Diagrammatic Reasoning for Asynchronous Circuits

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How I met your Samson

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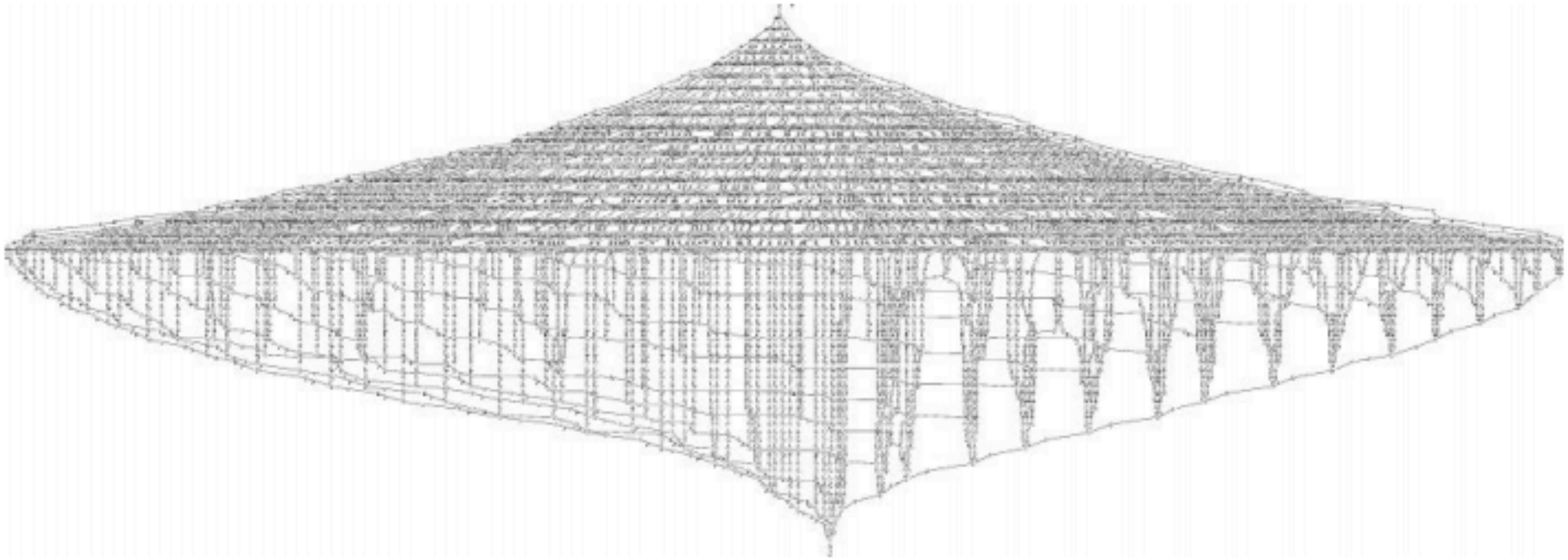


why async?

- very high speed (+)
- very low power (+)
- high footprint (-)
- *design and verification (-)*

MICROPIPELINES

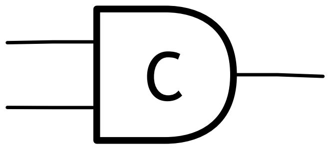
IVAN E. SUTHERLAND

The pipeline processor is a common paradigm for very high speed computing machinery. Pipeline processors provide high speed because their separate stages can operate concurrently, much as different people on a manufacturing assembly line work concurrently on material passing down the line. Although the concurrency of pipeline processors makes their design a demanding task, they can be found in graphics processors, in signal processing devices, in integrated circuit components for doing arithmetic, and in the instruction interpretation units and arithmetic operations of general purpose computing machinery.

Because I plan to describe a variety of pipeline processors, I will start by suggesting names for their var- that they are elastic.

I assign the name micropipeline to a particularly simple form of event-driven elastic pipeline with or without internal processing. The micro part of this name seems appropriate to me because micropipelines contain very simple circuitry, because micropipelines are useful in very short lengths, and because micropipelines are suitable for layout in microelectronic form.

I have chosen micropipelines as the subject of this lecture for three reasons. First, micropipelines are simple and easy to understand. I believe that simple ideas are best, and I find beauty in the simplicity and symmetry of micropipelines. Second, I see confusion surrounding the design of FIFOs. I offer this description of



The *Muller C-element* is the typical synchronisation gate. It produces an output if it receives signals on both inputs.

Ebergen's model

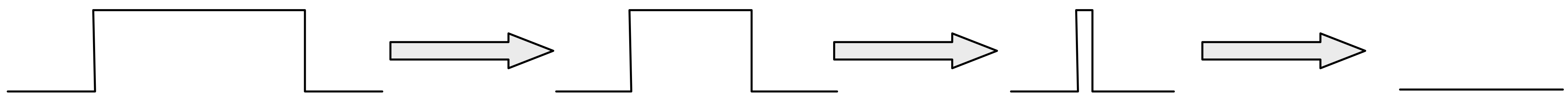
$$[\![W : A \to A']\!] = \left[(AA')^*\right].$$
$$[\![C : A_1 \otimes A_2 \to A']\!] = \left[((A_1 \mid A_2) \cdot A')^*\right]$$
$$[\![X : A_1 \otimes A_2 \to A']\!] = \left[(A_1 A' + A_2 A')^*\right]$$
$$[\![T : A \to A'_1 \otimes A'_2]\!] = \left[(AA'_1 + AA'_2)^*\right]$$
$$[\![F : A \to A'_1 \otimes A'_2]\!] = \left[(A \cdot (A'_1 \mid A'_2))^*\right].$$

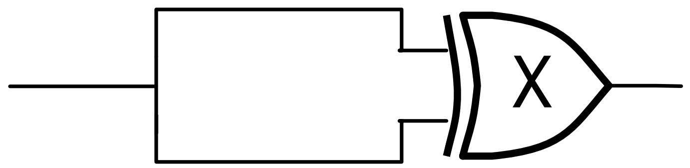
we want some kind of monoidal category

Wire is not identity

$$W : A1 \rightarrow A2$$
$$W' : A2 \rightarrow A3$$
$$W' o W \ni A1.A1.A3.A3$$

Wire is not realistic





can we have a
‘nice’ model?

