



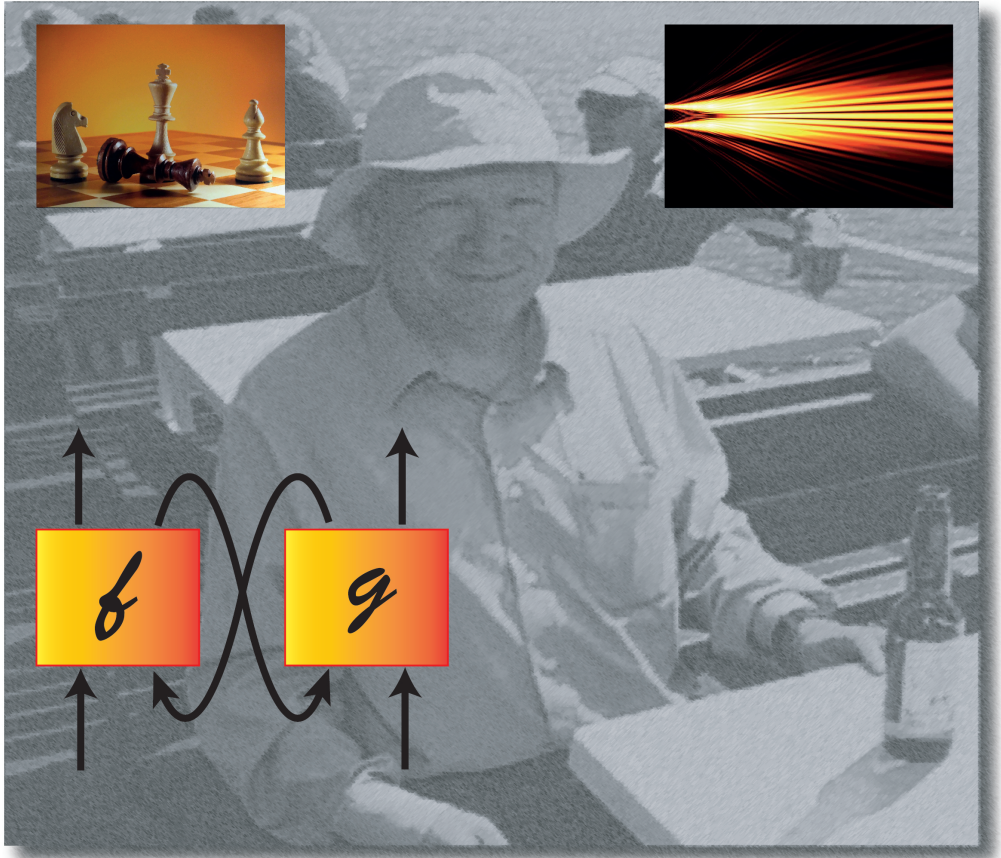
2009



2013

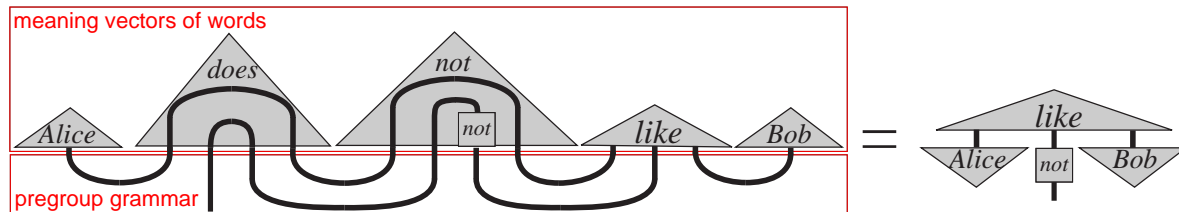
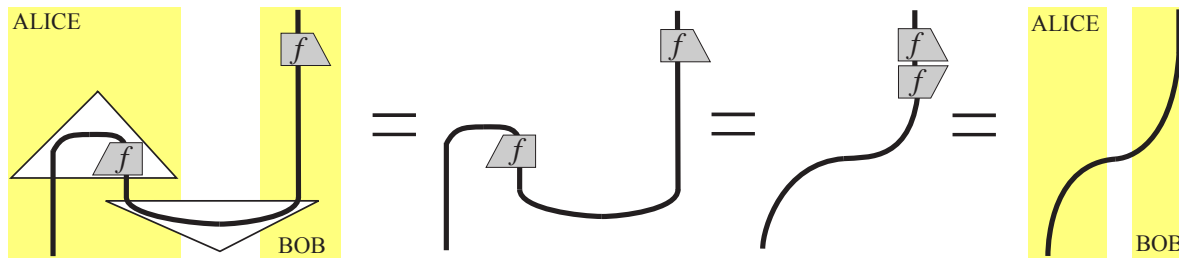






# The logic of quantum mechanics - take II

arXiv:1204.3458



— *genesis* —

— *genesis* —

[**von Neumann 1932**] Formalized quantum mechanics  
in “**Mathematische Grundlagen der Quantenmechanik**”

— *genesis* —

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[**von Neumann to Birkhoff 1935**] “I would like to  
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— *the mathematics of it* —

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**Hilbert space stuff:** continuum, field structure of complex numbers, vector space over it, inner-product, etc.

— *the mathematics of it* —

**Hilbert space stuff:** continuum, field structure of complex numbers, vector space over it, inner-product, etc.

**WHY?**

— *the mathematics of it* —

**Hilbert space stuff:** continuum, field structure of complex numbers, vector space over it, inner-product, etc.

**WHY?**

**von Neumann:** only used *it* since *it* was ‘available’.

— *the physics of it* —

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**von Neumann** crafted **Birkhoff-von Neumann Quantum ‘Logic’** to capture the concept of **superposition**.

— *the physics of it* —

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**Schrödinger (1935):** the stuff which is the true soul of quantum theory is **how quantum systems compose**.

— *the physics of it* —

**von Neumann** crafted **Birkhoff-von Neumann Quantum ‘Logic’** to capture the concept of **superposition**.

**Schrödinger (1935):** the stuff which is the true soul of quantum theory is **how quantum systems compose**.

**Quantum Computer Scientists:** Schrödinger is right!

— *the game plan* —

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**Task 0.** Solve:

$$\frac{\text{tensor product structure}}{\text{the other stuff}} = ???$$

— *the game plan* —

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i.e. **axiomatize “ $\otimes$ ” without reference to spaces.**

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**Task 1.** Investigate which assumptions (i.e. which structure) on  $\otimes$  is needed to deduce **physical phenomena.**

— *the game plan* —

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**Task 1.** Investigate which assumptions (i.e. which structure) on  $\otimes$  is needed to deduce **physical phenomena**.

**Task 2.** Investigate whether such an “interaction structure” appear elsewhere in “**our classical reality**”.

## **Outcome 1a:**

## Outcome 1a: “Sheer ratio of results to assumptions”

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Hans Halvorson (2010) Editorial to: *Deep Beauty: Understanding the Quantum World through Mathematical Innovation*, Cambridge University Press.

**Outcome 1a: “Sheer ratio of results to assumptions”**  
confirms that we are probing something very essential.

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**Outcome 1b:** Exposing this structure has already helped  
to **solve open problems elsewhere.**

---

E.g.: Ross Duncan & Simon Perdrix (2010) *Rewriting measurement-based quantum computations with generalised flow*. ICALP’10.

