A mathematical approach to defining the semantics of modelling languages

Jane Hillston
LFCS, University of Edinburgh

19th November 2016

A modelling language approach to defining mathematical structures via semantics

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Outline

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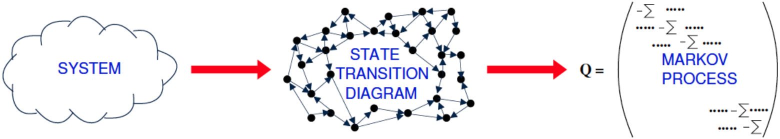
Mathematical models are needed to represent and analyse the dynamic behaviour to gain understanding and make predictions.

Quantitative Modelling using CTMC

Continuous Time Markov Chains are often the formalism of choice



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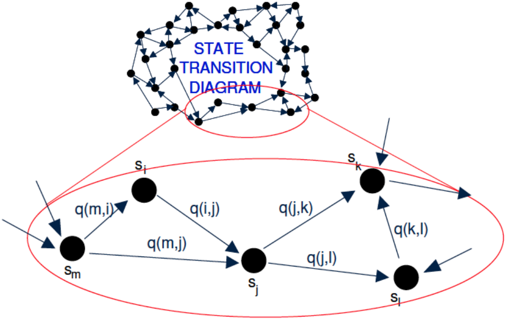

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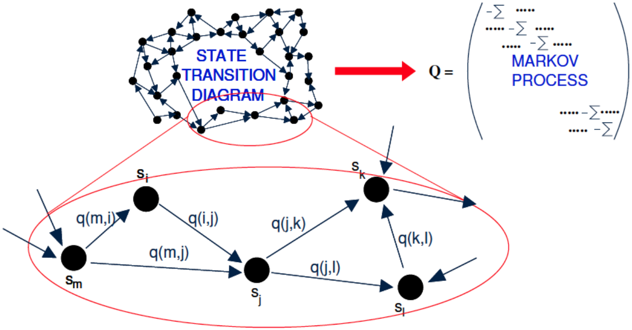

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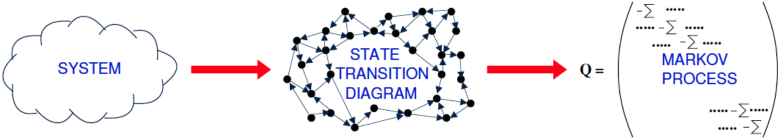
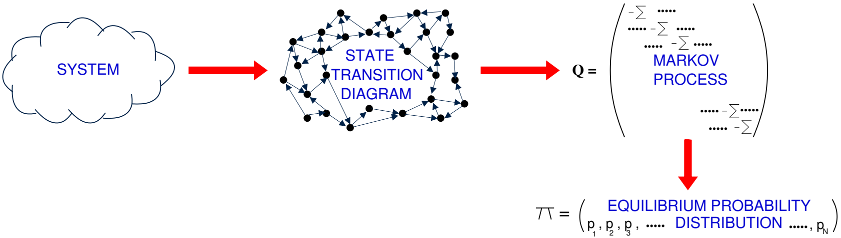