an affine model

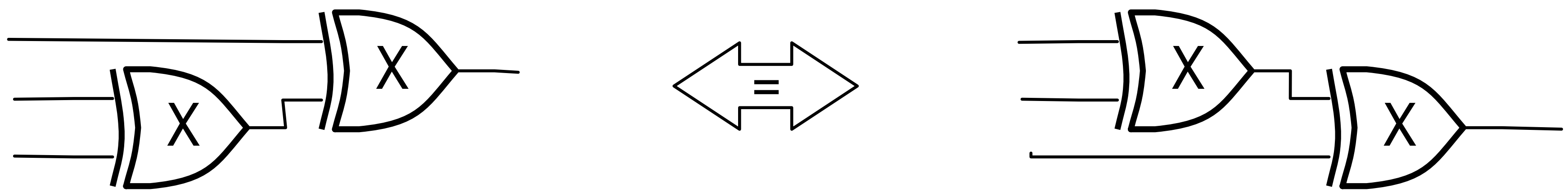
$$[\![C : A_1 \otimes A_2 \to A']\!] = (A_1 \mid A_2) \cdot A'$$
$$[\![X : A_1 \otimes A_2 \to A']\!] = A_1 A' + A_2 A'$$
$$[\![T : A \to A'_1 \otimes A'_2]\!] = A A'_1 + A A'_2$$
$$[\![F : A \to A'_1 \otimes A'_2]\!] = A \cdot (A'_1 \mid A'_2)$$
$$[\![W : A \to A']\!] = A A'$$
$$[\![U : \emptyset \to A]\!] = \epsilon$$
$$[\![E : A \to \emptyset]\!] = A$$
$$[\![P : \emptyset \to A]\!] = A.$$

a symmetric monoidal category

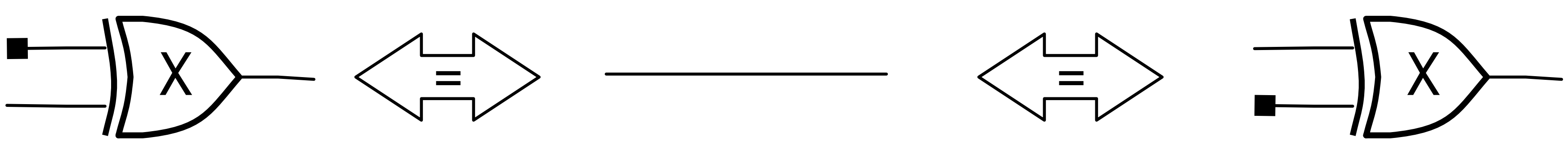
algebraic structure

1. (A, X, U) is a commutative monoid, with T a retract of X.

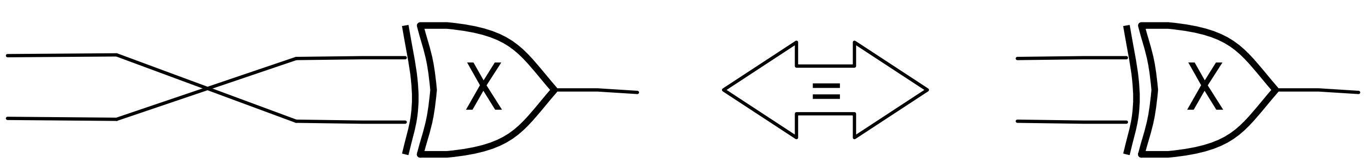
Associativity $(W \otimes X); X = (X \otimes W); X.$



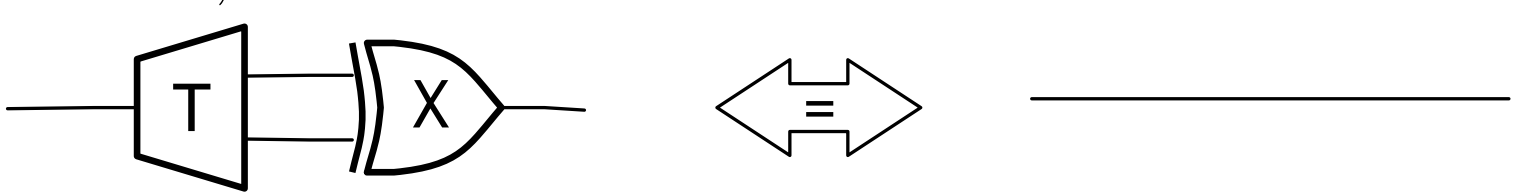
Unit $(U \otimes W); X = (W \otimes U); X = W.$