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**Outcome 1b:** Exposing this structure has already helped  
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**Outcome 1c: Framework is a simple intuitive (but  
rigorous) diagrammatic language,**

**Outcome 1a: “Sheer ratio of results to assumptions”**  
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**Outcome 1b:** Exposing this structure has already helped  
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**Outcome 1c: Framework is a simple intuitive (but  
rigorous) diagrammatic language**, meanwhile adopted  
by others e.g. Lucien Hardy in arXiv:1005.5164:

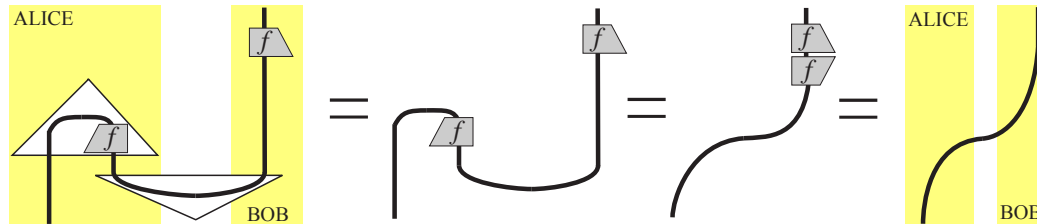
“... we join the *quantum picturalism* revolution [1]”

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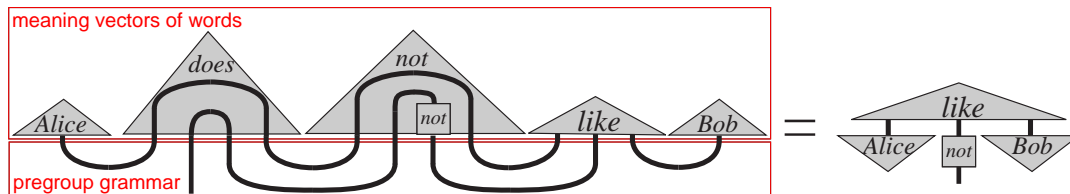
[1] BC (2010) *Quantum picturalism*. Contemporary Physics **51**, 59–83.

## Outcome 2a:

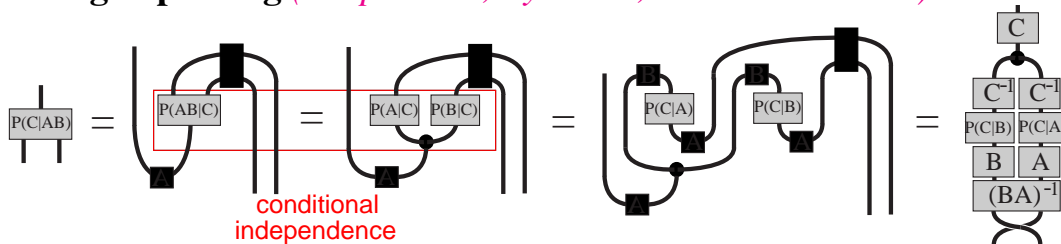
## Behaviors of matter (*Abramsky-C; LiCS'04, quant-ph/0402130*):



### Meaning in language (Clark-C-Sadrzadeh; Linguistic Analysis, arXiv:1003.4394):



### Knowledge updating (*C-Spekkens; Synthese, arXiv:1102.2368*):



— *the logic of it* —

— *the logic of it* —

**WHAT IS “LOGIC”?**

— *the logic of it* —

## WHAT IS “LOGIC”?

**Pragmatic option 1: Logic is structure in language.**

— *the logic of it* —

## WHAT IS “LOGIC”?

**Pragmatic option 1: Logic is structure in language.**

*“Alice and Bob ate everything or nothing, then got sick.”*

connectives ( $\wedge, \vee$ ) : *and, or*

negation ( $\neg$ ) : *not (cf. nothing = not something)*

entailment ( $\Rightarrow$ ) : *then*

quantifiers ( $\forall, \exists$ ) : *every(thing), some(thing)*

constants ( $a, b$ ) : *thing*

variable ( $x$ ) : *Alice, Bob*

predicates ( $P(x), R(x, y)$ ) : *eating, getting sick*

truth valuation ( $0, 1$ ) : *true, false*

$$(\forall z : Eat(a, z) \wedge Eat(b, z)) \wedge \neg(\exists z : Eat(a, z) \wedge Eat(b, z)) \Rightarrow Sick(a), Sick(b)$$

— *the logic of it* —

## WHAT IS “LOGIC”?

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**Pragmatic option 2: Logic lets machines reason.**

— *the logic of it* —

## WHAT IS “LOGIC”?

**Pragmatic option 1: Logic is structure in language.**

**Pragmatic option 2: Logic lets machines reason.**

Cf. the *soft* incarnation of AI in robotics, automated theorem proving, automated theory exploration, ...

**theory[mine]<sup>TM</sup>**  
Personalized mathematical theorems



1573 AD  
Pythagoras



1571 AD  
Kepler's



1879 AD  
Einstein

... you could be next!

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**[ 1 ] YOU** choose a  
name

**[ 2 ] WE** discover a  
theorem

**[ 3 ] THEY** get a  
great gift

— *the logic of it* —

## WHAT IS “LOGIC”?

**Pragmatic option 1: Logic is structure in language.**

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**Our framework appeals to both senses of logic, and moreover induces important new applications:**

— *the logic of it* —

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**From truth to meaning in natural language processing:**

—  (December 2010)

— *the logic of it* —

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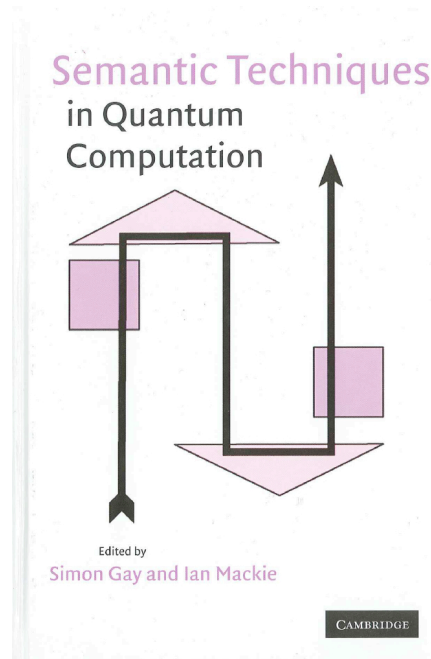
From truth to meaning in natural language processing:

—  (December 2010)

Automated theorem generation for graphical theories:

—  <http://sites.google.com/site/quantomatic/>

# MINIMAL QUANTUM PROCESS LANGUAGE

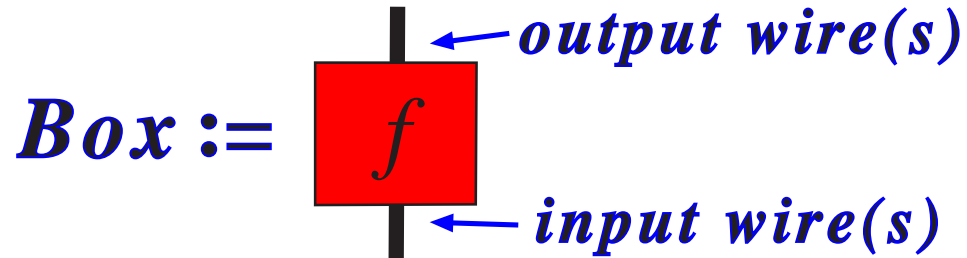


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Samson Abramsky & BC (2004) *A categorical semantics for quantum protocols*. In: IEEE-LICS'04. [quant-ph/0402130](#)

BC (2005) *Kindergarten quantum mechanics*. [quant-ph/0510032](#)