Deriving Performance Measures



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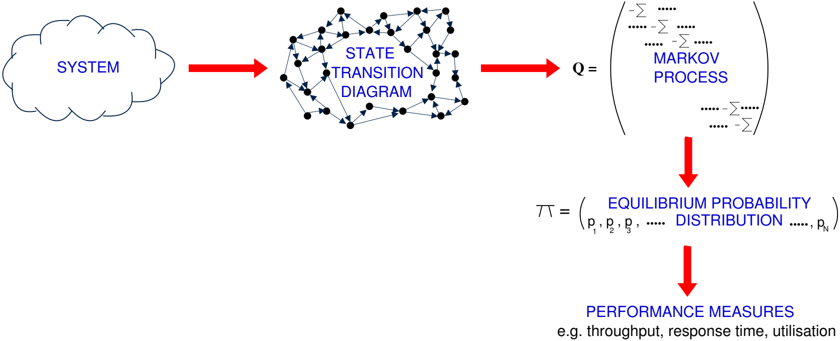
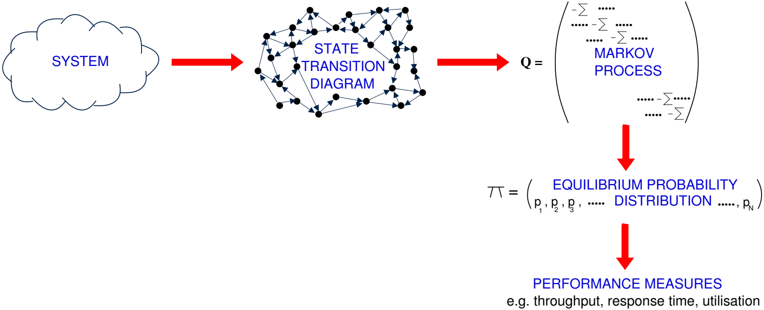


Linear algebra is used to derive a transient or steady state probability distribution — the probability that the system is in each particular state at a given time.

From the probability distribution the measures such a throughput, response time and utilisation can be straightforwardly derived

Difficulties of working with Markov processes



Whilst Markov process-based modelling has many advantages, working directly in terms of the state transition diagram or infinitesimal generator matrix is at best time-consuming and error prone, and often simply infeasible.

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- Performance Evaluation Process Algebra (PEPA) sought to address these problems by the introduction of a suitable process algebra.
- We have sought to investigate and exploit the interplay between the process algebra and the continuous time Markov chain (CTMC).

Outline

Process Algebra

- Models consist of agents which engage in actions.

$$\alpha.P$$

action type or name

agent/ component

- The structured operational (interleaving) semantics of the language is used to generate a labelled transition system.

$$\text{Process algebra model} \xrightarrow{\text{SOS rules}} \text{Labelled transition system}$$

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Process algebra model $\xrightarrow{\text{SOS rules}}$ Labelled transition system

- For quantitative modelling we need to incorporate quantitative information — stochastic process algebra (SPA).

SPA Languages

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integrated time

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orthogonal time

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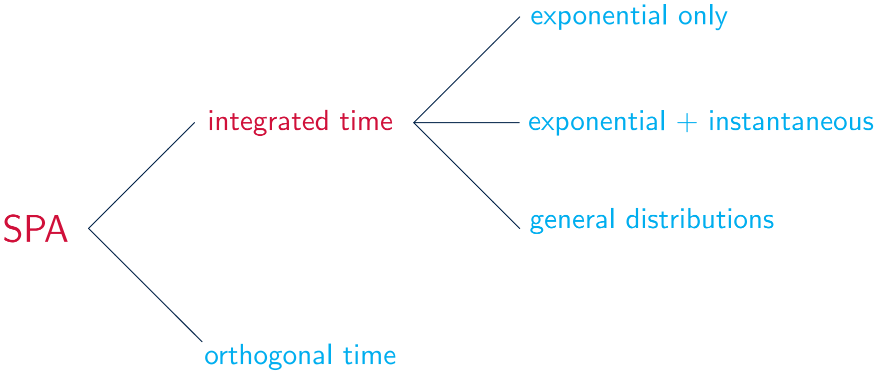

SPA Languages

- SPA
  - integrated time
    - exponential only
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    - general distributions
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$$\text{PEPA MODEL} \xrightarrow{\text{SOS rules}} \text{LABELLED TRANSITION SYSTEM} \xrightarrow[\text{diagram}]{\text{state transition}} \text{CTMC } \mathbf{Q}$$

PEPA

$$
\begin{aligned}
S &::= (\alpha, r).S \mid S + S \mid A \\
P &::= S \mid P \underset{L}{\bowtie} P \mid P/L
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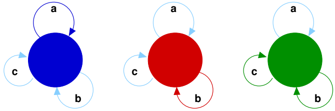
HIDING:	P/L	abstraction $\alpha \in L \Rightarrow \alpha \rightarrow \tau$

Solving discrete state models



Under the SOS semantics a SPA model is mapped to a CTMC with global states determined by the local states of all the participating components.