Commutativity $\gamma_A; X = X.$



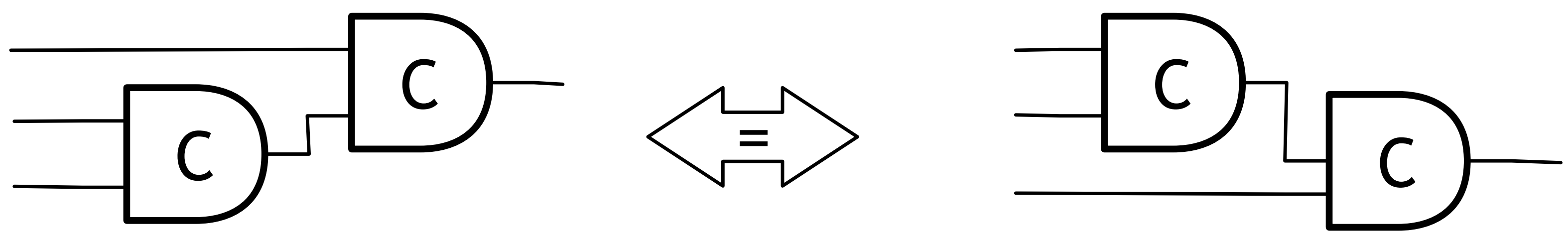
Retract $T; X = W.$



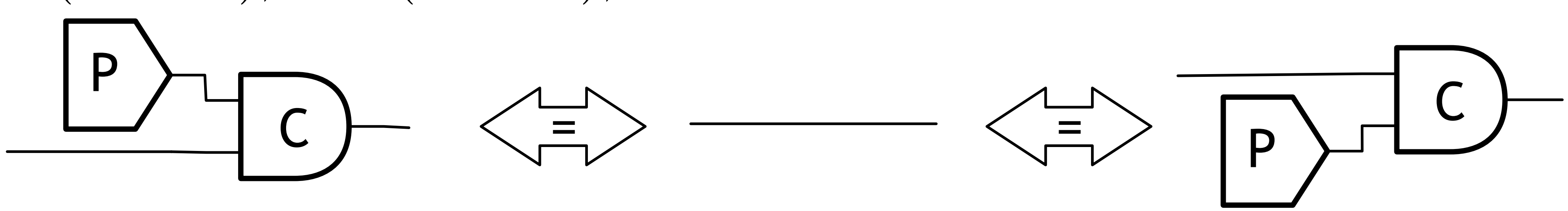
more algebraic structure

2. (A, C, P) is a commutative monoid with U an absorbing element.

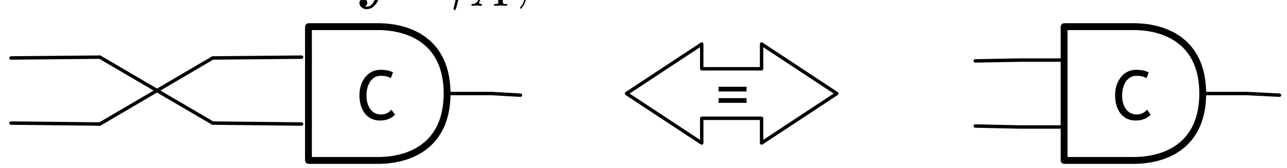
Associativity $(W \otimes C); C = (C \otimes W); C.$



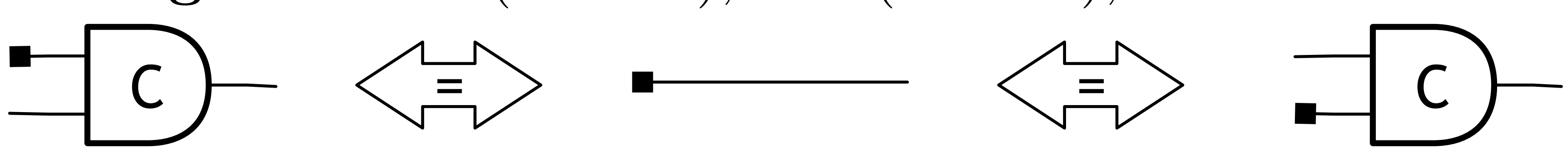
Unit $(P \otimes W); C = (W \otimes P); C = W$



Commutativity $\gamma_A; C = C.$



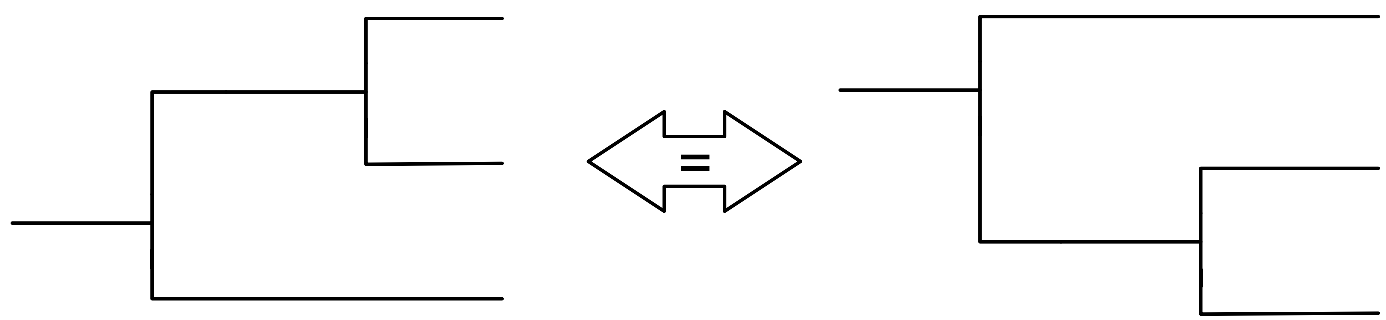
Absorbing element $(W \otimes U); C = (U \otimes W); C = U$



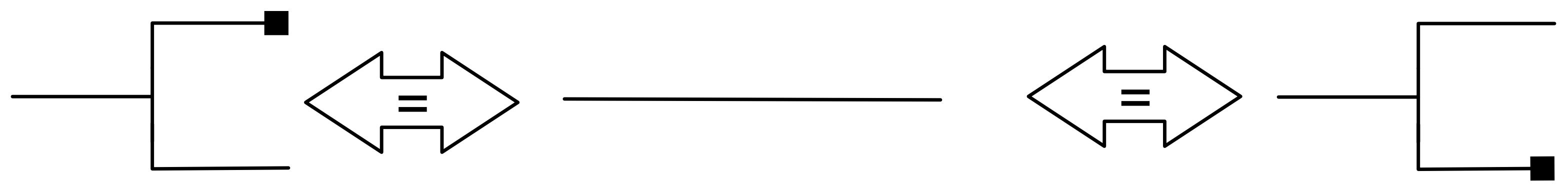
yet more algebraic structure

3. (A, F, E) *is a co-commutative co-monoid, with* C *a section of* F*.*

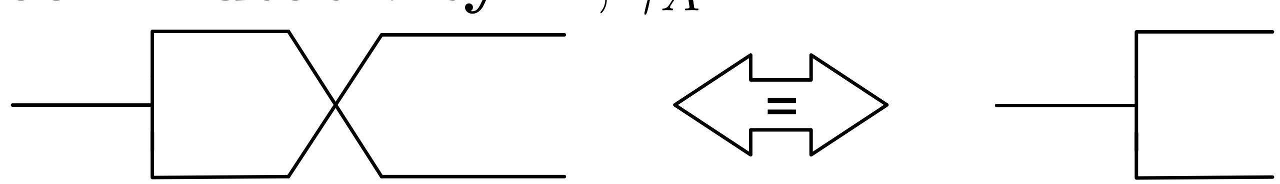
Co-associativity $F;(F \otimes W) = F;(W \otimes F)$.



