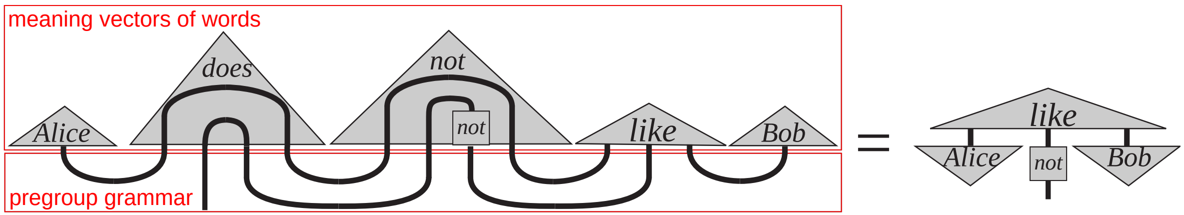
The logic of quantum mechanics - take II

arXiv:1204.3458





EPSRC
Engineering and Physical Sciences Research Council
Department of the Navy
ONR
Science & Technology
QICS
Information Society Technologies
CORDIS
Sixth Framework Programme
fq(x) Foundational Questions Institute
John Templeton Foundation
The Leverhulme Trust
AXA
PROTECTION FINANCIÈRE

— *genesis* —

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[1936 – 2000] many followed them, ... and **FAILED**.

— *the mathematics of it* —

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WHY?

— *the mathematics of it* —

Hilbert space stuff: continuum, field structure of complex numbers, vector space over it, inner-product, etc.

WHY?

von Neumann: only used *it* since *it* was ‘available’.

— *the physics of it* —

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Quantum Computer Scientists: Schrödinger is right!

— *the game plan* —

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$$\frac{\texttt{tensor product structure}}{\texttt{the other stuff}} = ???$$

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Task 1. Investigate which assumptions (i.e. which structure) on $\otimes$ is needed to deduce **physical phenomena**.

Task 2. Investigate wether such an "interaction structure" appear elsewhere in **"our classical reality"**.

Outcome 1a:

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Hans Halvorson (2010) Editorial to: *Deep Beauty: Understanding the Quantum World through Mathematical Innovation*, Cambridge University Press.

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Outcome 1b: Exposing this structure has already helped to **solve open problems elsewhere**.

E.g.: Ross Duncan & Simon Perdrix (2010) *Rewriting measurement-based quantum computations with generalised flow*. ICALP'10.

Outcome 1a: **“Sheer ratio of results to assumptions”** confirms that we are probing something very essential.

Outcome 1b: Exposing this structure has already helped to **solve open problems elsewhere**.

Outcome 1c: **Framework is a simple intuitive (but rigorous) diagrammatic language**,

Outcome 1a: **"Sheer ratio of results to assumptions"** confirms that we are probing something very essential.

Outcome 1b: Exposing this structure has already helped to **solve open problems elsewhere**.

Outcome 1c: **Framework is a simple intuitive (but rigorous) diagrammatic language**, meanwhile adopted by others e.g. Lucien Hardy in arXiv:1005.5164:

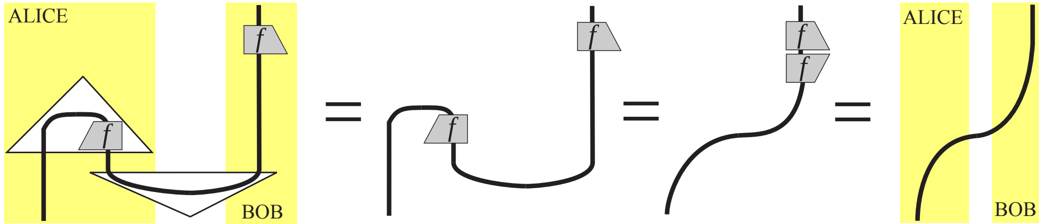
"... we join the ***quantum picturalism*** revolution [1]"

---

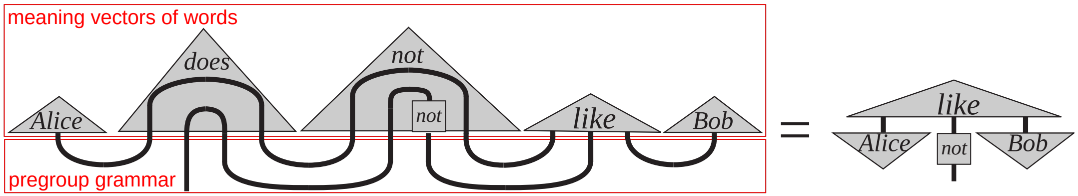
[1] BC (2010) ***Quantum picturalism***. Contemporary Physics **51**, 59–83.

Outcome 2a:

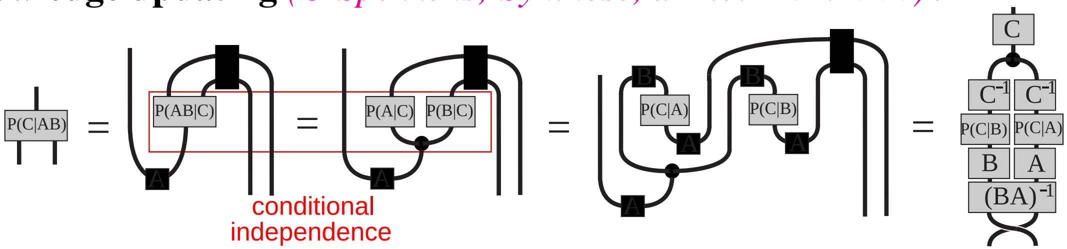
Behaviors of matter *(Abramsky-C; LiCS'04, quant-ph/0402130)* :



Meaning in language *(Clark-C-Sadrzadeh; Linguistic Analysis, arXiv:1003.4394)* :



Knowledge updating *(C-Spekkens; Synthese, arXiv:1102.2368)* :



— *the logic of it* —

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WHAT IS “LOGIC”?

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Pragmatic option 1: Logic is structure in language.

— *the logic of it* —

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"Alice and Bob ate everything or nothing, then got sick."

connectives $(\wedge, \vee)$: *and, or*
negation $(\neg)$: *not (cf. nothing = not something)*
entailment $(\Rightarrow)$: *then*
quantifiers $(\forall, \exists)$: *every(thing), some(thing)*
constants (a, b) : *thing*
variable (x) : *Alice, Bob*
predicates $(P(x), R(x, y))$: *eating, getting sick*
truth valuation $(0, 1)$: ***true, false***

$$(\forall z : Eat(a, z) \wedge Eat(b, z)) \wedge \neg(\exists z : Eat(a, z) \wedge Eat(b, z)) \Rightarrow Sick(a), Sick(b)$$

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Cf. the *soft* incarnation of AI in robotics, automated theorem proving, automated theory exploration, ...