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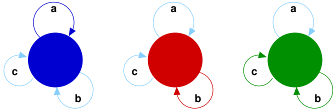

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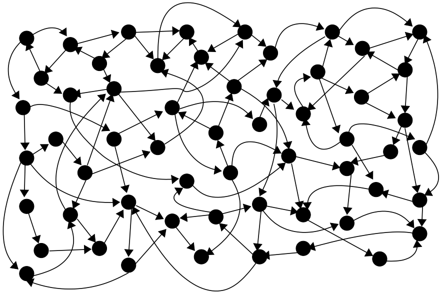

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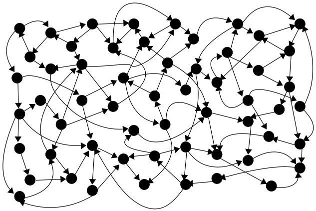

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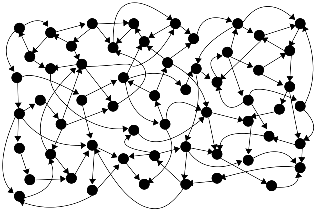

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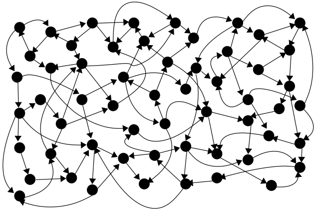

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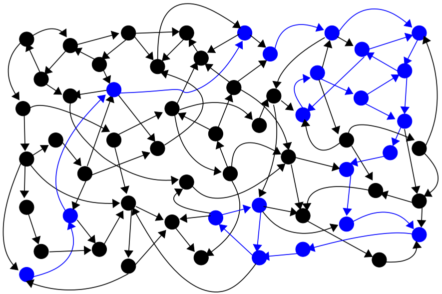
$$\pi(t) = (\pi_1(t), \pi_2(t), \ldots, \pi_N(t))$$

$$\pi(\infty)Q = 0$$

Solving discrete state models

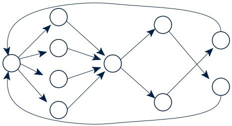
Alternatively they may be studied using stochastic simulation. Each run generates a single trajectory through the state space. Many runs are needed in order to obtain average behaviours.



Benefits of using a language

- There are clear benefits for model construction in using a modelling language and its semantics to build the required CTMC.
- But the language also allows you to characterise properties of the CTMC, previously described as properties of the infinitesimal generator matrix, as easily checked syntactic conditions.
- This supports automatic model reductions and model manipulations to improve the efficiency of solution.

Aggregation and lumpability



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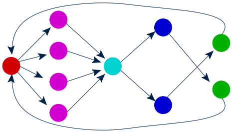

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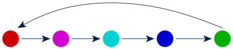

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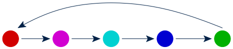

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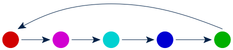

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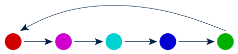
Aggregation and lumpability



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- This is not as straightforward as it may seem if we wish the aggregated process to still be a Markov process — an arbitrary partition will not in general preserve the Markov property.
- In order to preserve the Markov property we must ensure that the partition satisfies a condition called lumpability.
- Use a behavioural equivalence in the process algebra to form the partitions; moreover this is a congruence allowing the reduction to be carried out compositionally.

State space explosion

Unfortunately, as the size of the state space becomes large it becomes infeasible to carry out numerical solution and extremely time-consuming to conduct stochastic simulation.

Even with sophisticated model reduction and aggregation techniques discrete approaches are defeated by the scale of many dynamic systems.

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These are models which consist of populations.

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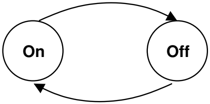
Furthermore we make a continuous approximation of how the proportions vary over time.

