Programming Research Group

A REPRESENTATION FOR CLINICAL PROCEDURES

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Abstract

In general, the management of a patient is a co-operative activity involving doctors, nurses, and many support staff such as paramedics and physiotherapists. There is currently much interest in modeling this process of care in order to manage resources better, to audit the quality of care actually provided, and to establish clinical guidelines. This paper presents a formal notation for clinical procedures; a procedure is represented by a goal-oriented hierarchy of simple flowcharts. General correctness properties of the procedure model can be specified in mathematical logic, and use of a computer enables the model to be validated against its specification. By means of a concealment operator, a model can be simplified to show the rôle of just one individual in the entire procedure. This enables, for example, separate clinical guidelines for doctors and nurses to be extracted from the overall model.

Introduction

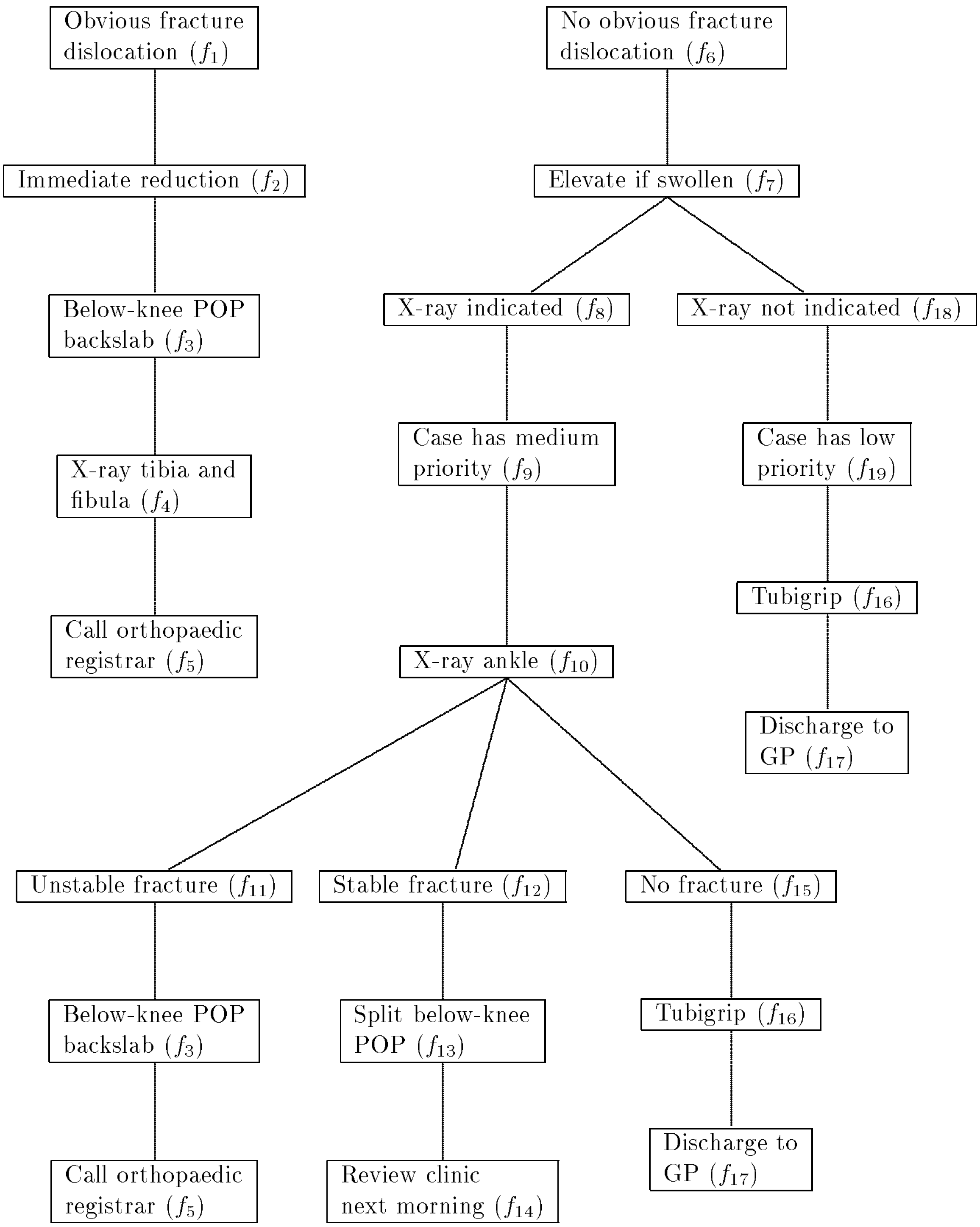
There is currently a great deal of interest in developing clinical guidelines for the medical practitioner; their introduction seems generally to have a beneficial effect on clinical practice [Gri93]. Typically, algorithms for clinical management are presented as flowcharts. Large numbers of these have been published, covering topics as diverse as the management of back pain [Wie88], the dressing of wounds [Tur91], and the diagnosis of neonatal heart disease [Fra91]. With so many flowcharts available, the practitioner has a potentially large reference library from which to seek help in the management of any particular case. However, flowcharts by different authors are likely to use different styles and terminology, which may be confusing. Partly to counteract this, standards have been proposed for flowchart notation [Soc92, Col93]. Nevertheless, although these help of course, it is still difficult to ensure that terminology is used consistently, and it is also difficult to update and maintain a large collection of flowcharts.