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From truth to meaning in natural language processing:

QUANTUM LINGUISTICS Leap forward for artificial intelligence

— NewScientist (December 2010)

WEEKLY 11 December 2010

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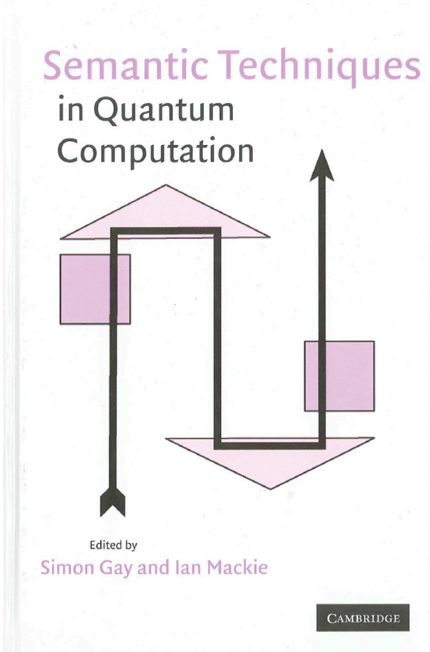
— QUANTUM LINGUISTICS Leap forward for artificial intelligence NewScientist WEEKLY 11 December 2010 (December 2010)

Automated theorem generation for graphical theories:

— `http://sites.google.com/site/quantomatic/`

MINIMAL QUANTUM PROCESS LANGUAGE

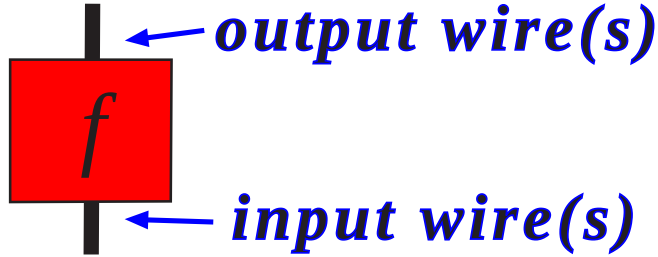


Samson Abramsky & BC (2004) *A categorical semantics for quantum protocols*. In: IEEE-LiCS'04. `quant-ph/0402130`

BC (2005) *Kindergarten quantum mechanics*. `quant-ph/0510032`

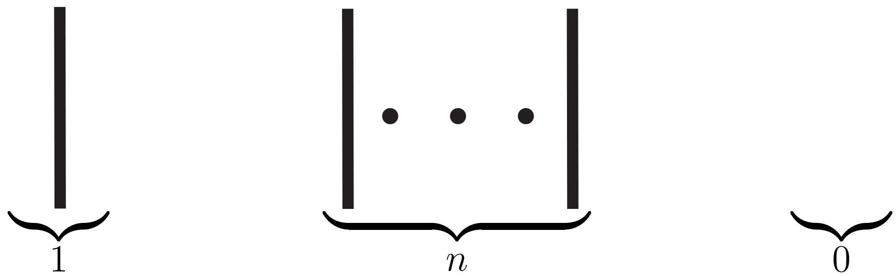
— *wire and box language* —

Box :=



Interpretation: wire := system ; box := **process**

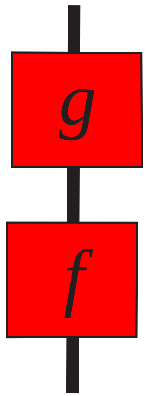
one system: $\quad$ n *sub*systems: $\quad$ no system:



— ***wire and box games*** —

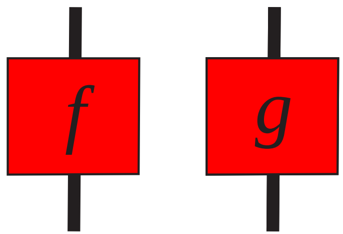
sequential or *causal* or *connected* composition:

$$g \circ f \equiv$$



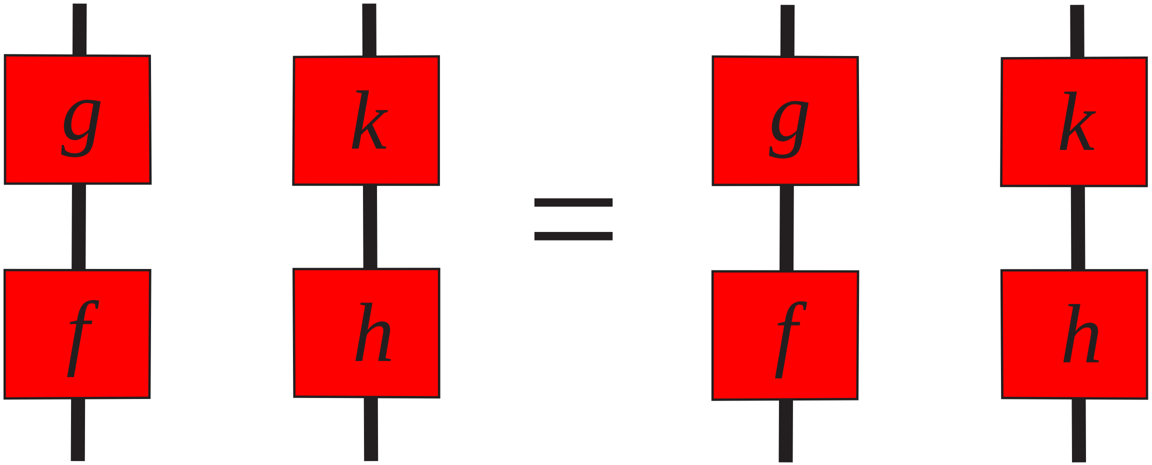
parallel or *acausal* or *disconnected* composition:

$$f \otimes g \equiv$$



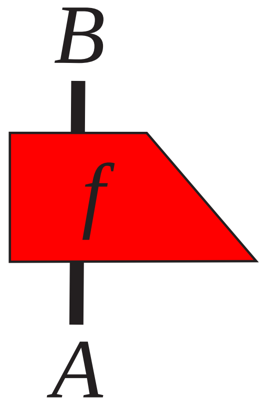
— *merely a new notation?* —

$$(g \circ f) \otimes (k \circ h) = (g \otimes k) \circ (f \otimes h)$$



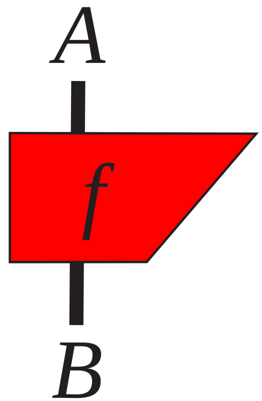
— ***quantitative metric*** —

$$f : A \to B$$

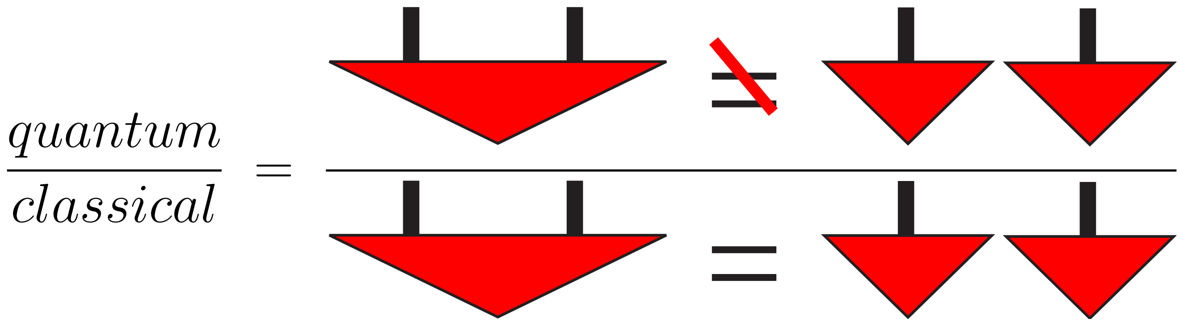


— ***quantitative metric*** —

$$f^{\dagger} : B \to A$$

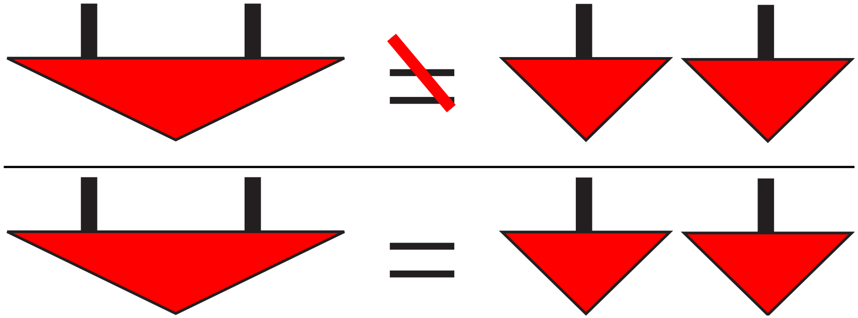


— *asserting (pure) entanglement* —

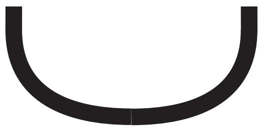


— *asserting (pure) entanglement* —

$$\frac{quantum}{classical} =$$



⇒ **introduce 'parallel wire' between systems:**

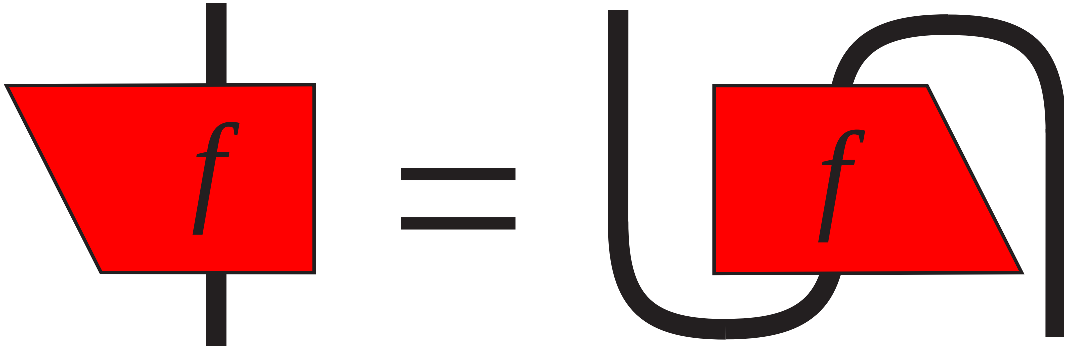


subject to: only topology matters!

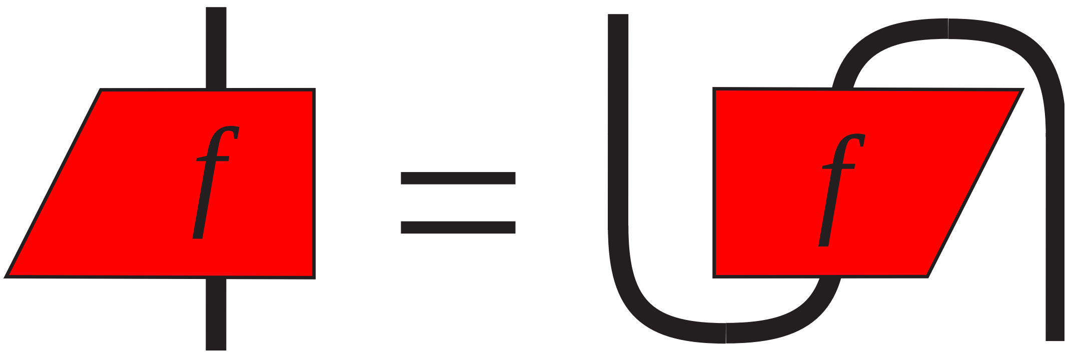
— *quantum-like* —

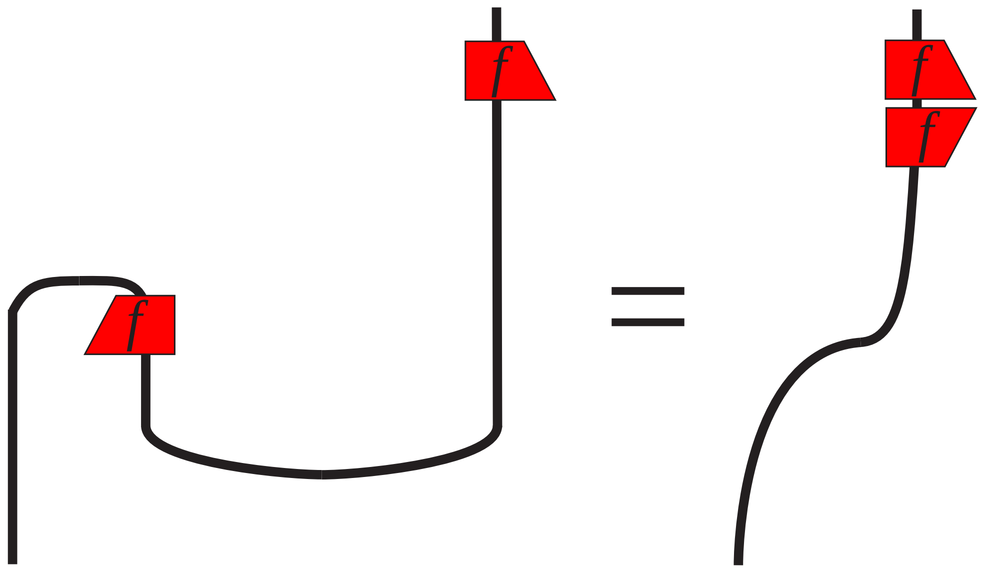
E.g.

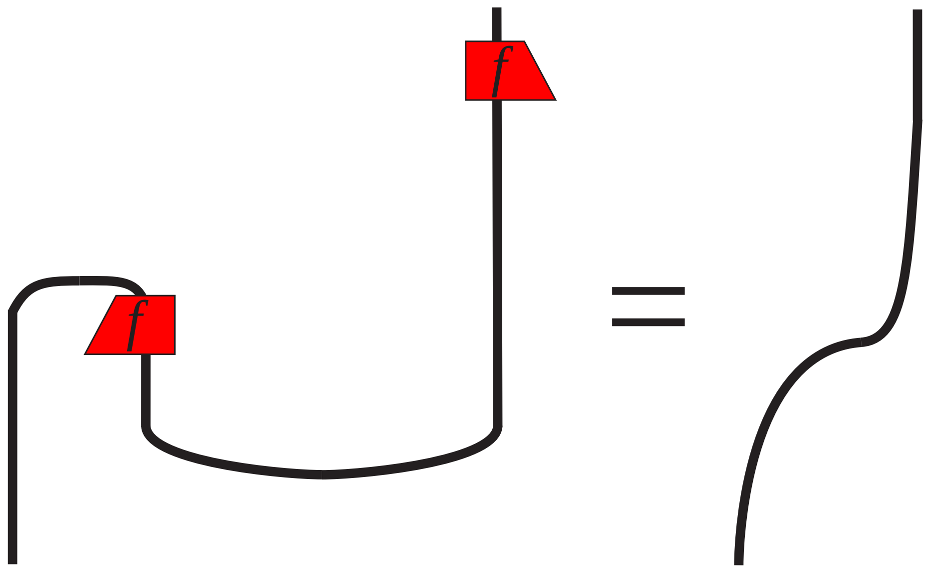
Transpose:

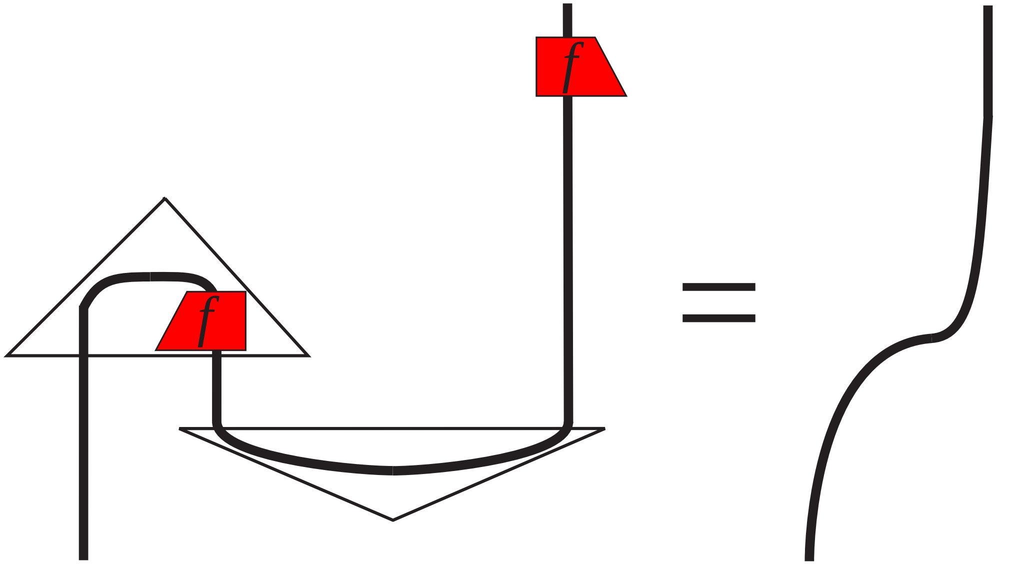


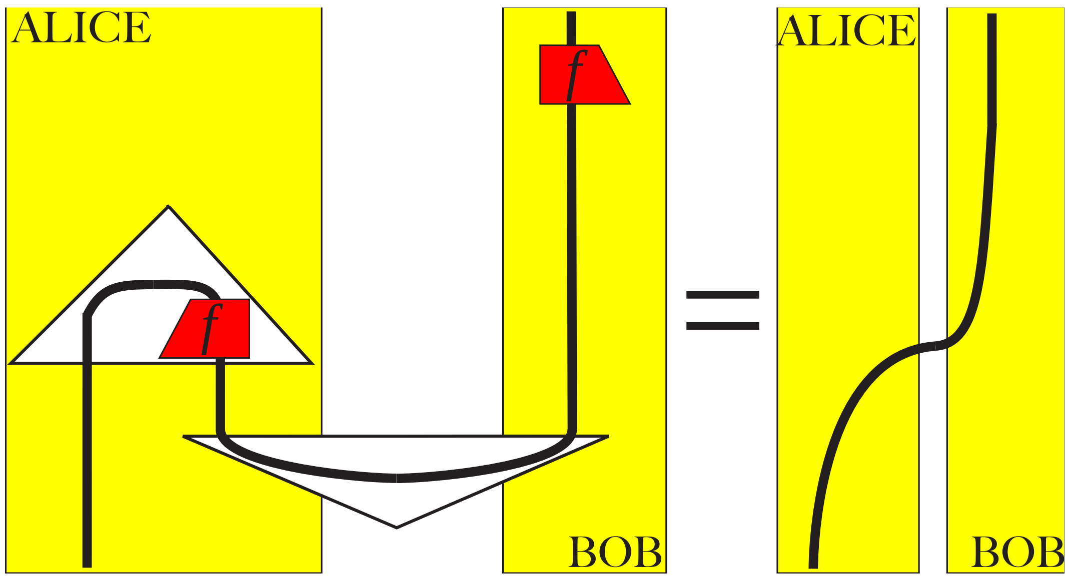
Conjugate:




f
f
=
f
f


f
f


f
f



⇒ quantum teleportation

— *symbolically: dagger compact categories* —

Thm. [Kelly-Laplaza '80; Selinger '05] *An equational statement between expressions in dagger compact categorical language holds if and only if it is derivable in the graphical notation via homotopy*.

Thm. [Hasegawa-Hofmann-Plotkin; Selinger '08] *An equational statement between expressions in dagger compact categorical language holds if and only if it is derivable in the dagger compact category of finite dimensional Hilbert spaces, linear maps, tensor product and adjoints*.

— *symbolically: dagger compact categories* —

In words: *Any equation involving:*

- *states, operations, effects*
- *unitarity, adjoints (e.g. self-adjoint), projections*
- *Bell-states/effects, transpose, conjugation*
- *inner-product, trace, Hilbert-Schmidt norm*
- *positivity, completely positive maps*, ...

holds in quantum theory if and only if it can be derived in the graphical language via homotopy.

— *kindergarten quantum mechanics: the experiment* —

Contest in problem solving between:

- **Children using quantum picturalism**

- **Physics teachers using ordinary QM**

The children will win!

[1] BC (2010) *Quantum picturalism*. Contemporary Physics 51, 59–83.

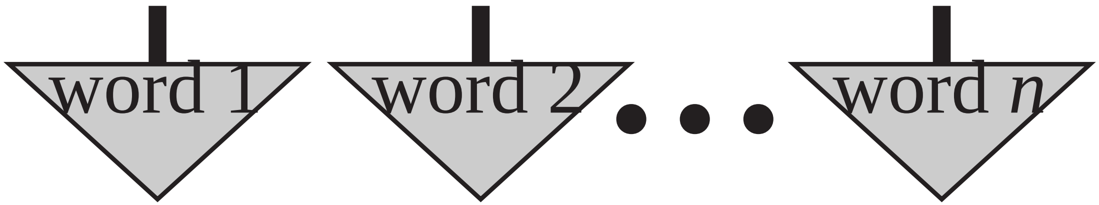
A SLIGHTLY DIFFERENT LANGUAGE FOR NATURAL LANGUAGE MEANING

BC, Mehrnoosh Sadrzadeh & Stephen Clark (2010) *Mathematical foundations for a compositional distributional model of meaning*. `arXiv:1003.4394`

— the from-words-to-a-sentence process —

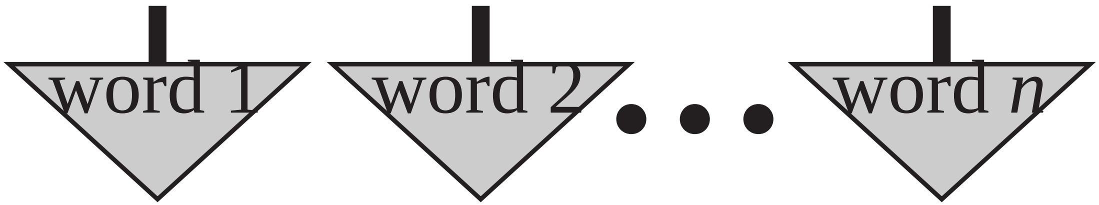
— *the from-words-to-a-sentence process* —