Population models — intuition



$Y(t)$

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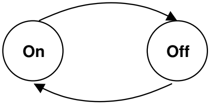

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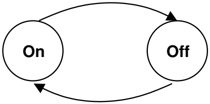

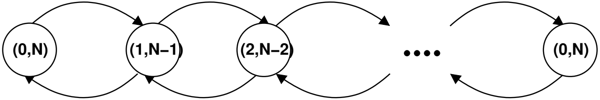

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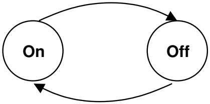

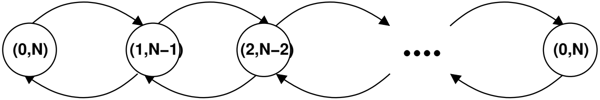

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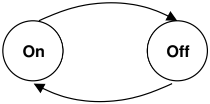

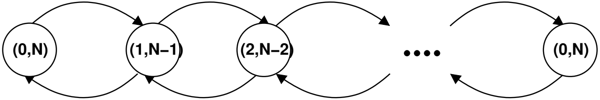

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$\mathbf{X}^{(N)}(t)$

$$X_j^{(N)} = \sum_{i=1}^{N} \mathbf{1}\{Y_i^{(N)} = j\}$$

- $Y(t)$, $Y_i^{(N)}(t)$ and $\mathbf{X}^{(N)}(t)$ are all CTMCs;
- As N increases we get a sequence of CTMCs, $\mathbf{X}^{(N)}(t)$

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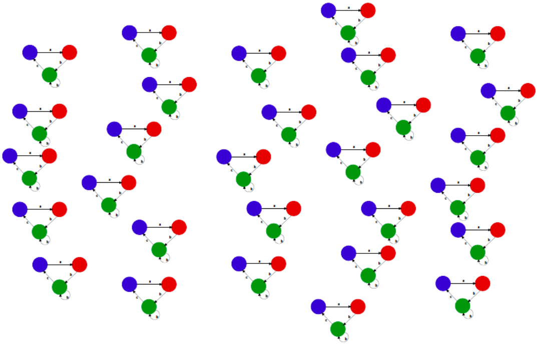

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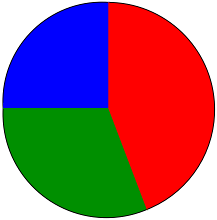

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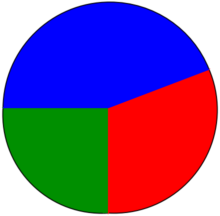

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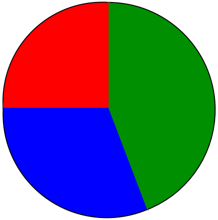


In the normalised CTMC $\hat{\mathbf{X}}^{(N)}(t)$ we are concerned with only the proportion of agents that exhibit the different possible states.

Kurtz's Deterministic Approximation Theorem

Kurtz established in the 1970s that for suitable sequences of CTMCs, in the limit, the behaviour becomes indistinguishable from a continuous evolution of the occupancy measures, governed by an appropriate set of ordinary differential equations.

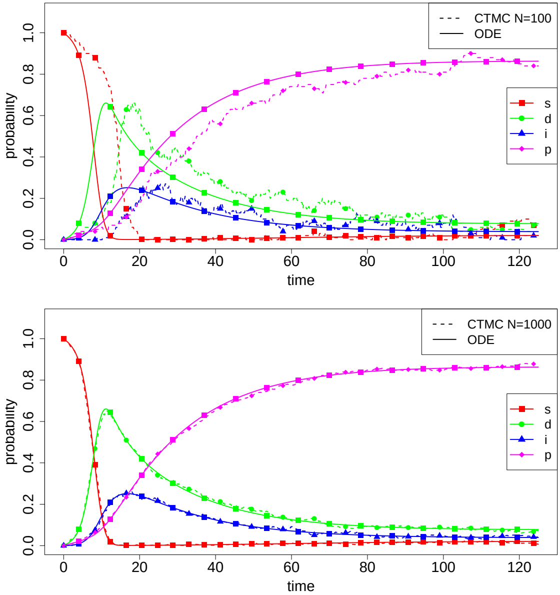
Deterministic Approximation Theorem (Kurtz)