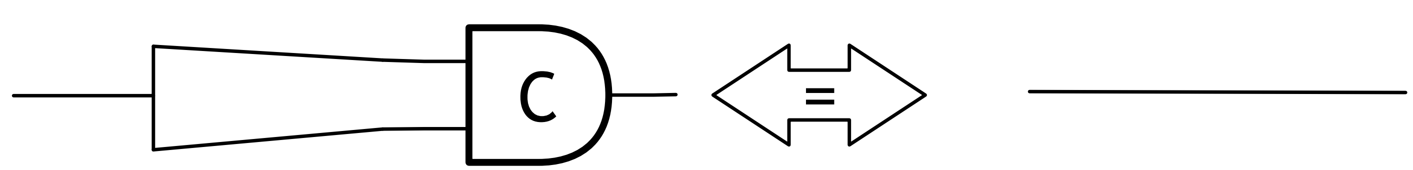
Co-unit $F;(W \otimes E) = F;(E \otimes W)$.



Co-commutativity $F;\gamma_A = F$.



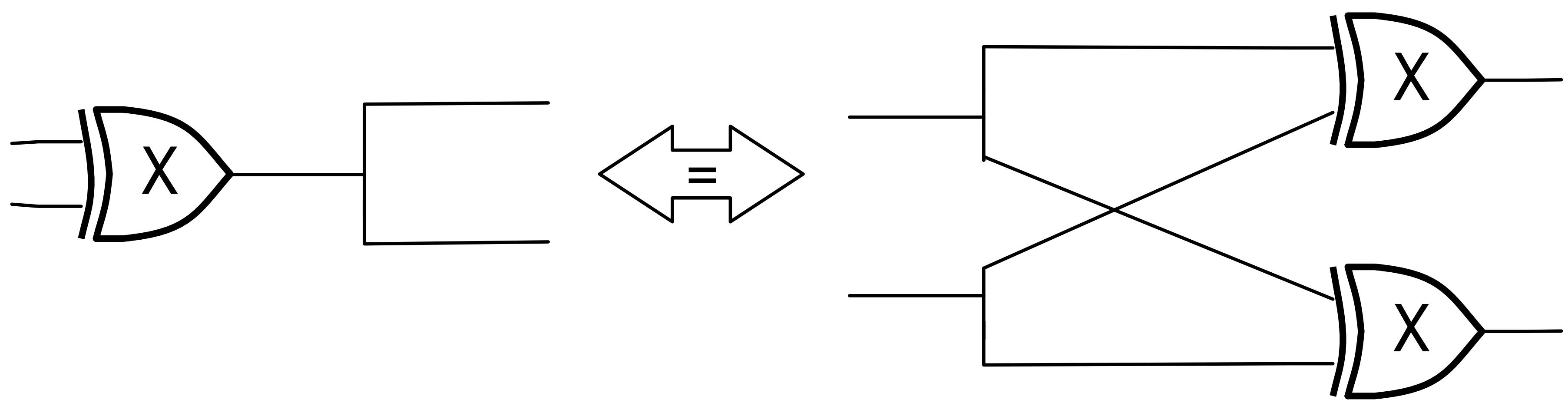
Section $F;C = W$



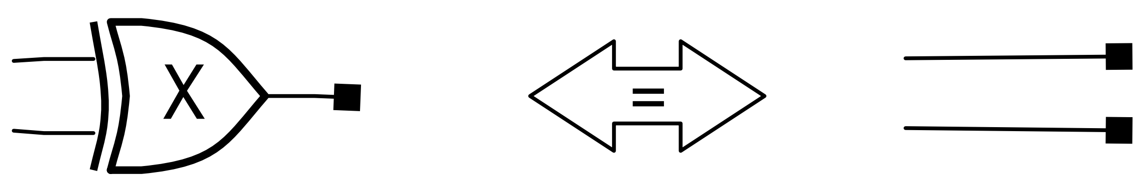
even more algebraic structure

Theorem 3.07 *1.* (A, X, E, F, U) *is a bialgebra.*

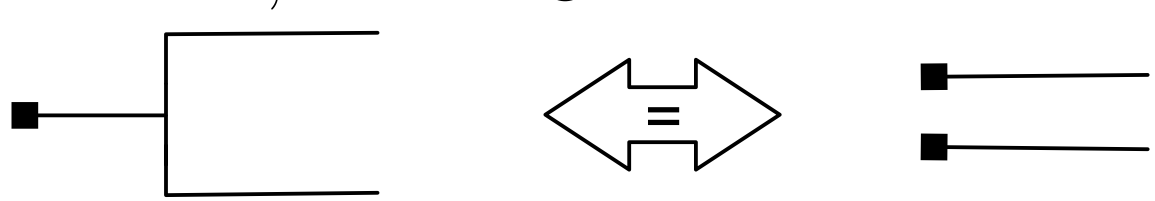
Distributivity $X; F = (F \otimes F); (W \otimes \gamma_A \otimes W); (X \otimes X)$.



Unit $E; F = E \otimes E$.