Several groups have investigated the feasibility of using computers to store, edit and display clinical protocols [Can80, Abe89, Des91]. Use of a computer makes it possible at the same time to check the correctness of the protocols to some extent. For this we need a mathematical model in order to define the meaning of 'correctness'. This paper presents such a formalism: it is a refinement and extension of an earlier one [Tod94, Tod96], drawing on the lessons of previous case studies [Led94].

Clinical Procedures

Consider the flowchart shown in Figure 1 for managing a patient who presents to an accident and emergency department with an acute ankle injury. If the patient has an obvious fracture-dislocation, then this requires immediate reduction and application of a below-knee Plaster of Paris backslab.

Figure 1: Flowchart protocol for the management of a patient attending an accident and emergency department with an acute ankle injury.



Only after this is done is an X-ray taken. The X-ray should include the full length of the tibia and fibula in case the distal fracture is associated with another more proximal one. The patient is then referred to the orthopaedic team for further management. Alternatively, if there is no obvious fracture-dislocation then the priority with which the patient needs to be seen by the doctor depends on whether an X-ray is required. Either way, a swollen ankle should be elevated while the patient is waiting to be seen. If an X-ray shows an unstable fracture, then a below-knee backslab is applied and the patient is referred to the orthopaedic team. However, if the fracture is stable then a split plaster is applied and the patient is referred for review on the following morning. If the X-ray shows no fracture, or if no X-ray is indicated, a tubigrip support is applied and the patient is discharged home under the care of the general practitioner. Please note that this flowchart serves for illustrative purposes only: although the described procedure is quite standard, and is based loosely on the more detailed one given in [Pac91], gross simplifications have been made for clarity. For example, the possibility of ligamentous injury is overlooked, analgesia is not considered, and the possible rôle of physiotherapy is omitted.

A flowchart offers various alternative management paths. Each involves establishing a set of facts, some relating to features of the case (e.g. f_1 — 'there is an obvious fracture-dislocation'), others relating to actions that are performed (e.g. f_{16} — 'a tubigrip support is provided'). For the purpose of our mathematical model we will be concerned only with the occurrence of facts along a given pathway, not in their sequence. We therefore identify a pathway with a set of facts. Similarly, we regard a procedure as a set of pathways, in other words a collection of sets of facts. Let 'Proc' be the set of all procedures. (We follow here the notational conventions of the 'Z' specification language [Spi88].)

$$\mathrm{Proc} \;\widehat{=}\; \mathbb{P}\,\mathbb{P}\,\mathrm{Fact} \tag{1}$$

The most trivial procedure of all is the empty set. This offers no pathway at all for managing the patient successfully. We refer to this as 'STOP'. Clearly we would never wish to implement such a useless procedure.

$$\mathsf{STOP} \;\widehat{=}\; \{\} \tag{2}$$

The largest procedure of all is the set of all paths. This is the most liberal of all: no guidelines at all are provided for managing the patient. All management pathways are equally valid. We refer to this procedure as 'FREE'.

$$\mathsf{FREE} \;\widehat{=}\; \mathbb{P}\,\mathrm{Fact} \tag{3}$$

A slightly more useful procedure is that containing just the empty pathway. This denotes the vacuous procedure which is to do precisely nothing. We refer to this procedure as 'SKIP'.