Assume that $\exists\, \mathbf{x_0} \in S$ such that $\hat{\mathbf{X}}^{(N)}(0) \to \mathbf{x_0}$ in probability. Then, for any finite time horizon $T < \infty$, it holds that as $N \longrightarrow \infty$:

$$\mathbb{P}\left\{ \sup_{0 \leq t \leq T} ||\hat{\mathbf{X}}^{(N)}(t) - \mathbf{x}(t)|| > \varepsilon \right\} \to 0.$$

T.G.Kurtz. *Solutions of ordinary differential equations as limits of pure jump Markov processes.* Journal of Applied Probability, 1970.

Illustrative trajectories



Comparison of the limit fluid ODE and a single stochastic trajectory of a network epidemic example, for total populations $N = 100$ and $N = 1000$.

Fluid semantics for Stochastic Process Algebras

- To apply these results in a stochastic process algebra we need to derive the right set of ODEs, from the model expression.

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- Embedding the approach in a formal language offers the possibility to establish the conditions for convergence at the language level via the semantics,
- This removes the requirement to fulfil the proof obligation on a model-by-model basis.
- Moreover the derivation of the ODEs can be automated in the implementation of the language.

Deriving a Fluid Approximation of a SPA model

The aim is to represent the CTMC implicitly (avoiding state space explosion), and to generate the set of ODEs which are the fluid limit of that CTMC.

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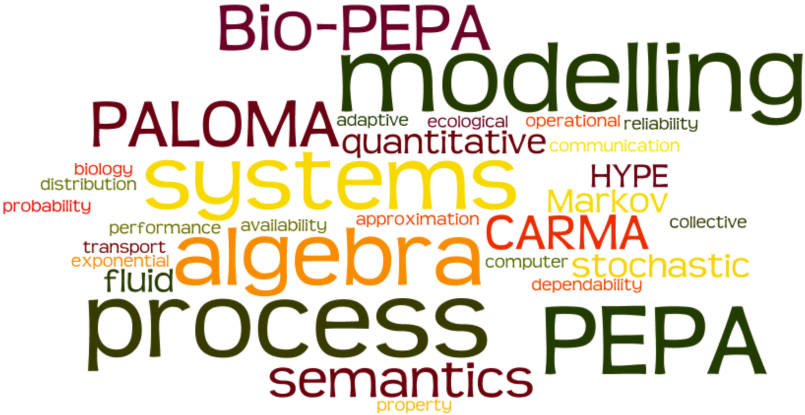
SPA MODEL $\xrightarrow{\text{SOS rules}}$ SYMBOLIC LABELLED TRANSITION SYSTEM $\xrightarrow[\text{functions}]{\text{generator}}$ ABSTRACT CTMC $\mathbf{Q}$ or ODEs $F_{\mathcal{M}}(x)$

M.Tribastone, S.Gilmore and J.Hillston. Scalable Differential Analysis of Process Algebra Models. IEEE TSE 2012.


Bio-PEPA
modelling
PALOMA
adaptive
ecological
operational
reliability
quantitative
communication
biology
distribution
systems
HYPE
probability
Markov
performance
availability
approximation
CARMA
collective
transport
exponential
algebra
computer
stochastic
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process
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semantics
property

Outline

Developing a probabilistic programming approach

SPA represent systems in which there is variability in behaviour but still with the assumption that all parameters (rates) in the model are known.

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- do all this in a formal context?

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- do all this in a formal context?

Starting from an existing process algebra (Bio-PEPA), we have developed a new language ProPPA that addresses these issues

A.Georgoulas, J.Hillston, D.Milios, G.Sanguinetti: *Probabilistic Programming Process Algebra*. QEST 2014.