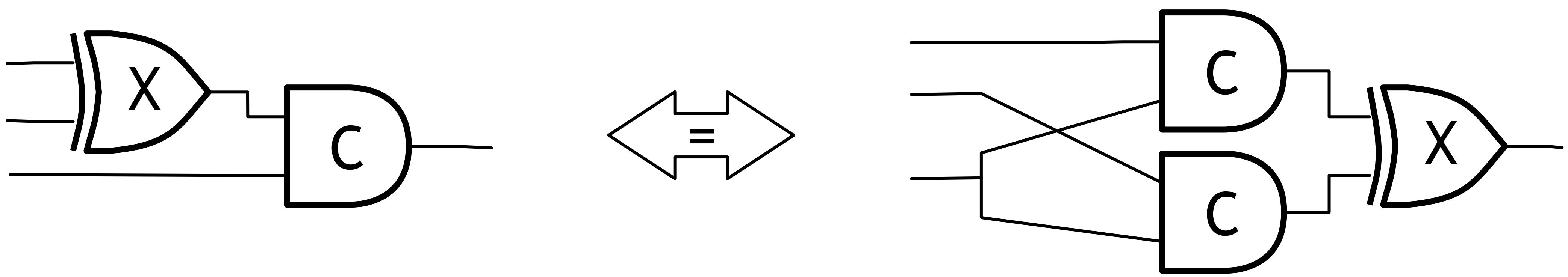
Co-unit $X; U = U \otimes U$.



further algebraic structure

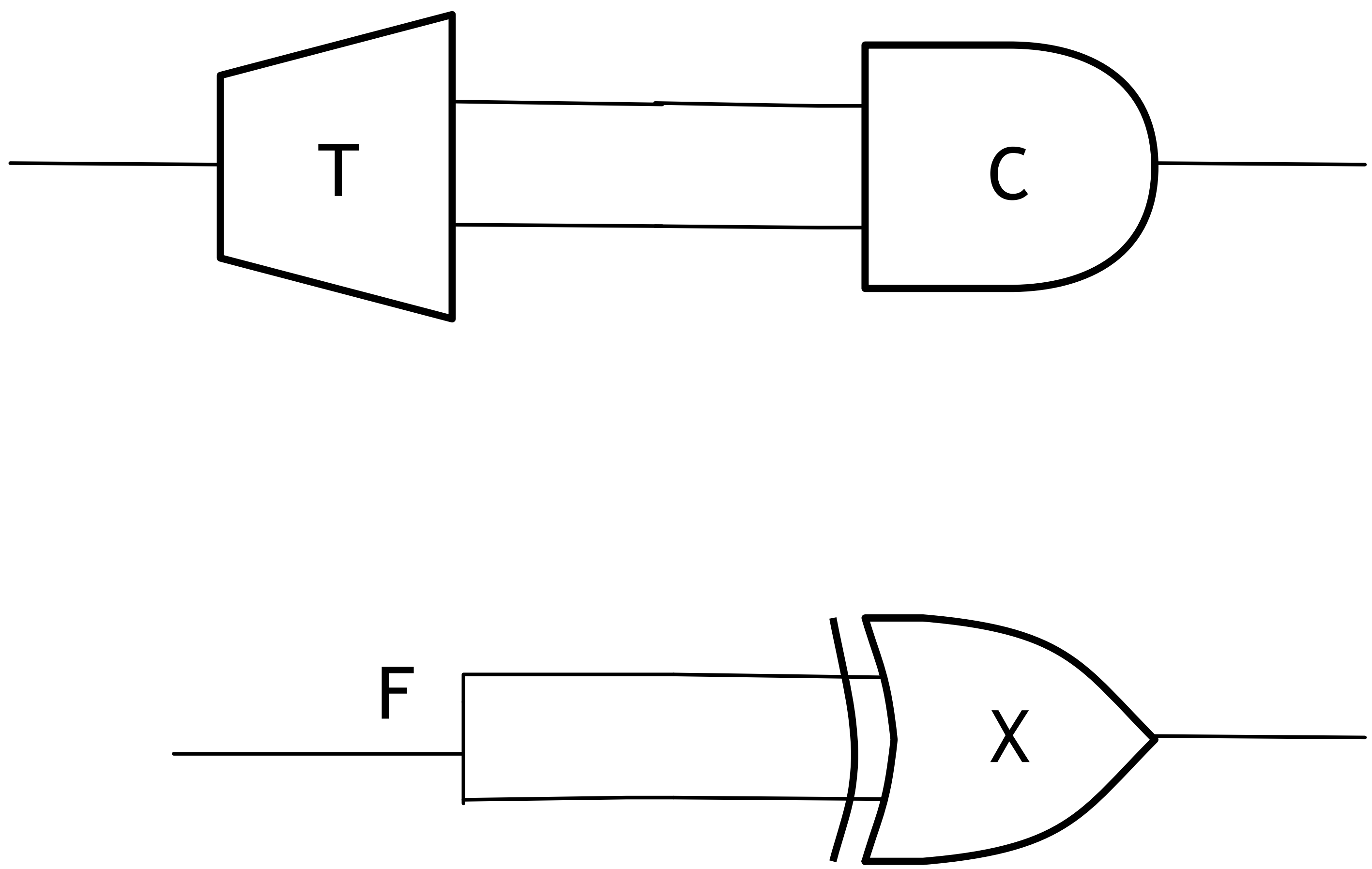
2. (A, C, F, X) *is a Laplace pairing (in the sense of Rota, as per [9]).*

$$(X \otimes W); C = (W \otimes W \otimes F); (W \otimes \gamma_A \otimes W); (C \otimes C); X.$$



fake algebraic structure

$$T;C = \emptyset = F;X$$



proofs are very easy calculations
(no iteration)

interleaved model with idealized wires

$f^0 = \emptyset, f^k = f \mid f^{k-1}, !f = \bigcup_{i \geq 0} f^i$. Note that if $f : X \to Y$ then $!f : X \to Y$. We define $\mathsf{C} = !C$, $\mathsf{X} = !X$, $\mathsf{T} = !T$, $\mathsf{F} = !F$, $\mathsf{W} = !W$, $\mathsf{U} = !U$, $\mathsf{E} = !E$,

Definition 4.11 *We say that* $f : X \to Y, g : Y \to Z$ compose safely *if and only if* $!(f;g) = !f;!g$.