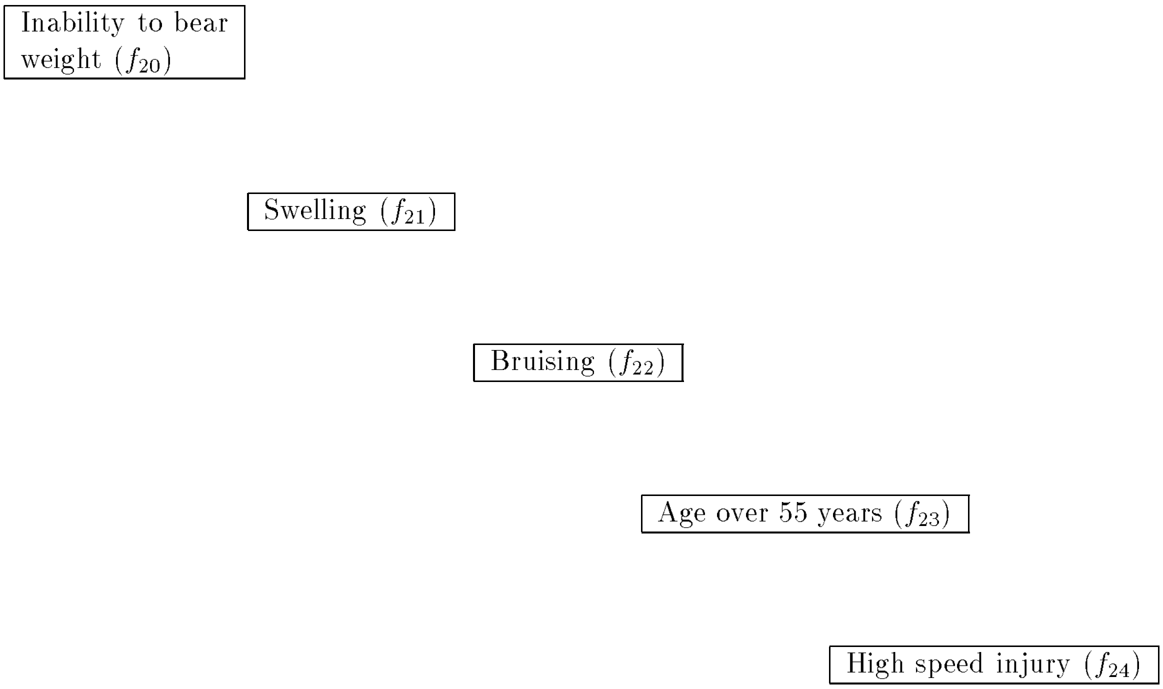
$$\mathsf{SKIP} \;\widehat{=}\; \{\{\}\} \tag{4}$$

In general, though, procedures are much more complicated than this. For example, the procedure P_{ANKLE} described by Figure 1 is

$$\begin{aligned} P_{\mathrm{ANKLE}} \;\widehat{=}\; \{&\{f_1, f_2, f_3, f_4, f_5\},\\ &\{f_6, f_7, f_8, f_9, f_{10}, f_{11}, f_3, f_5\},\\ &\{f_6, f_7, f_8, f_9, f_{10}, f_{12}, f_{13}, f_{14}\},\\ &\{f_6, f_7, f_8, f_9, f_{10}, f_{15}, f_{16}, f_{17}\},\\ &\{f_6, f_7, f_{18}, f_{19}, f_{16}, f_{17}\}\} \end{aligned} \tag{5}$$

Some of these facts are so abstract that it may not always be clear how to establish them. Each such fact (e.g. f_{18} — 'an X-ray is indicated') therefore requires its own explanatory flowchart. For

Figure 2: Flowchart protocol for establishing that an X-ray is indicated for an acute ankle injury.



example, Figure 2 shows a flowchart that details the indications for X-ray. Notice that there are again five alternative paths, but trivially each involves only a single fact. Thus the procedure P_{XRAY} for establishing f_{18} is

$$P_{\mathrm{XRAY}} \;\widehat{=}\; \{\{f_{20}\}, \{f_{21}\}, \{f_{22}\}, \{f_{23}\}, \{f_{24}\}\} \tag{6}$$

Clearly each of these new facts can *itself* be associated with an explanatory flowchart if necessary, and so on recursively. This is easily formalized. We refer to such a collection of flowcharts as a *theory*. A theory relates a fact to an acceptable management path for establishing the fact. The only requirement is that the theory is non-circular: no fact can be explained directly or indirectly in terms of itself.

$$\mathrm{Theory} \;\widehat{=}\; \{T : \mathrm{Fact} \leftrightarrow \mathbb{P}\,\mathrm{Fact} \mid \lessdot T \gtrdot^{+} \cap \mathsf{id} = \{\}\} \tag{7}$$

where for any two facts f and g, the relationship $f \lessdot T \gtrdot g$ holds precisely when f is explained directly in terms of g. Thus for any theory T,

$$\lessdot T \gtrdot \;\widehat{=}\; T \mathbin{⨾} \{S : \mathbb{P}\,\mathrm{Fact}; f : \mathrm{Fact} \mid f \in S\} \tag{8}$$

The predicate $\lessdot T \gtrdot^{+} \cap \mathsf{id} = \{\}$ in (7) thus ensures that no fact is directly or indirectly in terms of itself.