Consider meanings of **words**, e.g. as vectors (cf. Google):



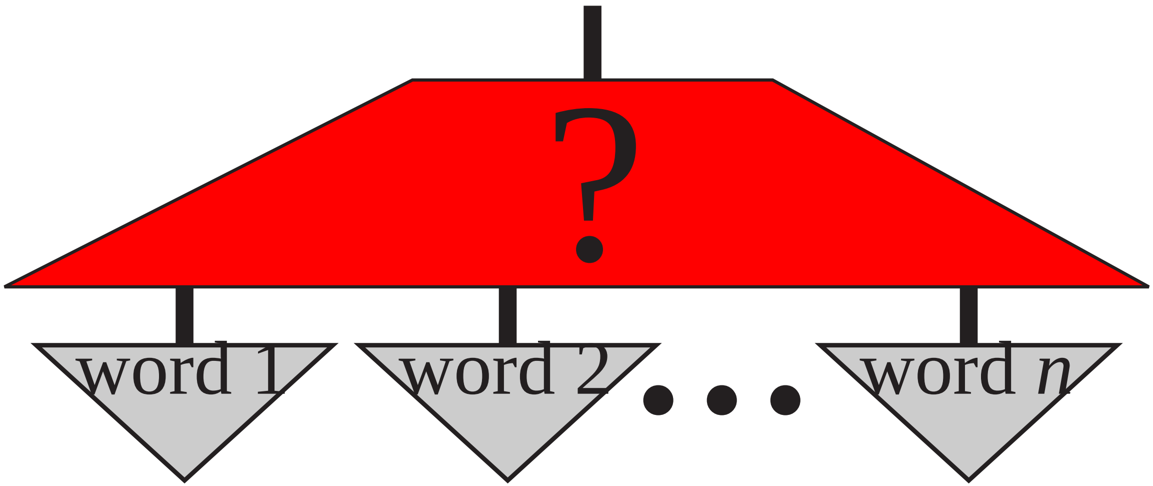
— *the from-words-to-a-sentence process* —

What is the meaning the **sentence** made up of these?



— *the from-words-to-a-sentence process* —

I.e. how do we/machines produce meanings of **sentences**?



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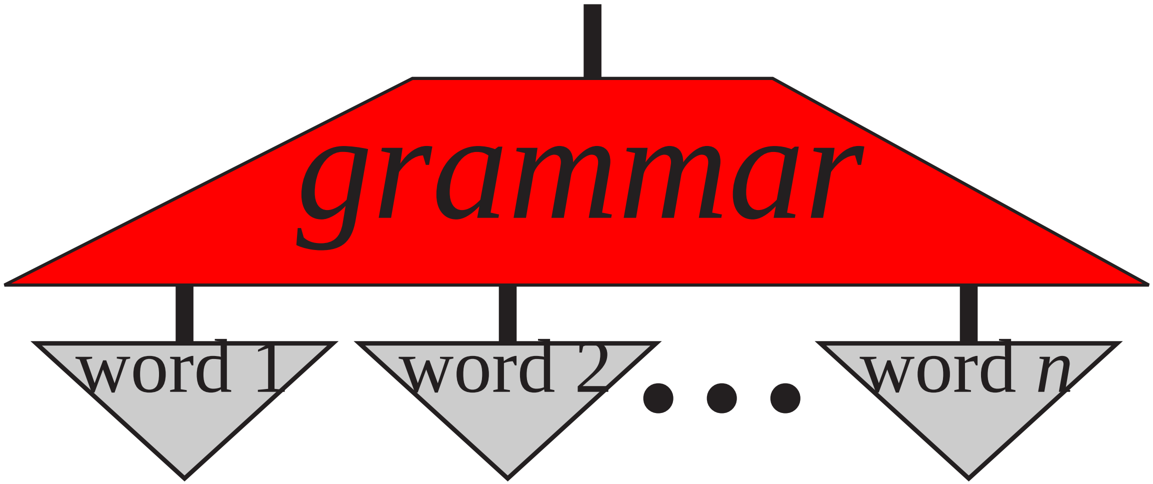
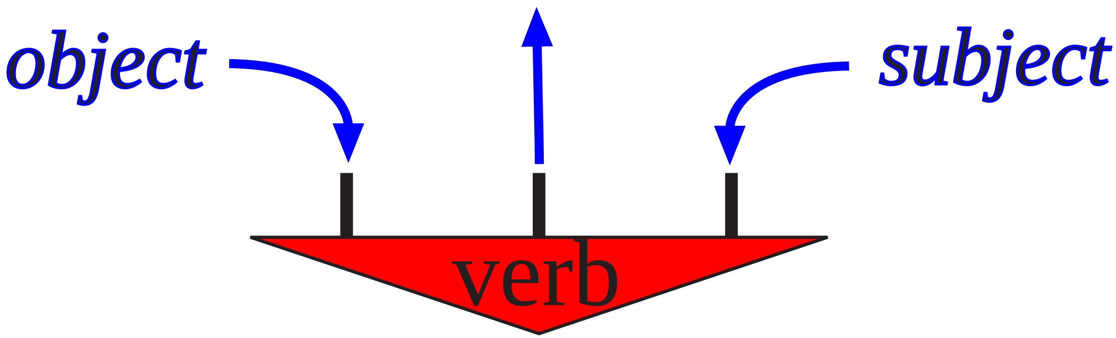


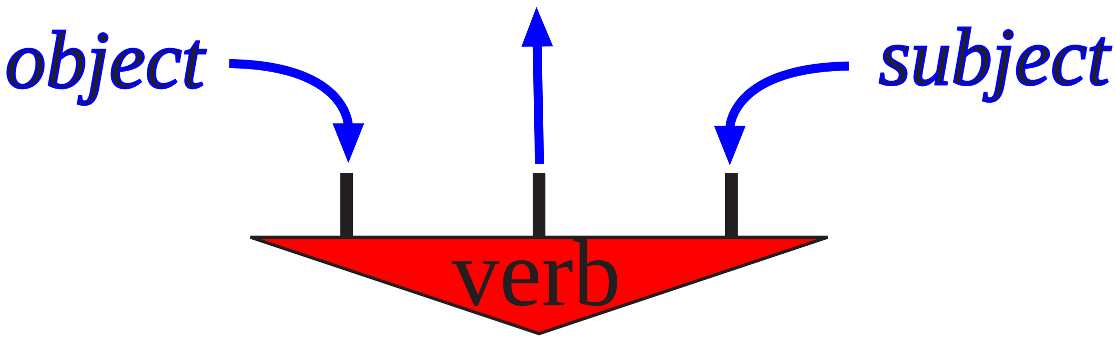
Gerald Gazdar (1996) *Paradigm merger in natural language processing*. In: Computing tomorrow: future research directions in computer science, eds., I. Wand and R. Milner, Cambridge University Press.