Lemma 4.12 *All the compositions in Thms. 3.06 and 3.07 are safe in the sense of Def. 4.11.*

Lemma 4.13 *If* $f : X \to Y, f' : X' \to Y'$ *then* $!(f \otimes g) = !f \otimes !g$.

$$\sout{T;C = \emptyset = F;X}$$

Theorem 4.14 *Asynchronous circuits with an interleaved model form a compact closed category, called* **IdAsy** *where*

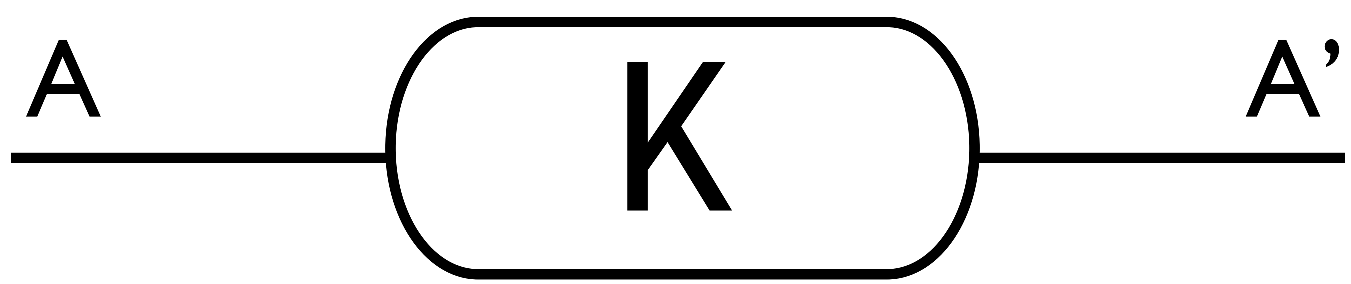
- *composition is defined as in* **AffAsy***;*
- *identity is* W*;*
- *the structural monoidal morphisms (associator, left identity, right identity, symmetry, unit, co-unit) are obtained by applying* $!-$ *to the corresponding structural morphisms in* **AffAsy***;*
- *objects are self-dual* $A^* = A$*;*
- *the unit* $\eta_A : I \to A_1^* \otimes A_2$ *and the co-unit* $\epsilon_A : A_1^* \otimes A_2 \to I$ *have the same sets of traces as the identity* $W : A_1 \to A_2$*.*

Theorem 4.16 *The algebraic structure of* **AffAsy** *is preserved by interleaving* $(!-)$ *in* **IdAsy***:*

- $(A, \mathsf{X}, \mathsf{U})$ *is a commutative monoid with* T *a retract of* X*.*
- $(A, \mathsf{C}, \mathsf{P})$ *is a commutative monoid with* U *an absorbing element.*
- $(A, \mathsf{F}, \mathsf{E})$ *is a co-commutative co-monoid with* C *a section of* F
- $(A, \mathsf{X}, \mathsf{E}, \mathsf{F}, \mathsf{U})$ *is a bialgebra.*
- $(A, \mathsf{C}, \mathsf{F}, \mathsf{X})$ *is a Laplace pairing.*

capacitive wires

$$[\![\mathsf{K}]\!] = !(AA' + AA).$$

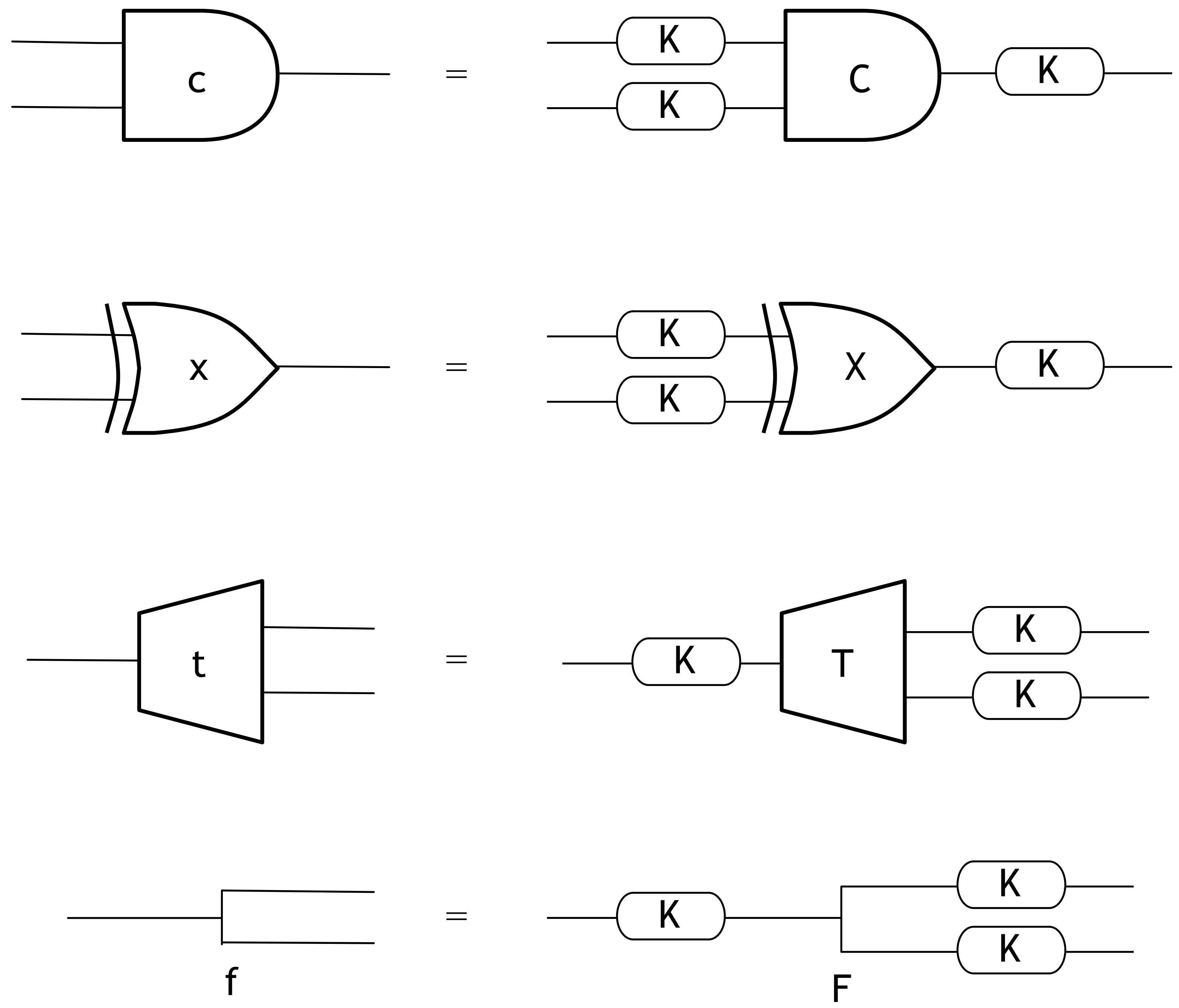


Lemma 4.18 *1.* $\mathsf{K} : A \to A$ *is idempotent, i.e.* $\mathsf{K};\mathsf{K} = \mathsf{K}$.