Probabilistic programming

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- Describe how the data is generated in syntax like a conventional programming language, but leaving some variables uncertain.
- Specify observations, which impose constraints on acceptable outputs of the program.
- Run program forwards: Generate data consistent with observations.
- Run program backwards: Find values for the uncertain variables which make the output match the observations.

ProPPA: Probabilistic Programming Process Algebra

The objective of ProPPA is to retain the features of the stochastic process algebra:

- simple model description in terms of components
- rigorous semantics giving an executable version of the model...

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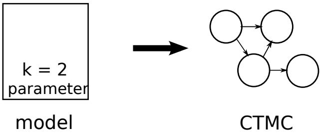
The objective of ProPPA is to retain the features of the stochastic process algebra:

- simple model description in terms of components
- rigorous semantics giving an executable version of the model...

... whilst also incorporating features of a probabilistic programming language:

- recording uncertainty in the parameters
- ability to incorporate observations into models
- access to inference to update uncertainty based on observations

Semantics



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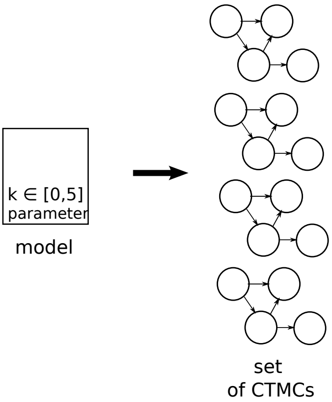

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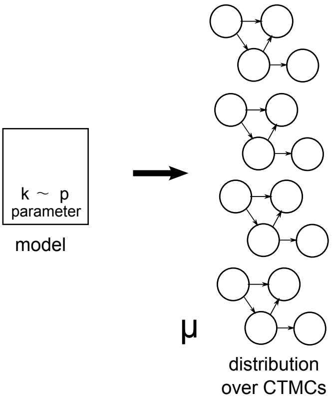

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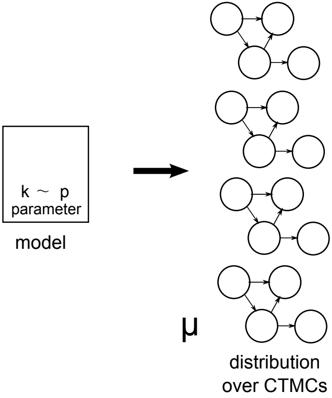

ProPPA models are given semantics in terms of Probabilistic Constraint Markov Chains, and a variety of inference algorithms are available to refine the prior distribution into the posterior.

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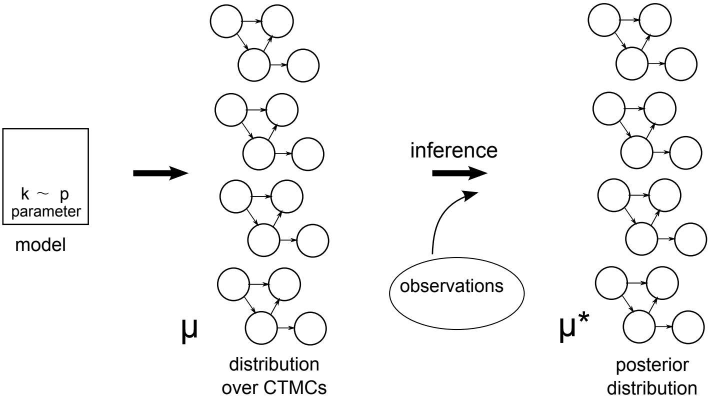

ProPPA models are given semantics in terms of Probabilistic Constraint Markov Chains, and a variety of inference algorithms are available to refine the prior distribution into the posterior.

The future?

The area for quantitative analysis and verification is a good example of Strachey's ideal of theory and practice intertwined.

New applications pose new challenges for both representation and analysis and we seek to design languages to support them.

Current challenges include

- Spatially constrained behaviour
- Heterogeneous populations of agents
- Collective adaptive systems where global behaviour is defined by but also influences the behaviour of individual agents.

Thank you!