On the Reality of Observable Properties

Shane Mansfield

Department of Computer Science, University of Oxford

SamsonFest

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Background: A Criterion for ‘Reality’ of the Wavefunction

Harrigan & Spekkens (2010):
Propose a mathematical distinction between *ontic* and *epistmic* interpretations of the wavefunction

Pusey, Barrett & Rudolph (2012):
Prove no-go result based on this

Ontic

Corresponds directly to reality

Epistemic

Corresponds to our state of knowledge about reality

Overview

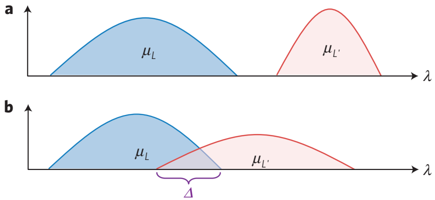
- Alternative definition for *ontic/epistemic*
    - Agrees with Harrigan & Spekkens

  But:
    - More general
    - Avoids measure-theoretic issues
    - Simple

- Application: *observable properties*
    - Novel characterisation of *non-locality/contextuality*
    - A weak Bell theorem

Harrigan-Spekkens Definition for the Wavefunction

- Assume a space Λ of *ontic states*
- Each $|\psi\rangle$ induces a probability distribution $\mu_{|\psi\rangle}$ over Λ
- *Ontic* if $\forall\, |\psi\rangle \neq |\phi\rangle$.
  $\mu_{|\psi\rangle}, \mu_{|\phi\rangle}$ have non-overlapping supports
- Otherwise *epistemic*



Alternative (General) Definition

Roughly

- Ontic properties are generated by functions $\hat{f} : \Lambda \to \mathcal{V}$
- Epistemic properties are inherently probabilistic

Carefully

A $\mathcal{V}$-*valued property* over Λ is a function $f : \Lambda \to \mathcal{D}(\mathcal{V})$, where $\mathcal{D}(\mathcal{V})$ is the set of probability distributions over $\mathcal{V}$.

- The property is *ontic* if $f(\lambda)$ is a delta function for all $\lambda \in \Lambda$.
- Otherwise it is *epistemic*.

Relating Definitions

A property f gives probability distributions over $\mathcal{V}$ conditioned on Λ. We can simply use Bayes' theorem

$$p(\lambda|v) = \frac{p(v|\lambda) \cdot p(\lambda)}{p(v)}$$

to obtain probability distributions over Λ conditioned on $\mathcal{V}$. Explicitly,

$$\mu_v(\lambda) := \frac{(f(\lambda))\,(v) \cdot p(\lambda)}{\int_\Lambda (f(\lambda'))\,(v) \cdot p(\lambda)\, d\lambda'}.$$

For finite Λ, we set $p(\lambda)$ to be uniform on Λ.

Proposition

A $\mathcal{V}$-valued property over finite Λ is ontic (present definition) *iff* the distributions $\{\mu_v\}_{v\in\mathcal{V}}$ have non-overlapping supports (Harrigan-Spekkens definition)

Ontological Models

We assume spaces:

Λ	ontic states
P	preparations
M	measurements
O	outcomes
$\mathcal{M} \subseteq \mathcal{P}(M)$	contexts

Ontological Models

An *ontological model* h over Λ specifies:

1. A distribution $h(\lambda|p)$ over Λ for each preparation $p \in P$;
2. For each $\lambda \in \Lambda$ and set of compatible measurements $\overline{m} \in \mathcal{M}$, a distribution $h(\overline{o}|\overline{m}, \lambda)$ over functional assignments $\overline{o} : \overline{m} \to O$ of outcomes to these measurements.

The *operational probabilities* are then prescribed by

$$h(\overline{o}|\overline{m}, p) = \int_\Lambda d\lambda \; h(\overline{o}|\overline{m}, \lambda) \; h(\lambda|p).$$

Ontological Models

λ-independence (*free will*)

$h(\lambda|p)$, not $h(\lambda|\overline{m},p)$

Determinism

$\forall \overline{m} \in \mathcal{M}, \lambda \in \Lambda.\ \exists \overline{o} \in \mathcal{E}(\overline{m})$ such that $h(\overline{o}|\overline{m},\lambda) = 1$

Parameter Independence

$\forall o \in O, \overline{m} \in \mathcal{M}, \lambda \in \Lambda$ the marginal probabilities $h(o|m,\lambda)$ are well-defined

Local Realism

Conjunction of the above

Characterising Locality

The *observable properties* of an ontological model h over Λ are the O-valued properties $f_m : \Lambda \to \mathcal{D}(O)$ given by

$$(f_m(\lambda))\,(o) := h(o|m, \lambda)$$

for each $m \in X$ such that the marginal $h(o|m, \lambda)$ is well-defined

Theorem

A model is local/non-contextual *iff*
all measurements are of ontic observable properties

We can use this as a route to a number of results:

- Canonical form for local models
- EPR argument
- Weak Bell theorem

Canonical Form for Local Models

Theorem

Local realistic ontological models can be expressed in a *canonical form*, with an ontic state space $\Omega := \mathcal{E}(X)$, and probabilities

$$h(\overline{o}|\overline{m}, \omega) = \prod_{m \in \overline{m}} \delta\left(\omega(m), \overline{o}(m)\right)$$

for all $\overline{m} \in \mathcal{M}$, $\overline{o} \in \mathcal{E}(\overline{m})$, and $\omega \in \Omega$

Use canonical transformation

$\{f_m : \Lambda \to \mathcal{O}\}_{m \in X} \longrightarrow \{\omega_\lambda : X \to O\}_{\lambda \in \Lambda}$

EPR: ψ-complete Quantum Mechanics

- The quantum wavefunction itself is taken to be the ontic state
- A preparation produces a density matrix (a distribution on the projective Hilbert space)
- By construction, operational probabilities agree with Born Rule

EPR

Proposition

Any non-trivial quantum mechanical observable is epistemic with respect to ψ-complete quantum mechanics

PROOF (OUTLINE): Take some $\hat{A} \neq \mathbb{1}$ and any $|\psi\rangle$ that's not an eigenvector. Then $\left(f_{\hat{A}}(\lambda)\right)(o_1) = h(o_1|\hat{A}, \lambda) = |\langle v_1|\psi\rangle|^2 > 0$, and similarly $\left(f_{\hat{A}}(\lambda)\right)(o_2) > 0$

Corollary (EPR)

Assuming locality/non-contextuality, quantum mechanics cannot be ψ-complete

A Weak Bell Theorem

Theorem

There exist quantum correlations that cannot be realised by any local/non-contextual ontological model for which the wavefunction is ontic

Proof (outline): there exists a function $\Psi : \Lambda \to \mathcal{H}$, specifying the wavefunction associated with each ontic state. For any $\lambda \in \Psi^{-1}(|\psi\rangle)$,

$$\left(f_{\hat{A}}(\lambda)\right)(o_1) = h(o_1|\hat{A}, \lambda) = |\langle v_1|\psi\rangle|^2 > 0,$$

and similarly $\left(f_{\hat{A}}(\lambda)\right)(o_2) > 0$

Theorem

Quantum mechanics is not realisable by any preparation independent, local/non-contextual ontological theory

Summary

- Alternative definition
    - More general
    - Avoids measure-theoretic issues
    - Simple
- A first application: *observable properties*
    - Novel characterisation of *non-locality/contextuality*
    - Makes contact with sheaf-theoretic approach
- Weak Bell theorem
    - A non-locality/contextuality test?
    - Question strength of *preparation independence*?

Rui at Rue Samson

(Post-release)

Photo credit: Nadish