On the Reality of Observable Properties

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Background:
A Criterion for ‘Reality’ of the Wavefunction

Harrigan & Spekkens (2010):
Propose a mathematical distinction between *ontic* and *epistemic* interpretations of the wavefunction

Pusey, Barrett & Rudolph (2012):
Prove no-go result based on this

<table>
<thead>
<tr>
<th><strong>Ontic</strong></th>
<th>Corresponds directly to reality</th>
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<tr>
<td><strong>Epistemic</strong></td>
<td>Corresponds to our state of knowledge about reality</td>
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Overview

- Alternative definition for *ontic/epistemic*
  - Agrees with Harrigan & Spekkens

But:
  - More general
  - Avoids measure-theoretic issues
  - Simple

- Application: *observable properties*
  - Novel characterisation of *non-locality/contextuality*
  - A weak Bell theorem
Assume a space $\Lambda$ of *ontic states*

- Each $|\psi\rangle$ induces a probability distribution $\mu_{|\psi\rangle}$ over $\Lambda$
- *Ontic* if $\forall |\psi\rangle \neq |\phi\rangle$.
  $\mu_{|\psi\rangle}, \mu_{|\phi\rangle}$ have non-overlapping supports
- Otherwise *epistemic*
Roughly

- Ontic properties are generated by functions $\hat{f} : \Lambda \rightarrow \mathcal{V}$
- Epistemic properties are inherently probabilistic

Carefully

A $\mathcal{V}$-valued property over $\Lambda$ is a function $f : \Lambda \rightarrow \mathcal{D}(\mathcal{V})$, where $\mathcal{D}(\mathcal{V})$ is the set of probability distributions over $\mathcal{V}$.

- The property is *ontic* if $f(\lambda)$ is a delta function for all $\lambda \in \Lambda$.
- Otherwise it is *epistemic*.
A property $f$ gives probability distributions over $\mathcal{V}$ conditioned on $\Lambda$. We can simply use Bayes’ theorem

$$p(\lambda|v) = \frac{p(v|\lambda) \cdot p(\lambda)}{p(v)}$$

to obtain probability distributions over $\Lambda$ conditioned on $\mathcal{V}$. Explicitly,

$$\mu_v(\lambda) := \frac{(f(\lambda))(v) \cdot p(\lambda)}{\int_{\Lambda} (f(\lambda'))(v) \cdot p(\lambda) \, d\lambda'}.$$ 

For finite $\Lambda$, we set $p(\lambda)$ to be uniform on $\Lambda$.

**Proposition**

A $\mathcal{V}$-valued property over finite $\Lambda$ is ontic (present definition) iff the distributions \{\mu_v\}_{v \in \mathcal{V}} have non-overlapping supports (Harrigan-Spekkens definition).
Ontological Models

We assume spaces:

\[ \Lambda \quad \text{ontic states} \]
\[ P \quad \text{preparations} \]
\[ M \quad \text{measurements} \]
\[ O \quad \text{outcomes} \]
\[ \mathcal{M} \subseteq \mathcal{P}(M) \quad \text{contexts} \]
An *ontological model* $h$ over $\Lambda$ specifies:

1. A distribution $h(\lambda|p)$ over $\Lambda$ for each preparation $p \in P$;
2. For each $\lambda \in \Lambda$ and set of compatible measurements $\overline{m} \in M$, a distribution $h(\overline{o}|\overline{m}, \lambda)$ over functional assignments $\overline{o} : \overline{m} \rightarrow O$ of outcomes to these measurements.

The *operational probabilities* are then prescribed by

$$h(\overline{o}|\overline{m}, p) = \int_\Lambda d\lambda \ h(\overline{o}|\overline{m}, \lambda) \ h(\lambda|p).$$
Ontological Models

\( \lambda \)-independence (free will)

\( h(\lambda | p) \), not \( h(\lambda | \overline{m}, p) \)

Determinism

\( \forall \overline{m} \in \mathcal{M}, \lambda \in \Lambda. \exists \overline{o} \in \mathcal{E}(\overline{m}) \) such that \( h(\overline{o} | \overline{m}, \lambda) = 1 \)

Parameter Independence

\( \forall o \in O, \overline{m} \in \mathcal{M}, \lambda \in \Lambda \) the marginal probabilities \( h(o | m, \lambda) \) are well-defined

Local Realism

Conjunction of the above
The observable properties of an ontological model $h$ over $\Lambda$ are the $O$-valued properties $f_m : \Lambda \to D(O)$ given by

$$(f_m(\lambda))(o) := h(o|m, \lambda)$$

for each $m \in X$ such that the marginal $h(o|m, \lambda)$ is well-defined.

**Theorem**

A model is local/non-contextual iff all measurements are of ontic observable properties.

We can use this as a route to a number of results:

- Canonical form for local models
- EPR argument
- Weak Bell theorem
Canonical Form for Local Models

**Theorem**

Local realistic ontological models can be expressed in a *canonical form*, with an ontic state space $\Omega := \mathcal{E}(X)$, and probabilities

$$h(\overline{o}|\overline{m}, \omega) = \prod_{m \in \overline{m}} \delta(\omega(m), \overline{o}(m))$$

for all $\overline{m} \in \mathcal{M}$, $\overline{o} \in \mathcal{E}(\overline{m})$, and $\omega \in \Omega$

Use canonical transformation

$$\{f_m : \Lambda \to \mathcal{O}\}_{m \in X} \longrightarrow \{\omega_\lambda : X \to O\}_{\lambda \in \Lambda}$$
The quantum wavefunction itself is taken to be the ontic state.

A preparation produces a density matrix (a distribution on the projective Hilbert space).

By construction, operational probabilities agree with Born Rule.
EPR

Proposition

Any non-trivial quantum mechanical observable is epistemic with respect to $\psi$-complete quantum mechanics

Proof (outline): Take some $\hat{A} \neq 1$ and any $|\psi\rangle$ that’s not an eigenvector. Then $(f_{\hat{A}}(\lambda))(o_1) = h(o_1|\hat{A},\lambda) = |\langle v_1|\psi\rangle|^2 > 0$, and similarly $(f_{\hat{A}}(\lambda))(o_2) > 0$

Corollary (EPR)

Assuming locality/non-contextuality, quantum mechanics cannot be $\psi$-complete
A Weak Bell Theorem

Theorem
There exist quantum correlations that cannot be realised by any local/non-contextual ontological model for which the wavefunction is ontic

Proof (Outline): there exists a function $\Psi : \Lambda \rightarrow \mathcal{H}$, specifying the wavefunction associated with each ontic state. For any $\lambda \in \Psi^{-1} (|\psi\rangle)$,

$$(f_{\hat{A}}(\lambda))(o_1) = h(o_1|\hat{A}, \lambda) = |\langle v_1|\psi\rangle|^2 > 0,$$

and similarly $(f_{\hat{A}}(\lambda))(o_2) > 0$

Theorem
Quantum mechanics is not realisable by any preparation independent, local/non-contextual ontological theory
Summary

- Alternative definition
  - More general
  - Avoids measure-theoretic issues
  - Simple

- A first application: *observable properties*
  - Novel characterisation of *non-locality/contextuality*
  - Makes contact with sheaf-theoretic approach

- Weak Bell theorem
  - A non-locality/contextuality test?
  - Question strength of *preparation independence*?
Rui at Rue Samson  
(Post-release)  
Photo credit: Nadish