— *wire and box language* —

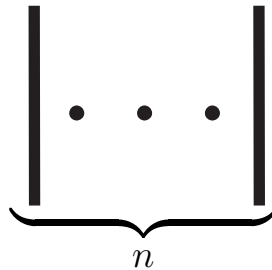


*Interpretation:* wire := **system** ; box := **process**

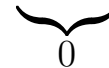
one system:



$n$  subsystems:

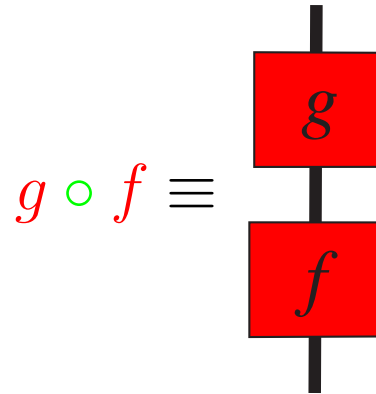


no system:

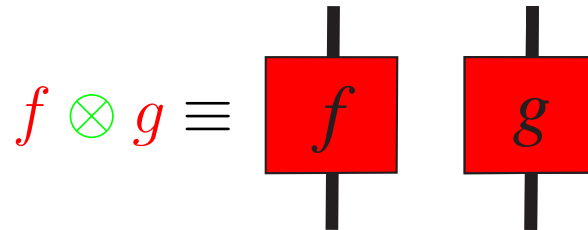


— *wire and box games* —

*sequential* or *causal* or *connected* composition:

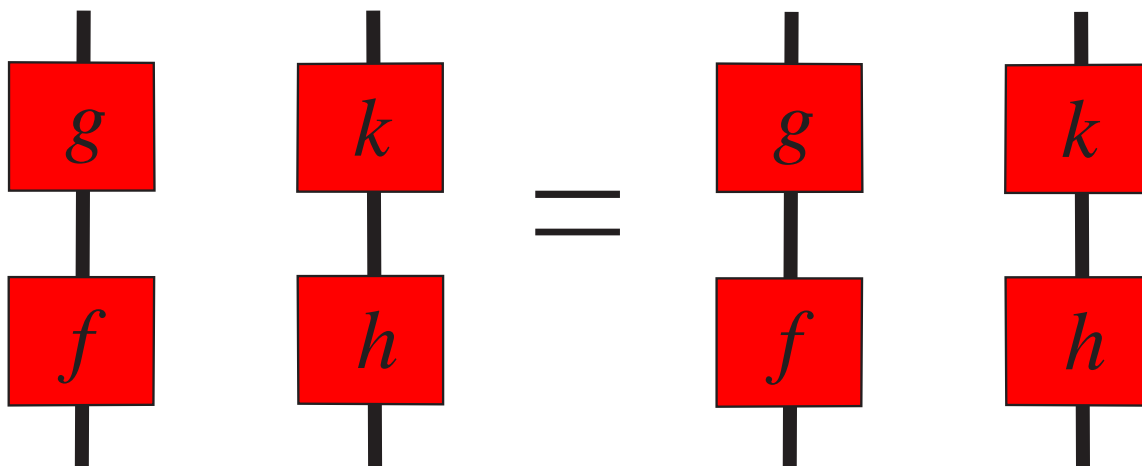


*parallel* or *acausal* or *disconnected* composition:



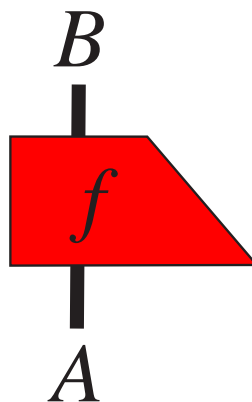
— *merely a new notation?* —

$$(g \circ f) \otimes (k \circ h) = (g \otimes k) \circ (f \otimes h)$$



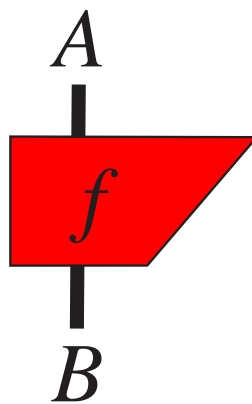
— *quantitative metric* —

$$f : A \rightarrow B$$

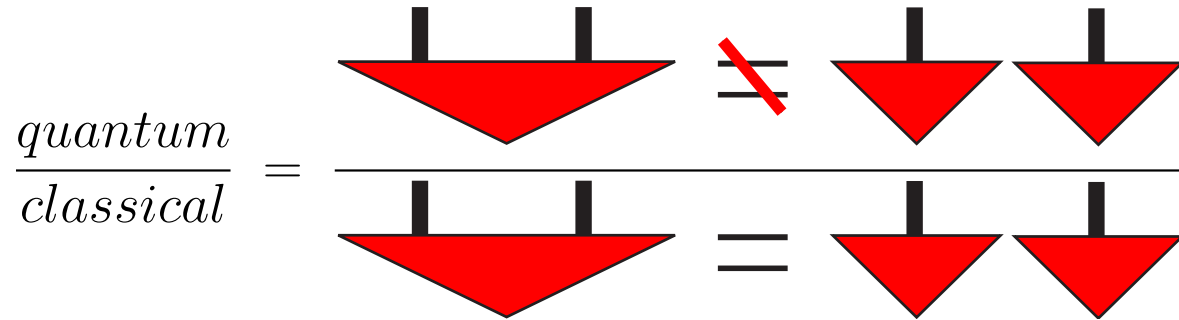


— *quantitative metric* —

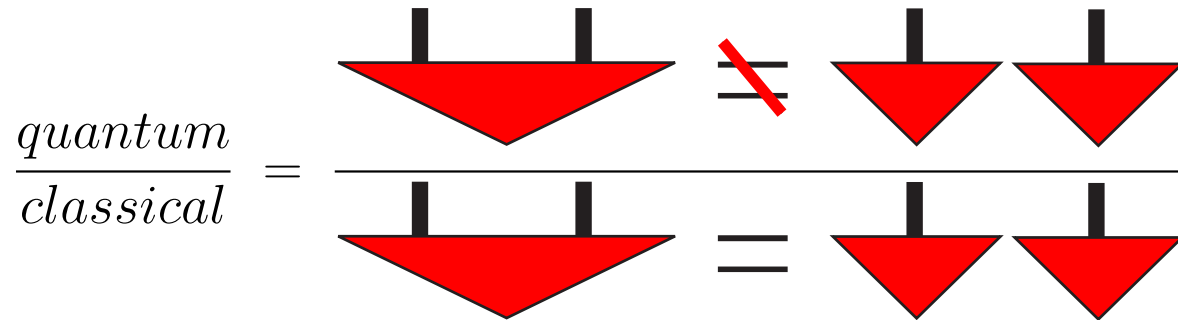
$$f^\dagger: B \rightarrow A$$



— *asserting (pure) entanglement* —



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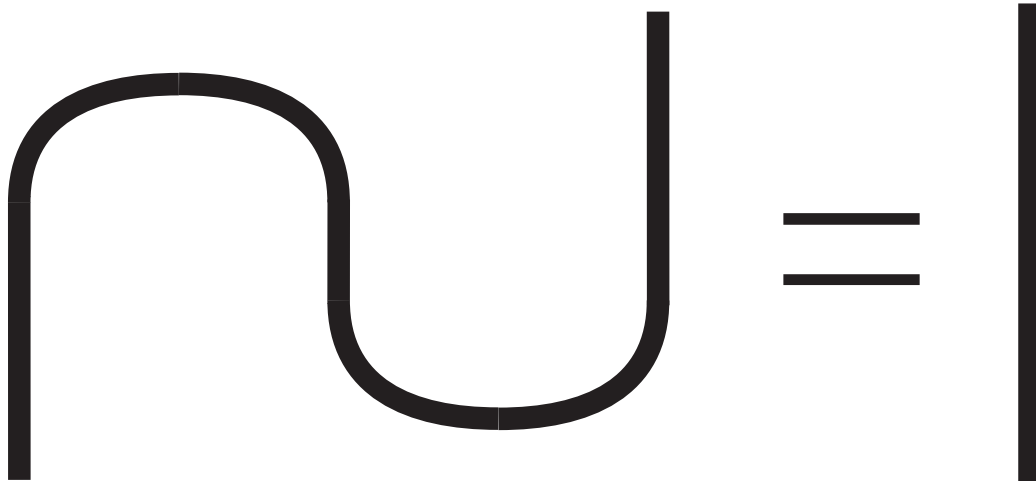
⇒ introduce ‘parallel wire’ between systems:



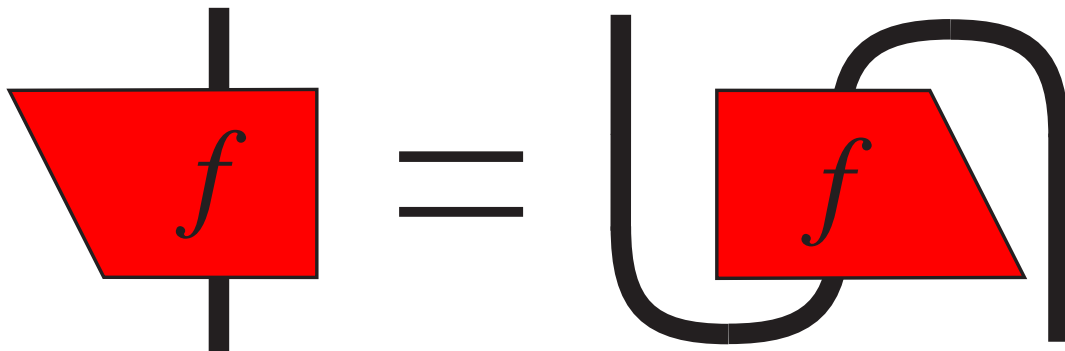
subject to: only topology matters!

— *quantum-like* —

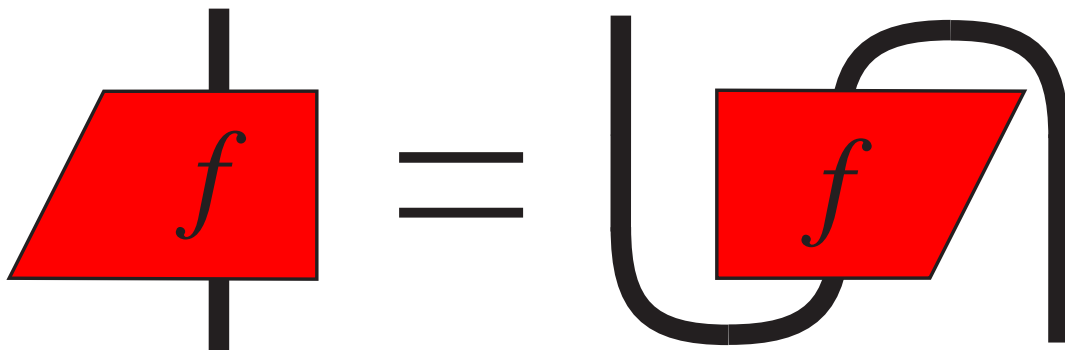
E.g.

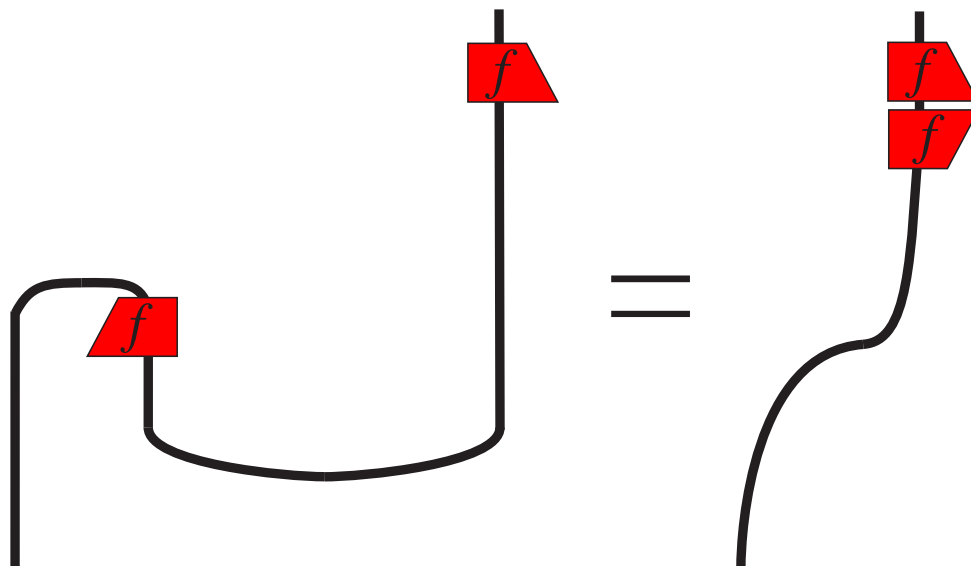


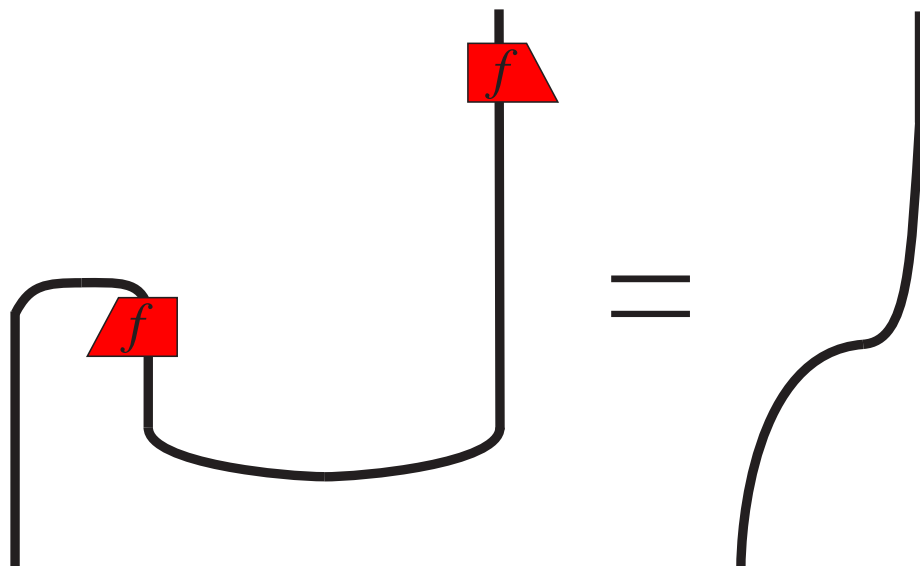
Transpose:

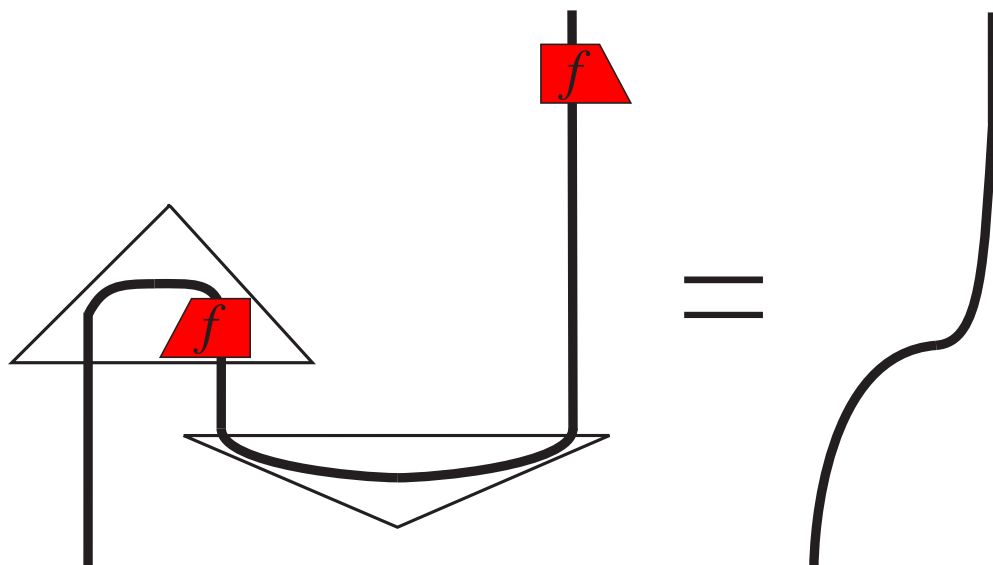


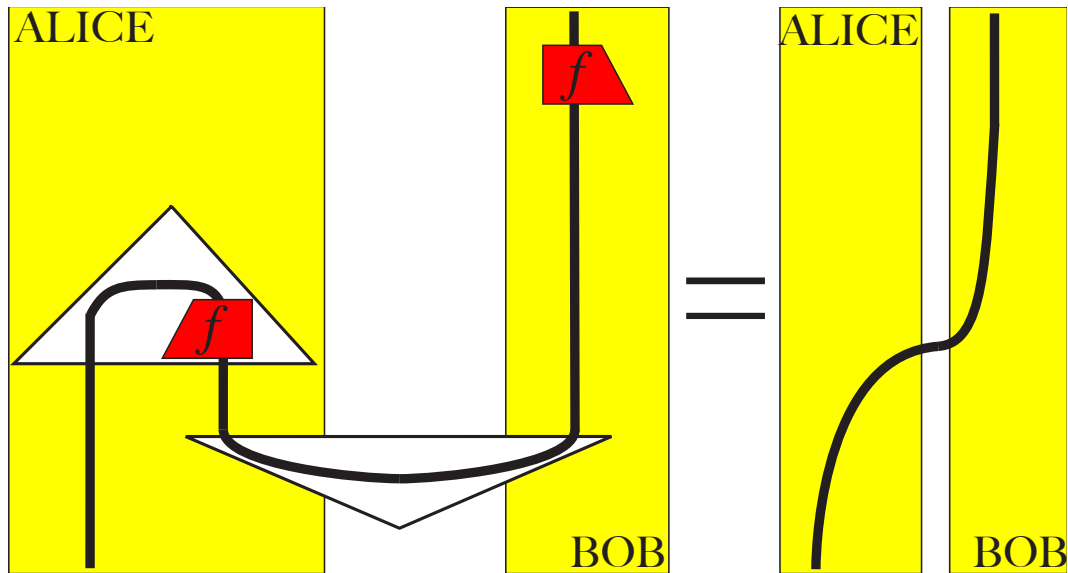
Conjugate:











$\Rightarrow$  quantum teleportation

— *symbolically: dagger compact categories* —

**Thm.** [Kelly-Laplaza '80; Selinger '05] *An equational statement between expressions in dagger compact categorical language holds if and only if it is derivable in the graphical notation via homotopy.*

**Thm.** [Hasegawa-Hofmann-Plotkin; Selinger '08] *An equational statement between expressions in dagger compact categorical language holds if and only if it is derivable in the dagger compact category of finite dimensional Hilbert spaces, linear maps, tensor product and adjoints.*

— *symbolically: dagger compact categories* —

**In words:** *Any equation involving:*

- *states, operations, effects*
- *unitarity, adjoints (e.g. self-adjoint), projections*
- *Bell-states/effects, transpose, conjugation*
- *inner-product, trace, Hilbert-Schmidt norm*
- *positivity, completely positive maps, ...*

*holds in quantum theory if and only if it can be derived in the graphical language via homotopy.*

— *kindergarten quantum mechanics: the experiment* —

**Contest in problem solving between:**

- **Children using quantum pictorialism**
- **Physics teachers using ordinary QM**

**The children will win!**

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[1] BC (2010) *Quantum pictorialism*. Contemporary Physics 51, 59–83.

# A SLIGHTLY DIFFERENT LANGUAGE FOR NATURAL LANGUAGE MEANING



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BC, Mehrnoosh Sadrzadeh & Stephen Clark (2010) *Mathematical foundations for a compositional distributional model of meaning*. [arXiv:1003.4394](https://arxiv.org/abs/1003.4394)

— *the from-words-to-a-sentence process* —

— *the from-words-to-a-sentence process* —

Consider meanings of **words**, e.g. as vectors (cf. Google):



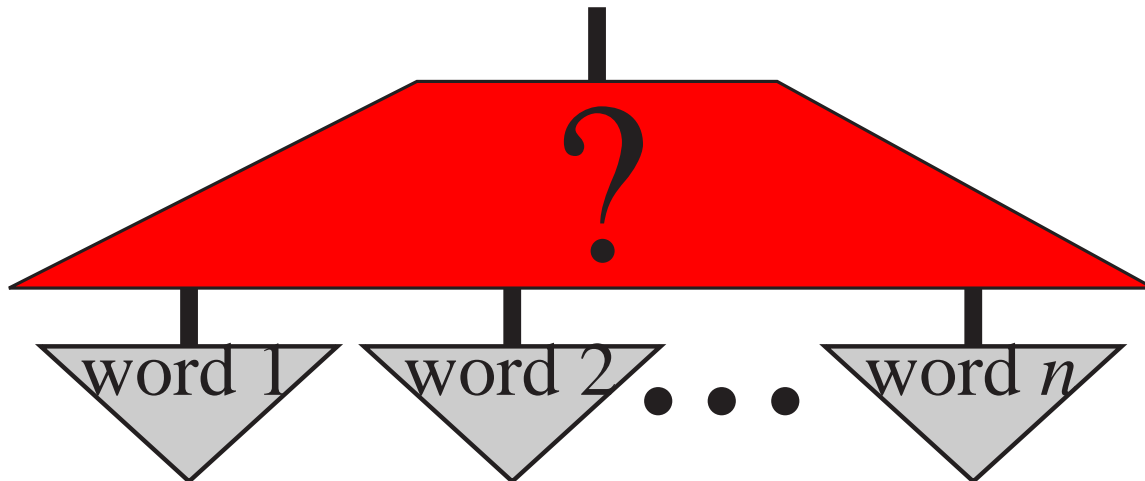
— *the from-words-to-a-sentence process* —

What is the meaning the **sentence** made up of these?



— *the from-words-to-a-sentence process* —

I.e. how do we/machines produce meanings of **sentences**?



— *the from-words-to-a-sentence process* —

I.e. how do we/machines produce meanings of **sentences**?