Information flow within a verb:

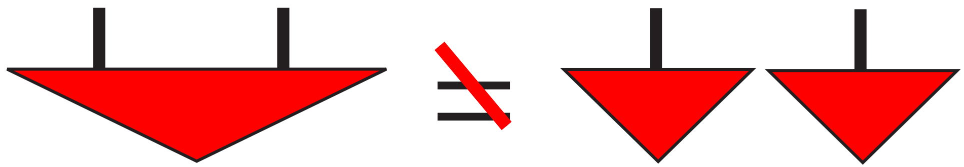


— ***the from-words-to-a-sentence process*** —

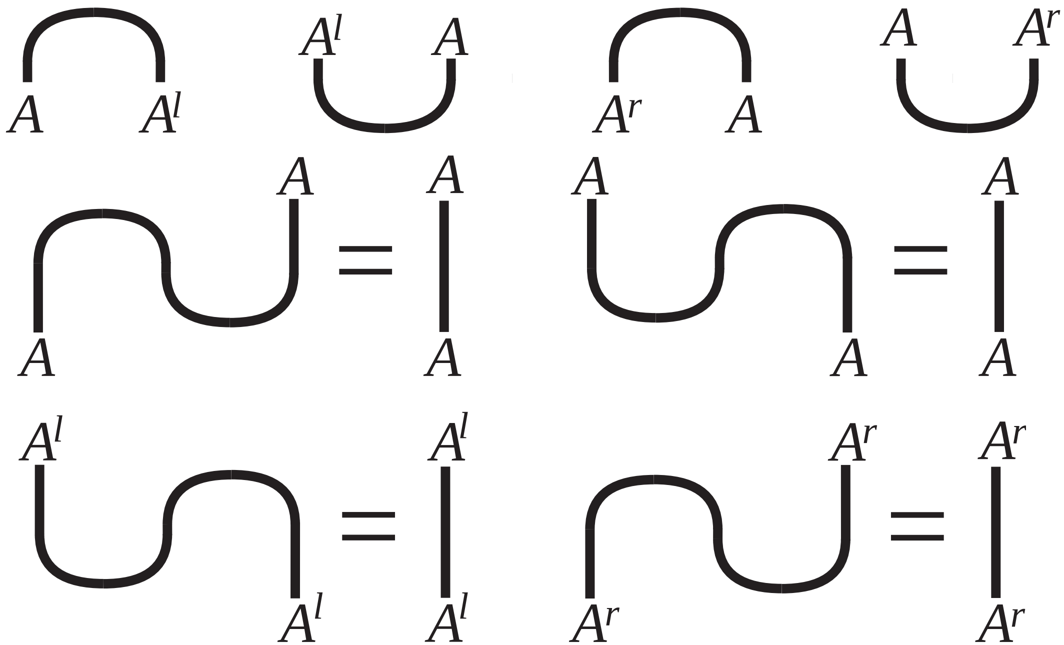
Information flow within a verb:



Again we have:



— *grammar as pregroups – Lambek '99* —



— *grammar as pregroups – Lambek '99* —

For noun type n, verb type is ${}^{-1}(n) \cdot s \cdot (n)^{-1}$, so:

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$$n \cdot {}^{-1}(n) \cdot s \cdot (n)^{-1} \cdot n = s$$

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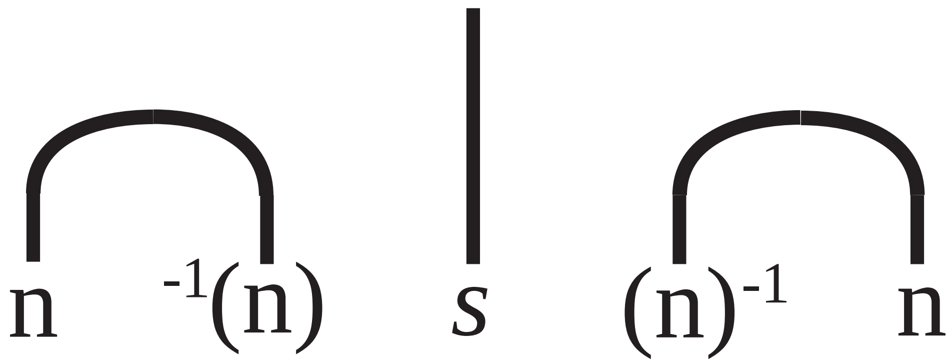
Diagrammatic typing:



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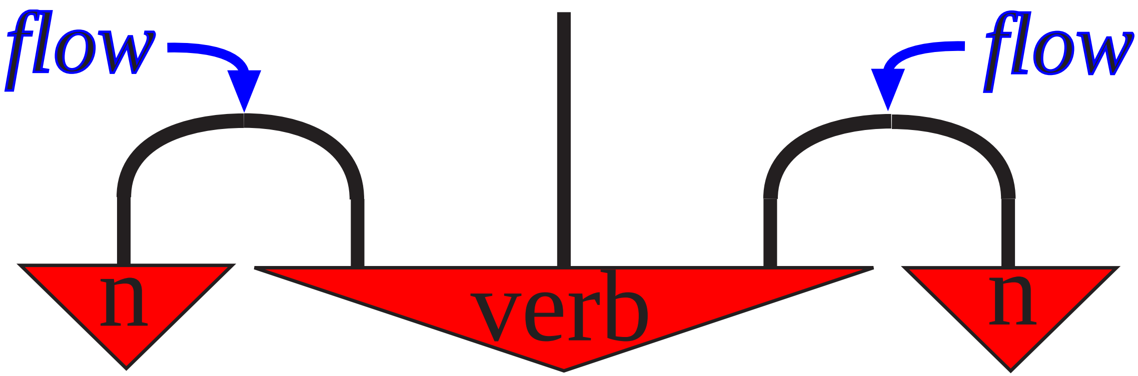
Diagrammatic meaning:



$$— \overrightarrow{Alice} \otimes \overrightarrow{does} \otimes \overrightarrow{not} \otimes \overrightarrow{like} \otimes \overrightarrow{Bob} —$$

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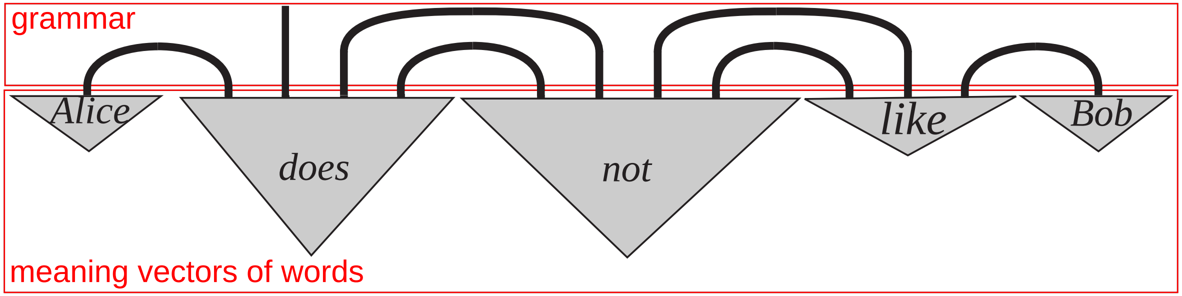
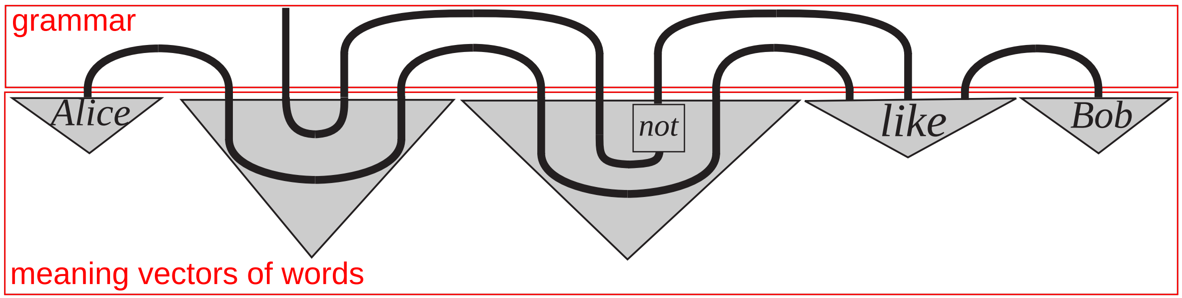


$$\overrightarrow{Alice} \otimes \overrightarrow{does} \otimes \overrightarrow{not} \otimes \overrightarrow{like} \otimes \overrightarrow{Bob}$$