towards physical realism:
remove the idealized wire component
W

recovering the categorical structure

Definition 4.21 *The* Karoubi envelope *of category* $\mathbf{C}$, *sometimes written* $\mathbf{Split}(\mathbf{C})$, *is the category whose objects are pairs of the form* (A, e) *where* A *is an object of* $\mathbf{C}$ *and* $e : A \to A$ *is an idempotent of* $\mathbf{C}$, *and whose morphisms are triples of the form* $(e, f, e') : (A, e) \to (A, e')$ *where* $f : A \to A'$ *is a morphism of* $\mathbf{C}$ *satisfying* $f = e; f; e'$.



c
=
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x
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=
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K
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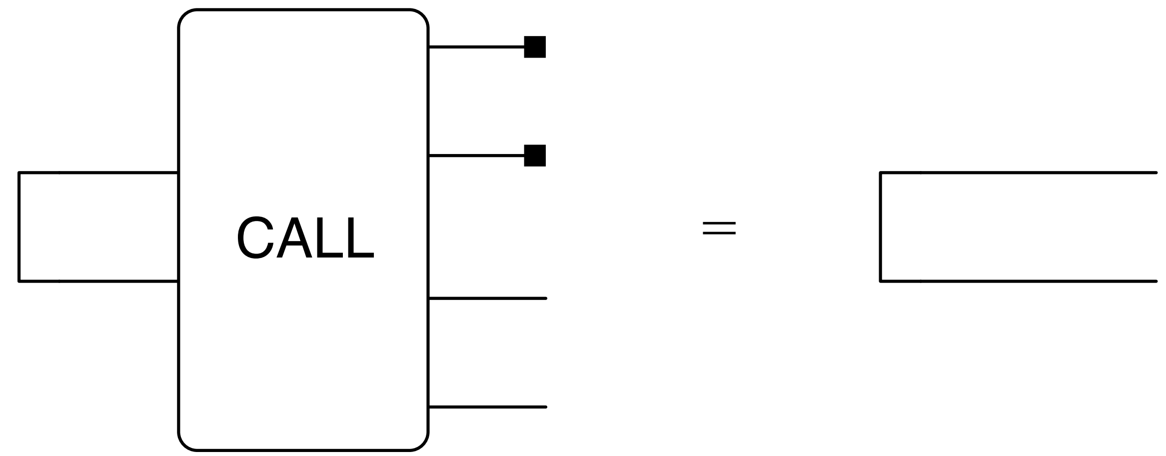
F

Theorem 4.25 *The category of delay-insensitive asynchronous circuits* DIAsy *is compact closed with*

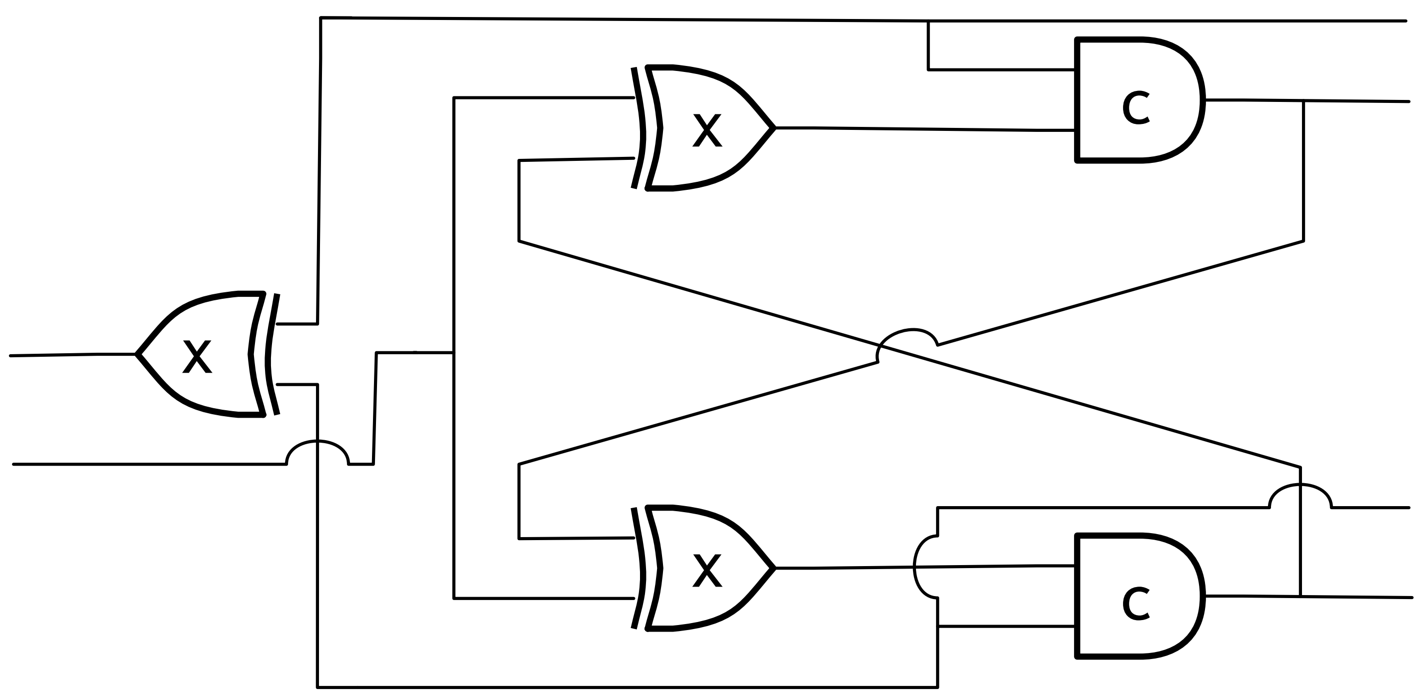
1. *dual objects* A, K^{*} A^{*}, K^{*}
2. *unit* $\bar{\eta}_A$ $I \to A^{*} \otimes A$ *defined as* $\bar{\eta}_A$ η_A $K^{*} \otimes K$ *;*
3. *co-unit* $^{-}{}_A$ $A^{*} \otimes A \to I$ *defined as* $^{-}{}_A$ $K^{*} \otimes K$ $_A$*.*