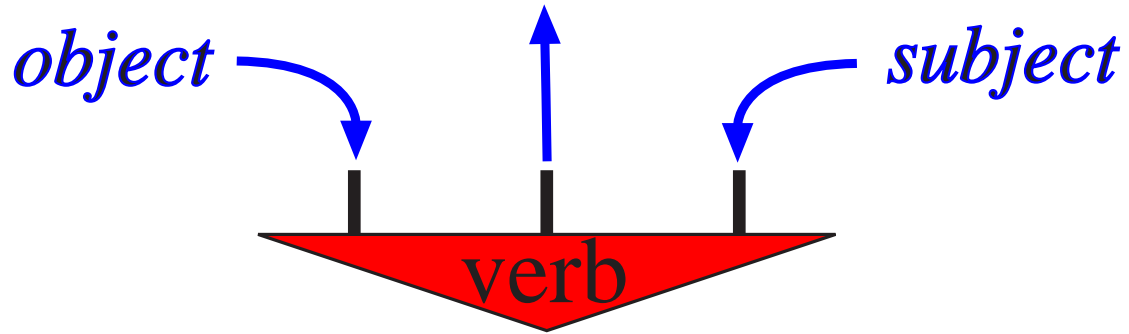


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Gerald Gazdar (1996) *Paradigm merger in natural language processing*. In: *Computing tomorrow: future research directions in computer science*, eds., I. Wand and R. Milner, Cambridge University Press.

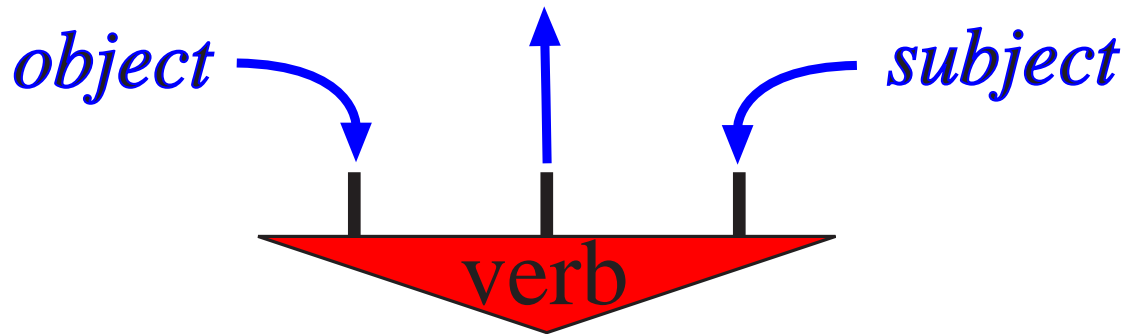
— *the from-words-to-a-sentence process* —

Information flow within a verb:



— *the from-words-to-a-sentence process* —

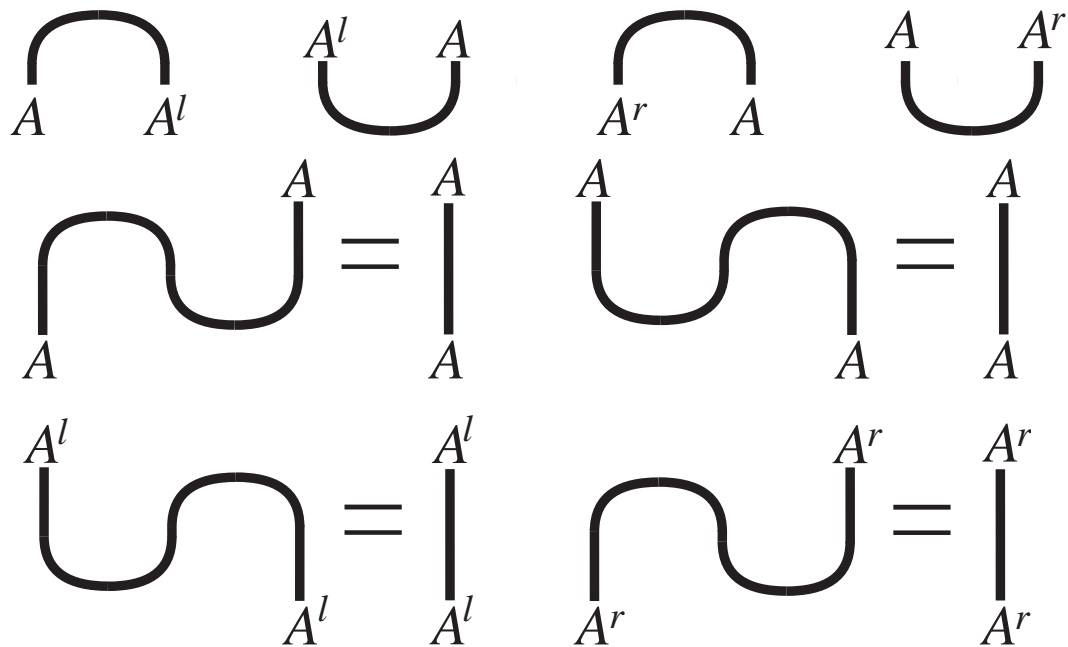
Information flow within a verb:



Again we have:



— *grammar as pregroups – Lambek '99* —



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For noun type  $n$ , verb type is  $^{-1}(n) \cdot s \cdot (n)^{-1}$ , so:

— *grammar as pregroups – Lambek '99* —

For noun type  $n$ , verb type is  $^{-1}(n) \cdot s \cdot (n)^{-1}$ , so:

$$n \cdot ^{-1}(n) \cdot s \cdot (n)^{-1} \cdot n = s$$

— *grammar as pregroups – Lambek '99* —

For noun type  $n$ , verb type is  $^{-1}(n) \cdot s \cdot (n)^{-1}$ , so:

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**Diagrammatic typing:**

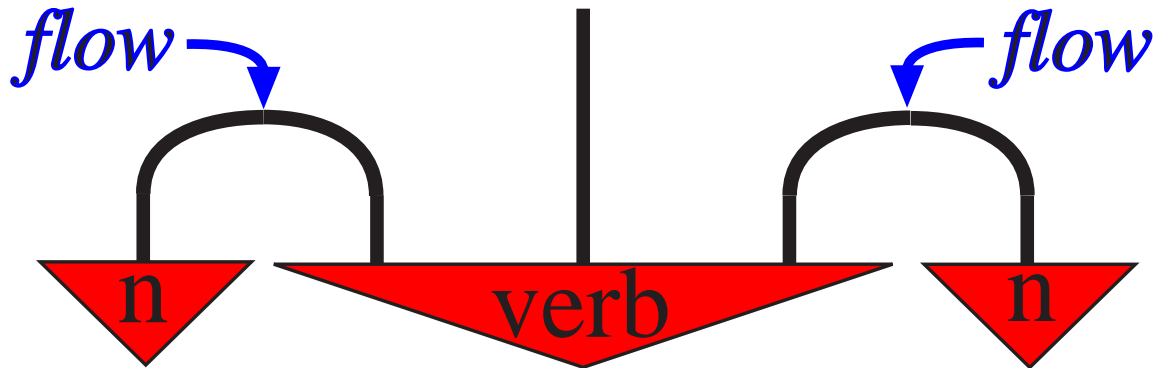


— *grammar as pregroups – Lambek '99* —

For noun type  $n$ , verb type is  $^{-1}(n) \cdot s \cdot (n)^{-1}$ , so:

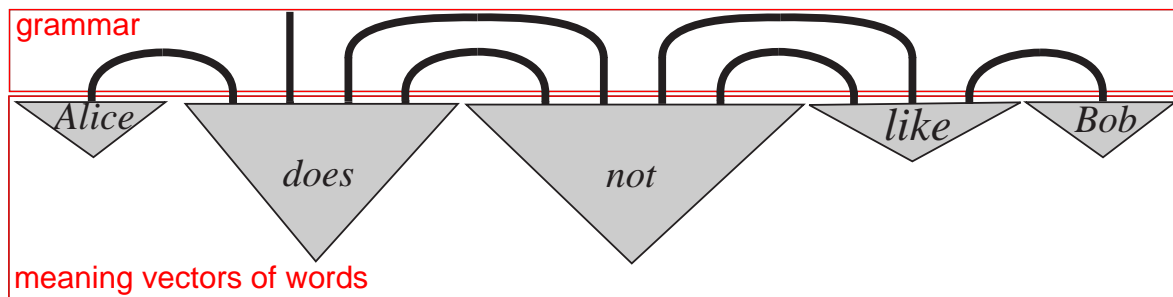
$$n \cdot ^{-1}(n) \cdot s \cdot (n)^{-1} \cdot n = s$$

**Diagrammatic meaning:**

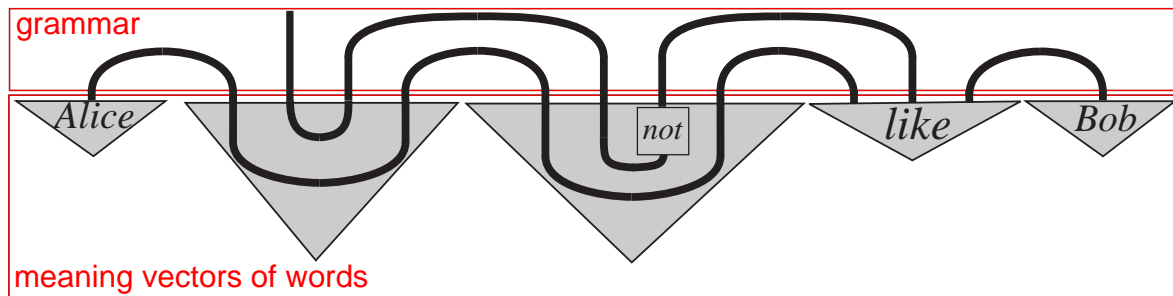


—  $\overrightarrow{Alice} \otimes \overrightarrow{does} \otimes \overrightarrow{not} \otimes \overrightarrow{like} \otimes \overrightarrow{Bob}$  —

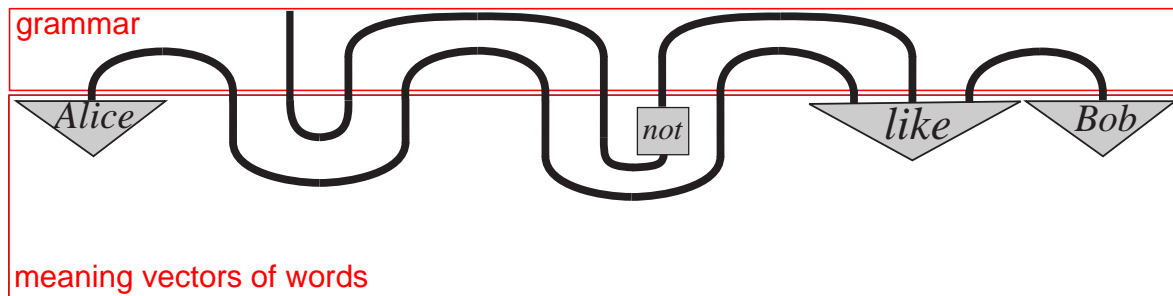
$$- \overrightarrow{Alice} \otimes \overrightarrow{does} \otimes \overrightarrow{not} \otimes \overrightarrow{like} \otimes \overrightarrow{Bob} -$$



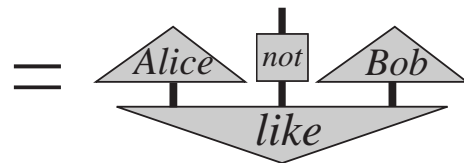
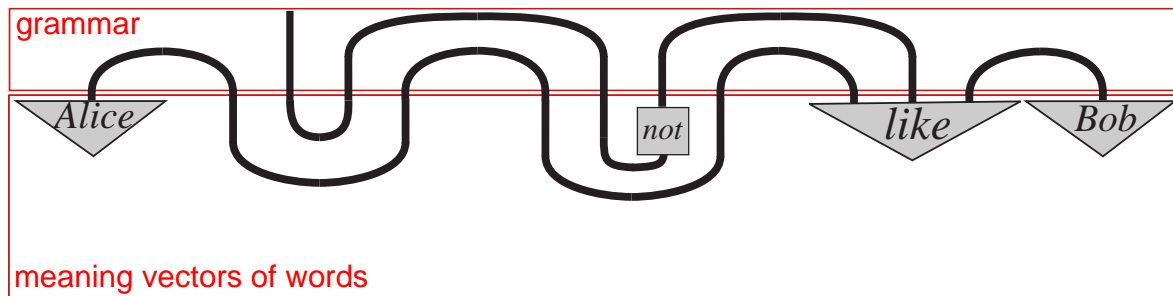
$$- \overrightarrow{Alice} \otimes \overrightarrow{does} \otimes \overrightarrow{not} \otimes \overrightarrow{like} \otimes \overrightarrow{Bob} -$$



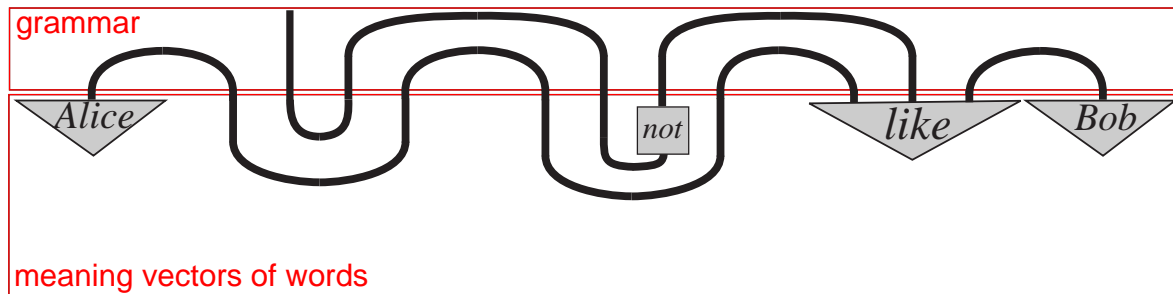
$$- \overrightarrow{Alice} \otimes \overrightarrow{does} \otimes \overrightarrow{not} \otimes \overrightarrow{like} \otimes \overrightarrow{Bob} -$$



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$$- \overrightarrow{Alice} \otimes \overrightarrow{does} \otimes \overrightarrow{not} \otimes \overrightarrow{like} \otimes \overrightarrow{Bob} -$$

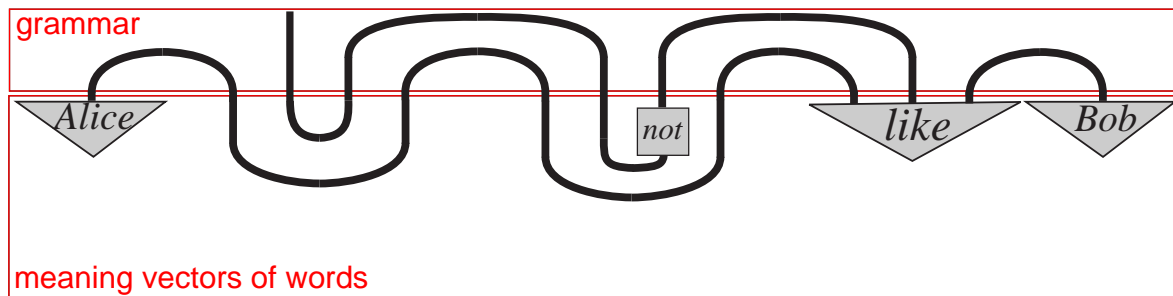


$$= \begin{array}{c} \triangle Alice \quad \square not \quad \triangle Bob \\ | \quad | \quad | \\ \triangle like \end{array} = \begin{array}{c} \square not \\ | \\ \triangle like \\ | \quad | \\ \triangle Alice \quad \triangle Bob \end{array}$$