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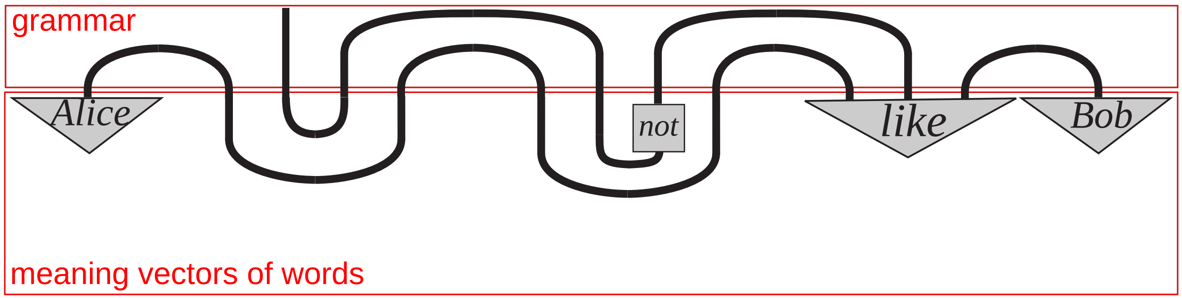

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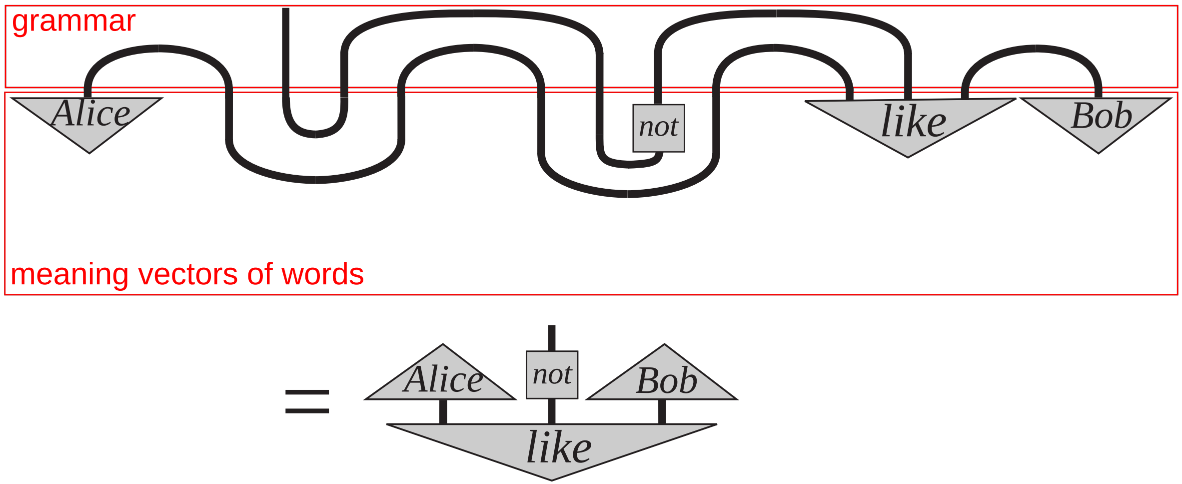

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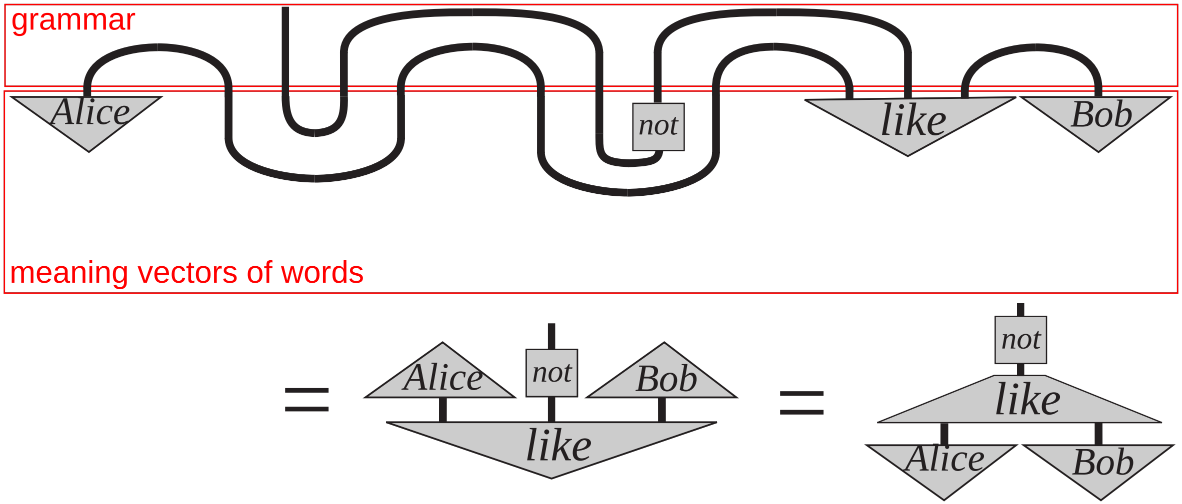
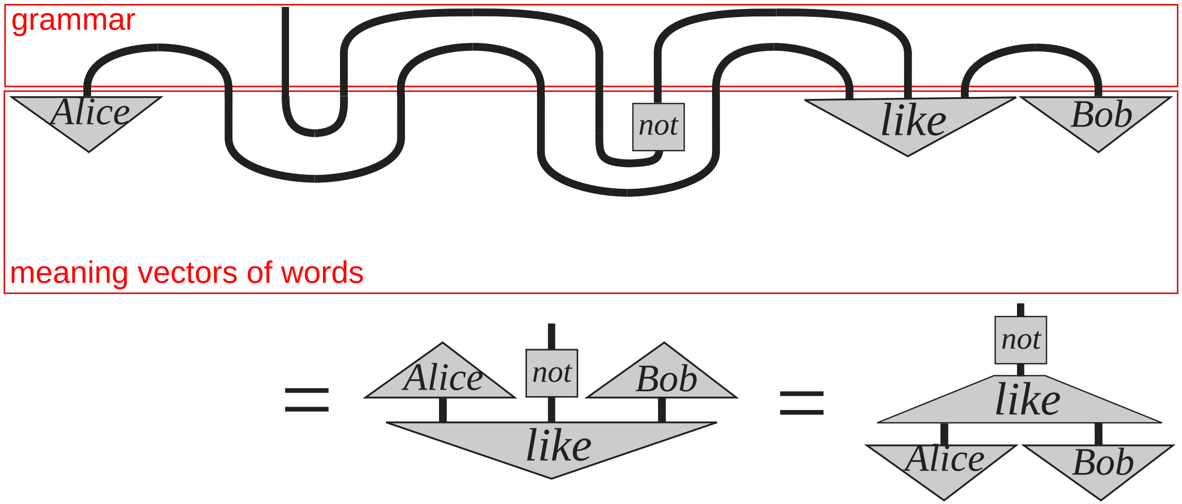


Using:

$$\overrightarrow{Alice} \otimes \overrightarrow{does} \otimes \overrightarrow{not} \otimes \overrightarrow{like} \otimes \overrightarrow{Bob}$$



$$= \vec{g}\left(\vec{f}(\vec{x},\vec{y})\right)$$

— *experiment: word disambiguation* —

E.g. what is "saw"" in: "Alice saw Bob with a saw".