Theorem 4.26 *The algebraic structure of* A□Asy *and* IdAsy *is preserved in* DIAsy*:*

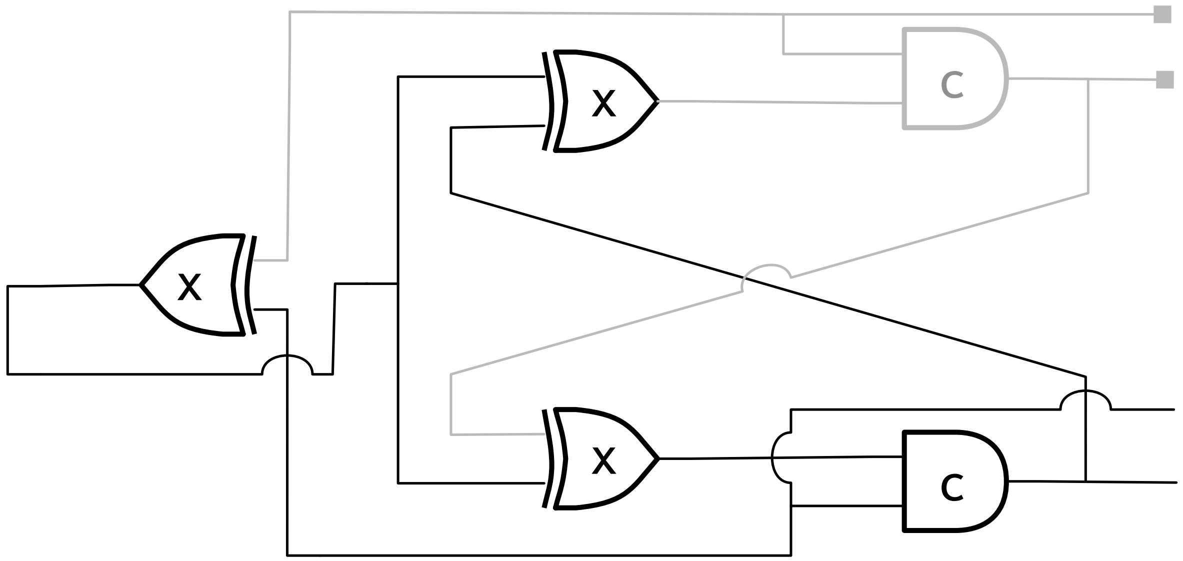
applications



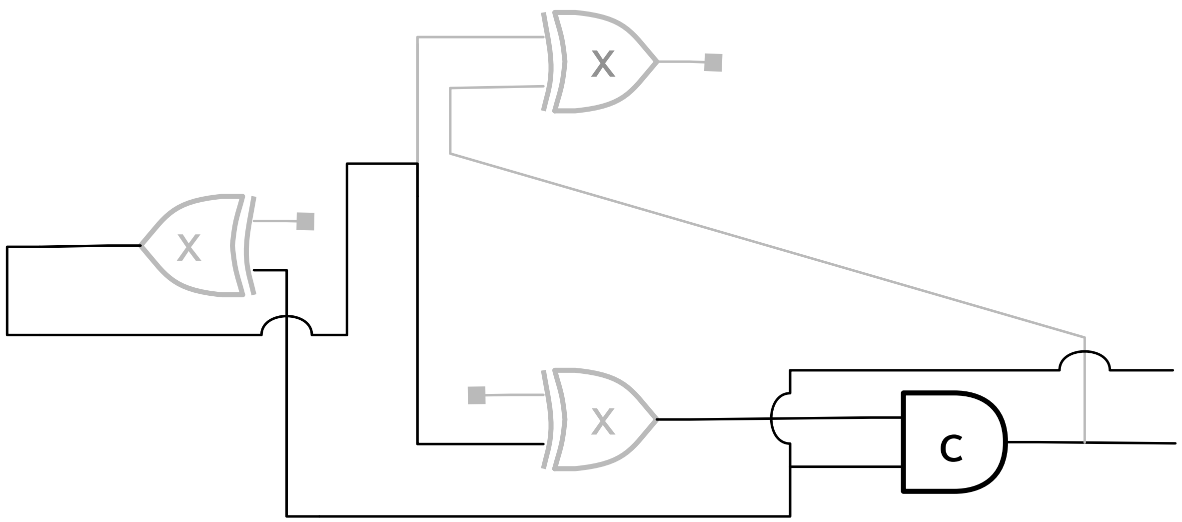
where
CALL is:



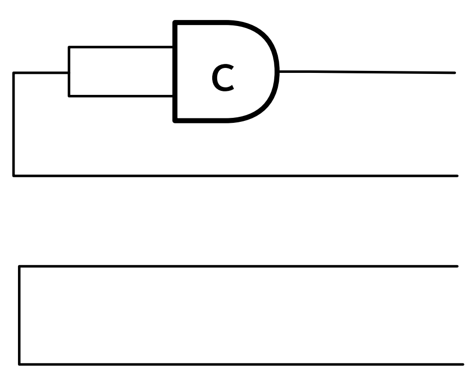
trace-level reasoning?



unit / counit
of
monoids / comonoids



compact-closed
structure



c section of f

conclusion

- very preliminary
- normal forms?
  - completeness
- model of feed-back?
  - causality