Using:

$$\begin{array}{c} | \\ \triangle like \\ | \end{array} = \begin{array}{c} \text{loop} \\ \triangle like \end{array} \quad \begin{array}{c} \text{loop} \\ \triangle like \end{array} = \begin{array}{c} | \quad | \quad | \\ \triangle like \end{array}$$

$$- \overrightarrow{Alice} \otimes \overrightarrow{does} \otimes \overrightarrow{not} \otimes \overrightarrow{like} \otimes \overrightarrow{Bob} -$$



$$= \begin{array}{c} \triangle Alice \quad \square not \quad \triangle Bob \\ | \quad | \quad | \\ \triangle like \end{array} = \begin{array}{c} \square not \\ | \\ \triangle like \\ | \quad | \\ \triangle Alice \quad \triangle Bob \end{array}$$

$$= \vec{g} \left( \vec{f}(\vec{x}, \vec{y}) \right)$$

— *experiment: word disambiguation* —

E.g. what is “saw” in: “Alice saw Bob with a saw”.

Model	High	Low	$\rho$
Baseline	0.47	0.44	0.16
Add	0.90	0.90	0.05
Multiply	0.67	0.59	0.17
<b>Categorical (1)</b>	<b>0.73</b>	<b>0.72</b>	<b>0.21</b>
<b>Categorical (2)</b>	<b>0.34</b>	<b>0.26</b>	<b>0.28</b>
UpperBound	4.80	2.49	0.62

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Edward Grefenstette & Mehrnoosh Sadrzadeh (2011) *Experimental support for a categorical compositional distributional model of meaning*. Accepted for: Empirical Methods in Natural Language Processing (EMNLP’11).

**WHERE DOES THE ANALOGY STOP?**

# — *Frobenius algebras* —

**quantum.1:** classical data/observables

$$\text{'spiders'} = \left\{ \begin{array}{c} m \\ \text{---} \\ \text{---} \\ \text{---} \\ n \end{array} \right\}$$

such that, for  $k > 0$ :

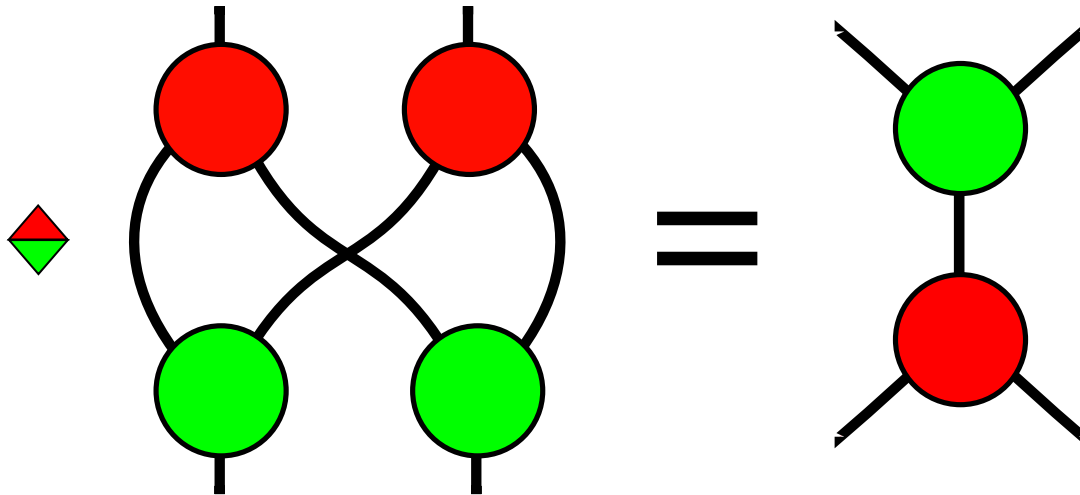
$$\begin{array}{c} m+m'-k \\ \text{---} \\ \text{---} \\ \text{---} \\ n+n'-k \end{array} = \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array}$$

BC & Dusko Pavlovic (2007) *Quantum measurement without sums*. In: Mathematics of Quantum Computing and Technology. quant-ph/0608035

BC, Dusko Pavlovic & Jamie Vicary (2008) *A new description of orthogonal bases*. Mathematical Structures in Computer Science. 0810.0812

— *Frobenius algebras* —

**quantum.2:** complementary quantum observables



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BC & Ross Duncan (2008) *Interacting quantum observables*. ICALP'08 & New Journal of Physics **13**, 043016. [arXiv:0906.4725](#)

Miriam Backens (2012) *The ZX-calculus is complete for stabilizer quantum mechanics*. In: Proc. Quantum Physics and Logic IX.

— *Frobenius algebras* —

**quantum.3:** entanglement classes

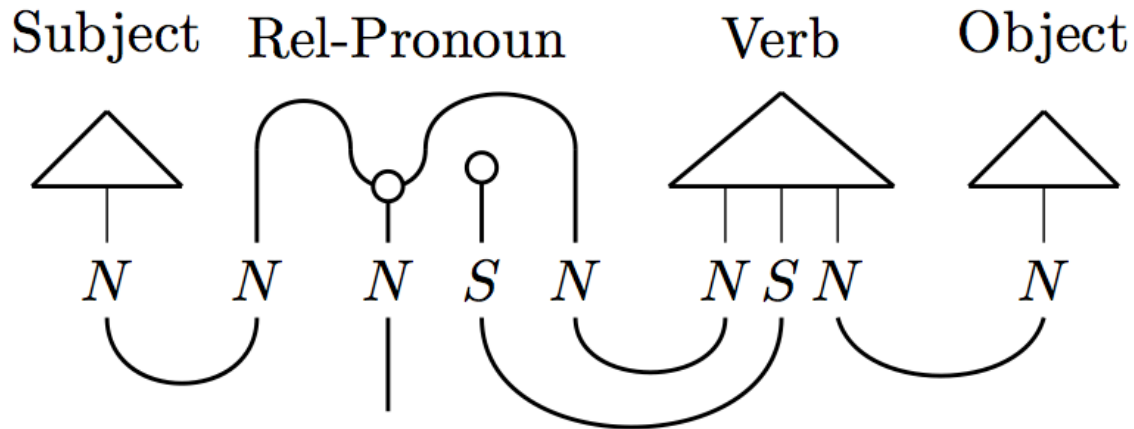
$$\begin{aligned}
 & \frac{GHZ = |000\rangle + |111\rangle}{W = |001\rangle + |010\rangle + |100\rangle} = \frac{\text{'special' CFAs}}{\text{'anti-special' CFAs}} \\
 & = \frac{\boxed{\text{Cup diagram} = \text{Cap diagram}}}{\boxed{\text{Cup diagram} = \text{Cap diagram}}} \\
 & = \frac{\times}{+} \Rightarrow \text{distributivity}
 \end{aligned}$$

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BC & Aleks Kissinger (2010) The compositional structure of multipartite quantum entanglement. ICALP'10. [arXiv:1002.2540](https://arxiv.org/abs/1002.2540)

— *Frobenius algebras* —

**Language-meaning:**



(the) man who Alice hates

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Stephen Clark, BC and Mehrnoosh Sadrzadeh (2013) *The Frobenius Anatomy of Relative Pronouns*. MOL'13.