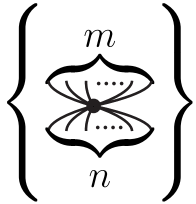
Model	High	Low	ρ
Baseline	0.47	0.44	0.16
Add	0.90	0.90	0.05
Multiply	0.67	0.59	0.17
Categorical (1)	**0.73**	**0.72**	**0.21**
Categorical (2)	**0.34**	**0.26**	**0.28**
UpperBound	4.80	2.49	0.62

Edward Grefenstette & Mehrnoosh Sadrzadeh (2011) *Experimental support for a categorical compositional distributional model of meaning*. Accepted for: Empirical Methods in Natural Language Processing (EMNLP'11).

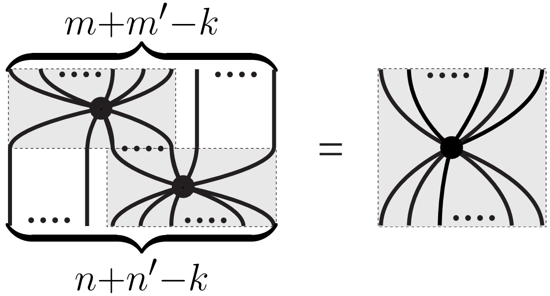
WHERE DOES THE ANALOGY STOP?

— *Frobenius algebras* —

quantum.1: classical data/observables

'*spiders*' = 

such that, for $k > 0$:



BC & Dusko Pavlovic (2007) *Quantum measurement without sums*. In: Mathematics of Quantum Computing and Technology. `quant-ph/0608035`

BC, Dusko Pavlovic & Jamie Vicary (2008) *A new description of orthogonal bases*. Mathematical Structures in Computer Science. `0810.0812`

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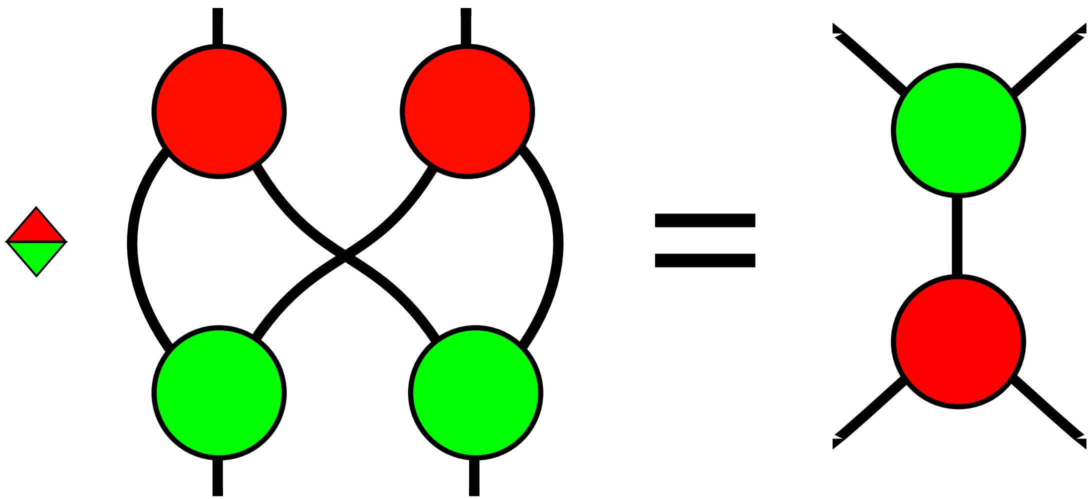
quantum.2: complementary quantum observables



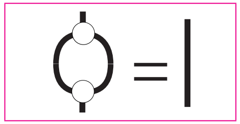
BC & Ross Duncan (2008) *Interacting quantum observables*. ICALP'08 & New Journal of Physics **13**, 043016. `arXiv:0906.4725`

Miriam Backens (2012) *The ZX-calculus is complete for stabilizer quantum mechanics*. In: Proc. Quantum Physic and Logic IX.

quantum.3: entangelement classes

$$\frac{GHZ = |000\rangle + |111\rangle}{W = |001\rangle + |010\rangle + |100\rangle} = \frac{\text{‘special’ CFAs}}{\text{‘anti-special’ CFAs}}$$

=



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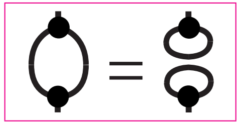

$$= \frac{\times}{+} \quad \Rightarrow \text{distributivity}$$

BC & Aleks Kissinger (2010) The compositional structure of multipartite quantum entanglement. ICALP'10. `arXiv:1002.2540`

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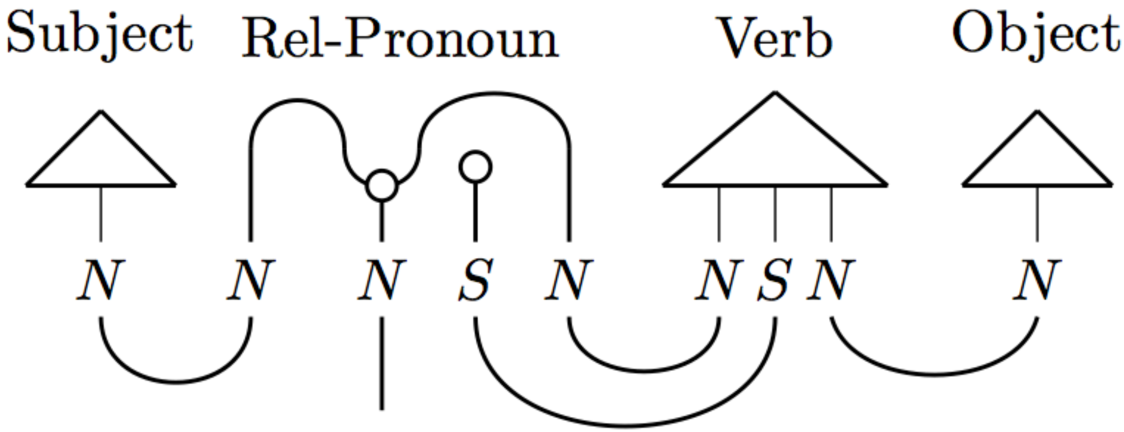
Language-meaning:



(the) man who Alice hates

Stephen Clark, BC and Mehrnoosh Sadrzadeh (2013) *The Frobenius Anatomy of Relative Pronouns*. MOL'13.