For example, if no fact in P_{ANKLE} other than f_{18} is associated with its own flowchart, and if f_0 denotes the fact 'an acute ankle injury is managed' which P_{ANKLE} itself describes, then the management of acute ankle injuries is given by the theory

$$\begin{array}{llll}
T_{\mathrm{ANKLE}} \;\widehat{=}\; \{ & f_0 & \mapsto & \{f_1, f_2, f_3, f_4, f_5\}, \\
& f_0 & \mapsto & \{f_6, f_7, f_8, f_9, f_{10}, f_{11}, f_3, f_5\}, \\
& f_0 & \mapsto & \{f_6, f_7, f_8, f_9, f_{10}, f_{12}, f_{13}, f_{14}\}, \\
& f_0 & \mapsto & \{f_6, f_7, f_8, f_9, f_{10}, f_{15}, f_{16}, f_{17}\}, \\
& f_0 & \mapsto & \{f_6, f_7, f_{18}, f_{19}, f_{16}, f_{17}\}, \\
& f_1 & \mapsto & \{\}, \\
& f_2 & \mapsto & \{\}, \\
& \vdots & \vdots & \vdots \\
& f_{17} & \mapsto & \{\}, \\
& f_{18} & \mapsto & \{f_{20}\}, \\
& f_{18} & \mapsto & \{f_{21}\}, \\
& f_{18} & \mapsto & \{f_{22}\}, \\
& f_{18} & \mapsto & \{f_{23}\}, \\
& f_{18} & \mapsto & \{f_{24}\}, \\
& f_{19} & \mapsto & \{\}, \\
& \vdots & \vdots & \vdots \\
& f_{24} & \mapsto & \{\}\}
\end{array} \tag{9}$$

Notice that if a fact has no explanatory flowchart then the fact is instead associated simply with the empty pathway: there is no specific precondition for establishing the particular fact. The procedure for establishing each fact is given by the image of the fact though the theory. Thus,

$$
\begin{array}{ll}
T_{\mathrm{ANKLE}}(\!| f_0 |\!) & = P_{\mathrm{ANKLE}} \\
T_{\mathrm{ANKLE}}(\!| f_1 |\!) & = \mathsf{SKIP} \\
\vdots & \\
T_{\mathrm{ANKLE}}(\!| f_{17} |\!) & = \mathsf{SKIP} \\
T_{\mathrm{ANKLE}}(\!| f_{18} |\!) & = P_{\mathrm{XRAY}} \\
T_{\mathrm{ANKLE}}(\!| f_{19} |\!) & = \mathsf{SKIP} \\
\vdots & \\
T_{\mathrm{ANKLE}}(\!| f_{24} |\!) & = \mathsf{SKIP}
\end{array}
\tag{10}
$$

The entire, detailed procedure for establishing a particular fact is given by recursive descent through each of the procedures for the various subgoals. For any theory T, let $T^\star$ be the corresponding theory that relates facts to complete paths followed by recursive descent.

$$
T^\star \;\widehat{=}\; \{ f : \mathrm{Fact}; S : \mathbb{P}\,\mathrm{Fact} \mid \exists F : \mathbb{P}\,T \bullet \mathsf{dom}\,F = \{f\} \cup S \;\wedge\; \bigcup \mathsf{ran}\,F = S \} \tag{11}
$$

Notice that acyclicity is preserved during expansion of a theory in this way. In the case of T_{ANKLE} the only fact in the top-level pathways that is explained in any further detail is f_{18}. The expansion of T_{ANKLE} is thus

$$
\begin{array}{rlll}
T_{\mathrm{ANKLE}}{}^\star = \{ & f_0 & \mapsto & \{f_1, f_2, f_3, f_4, f_5\}, \\
& f_0 & \mapsto & \{f_6, f_7, f_8, f_9, f_{10}, f_{11}, f_3, f_5\}, \\
& f_0 & \mapsto & \{f_6, f_7, f_8, f_9, f_{10}, f_{12}, f_{13}, f_{14}\}, \\
& f_0 & \mapsto & \{f_6, f_7, f_8, f_9, f_{10}, f_{15}, f_{16}, f_{17}\}, \\
& f_0 & \mapsto & \{f_6, f_7, f_{20}, f_{18}, f_{19}, f_{16}, f_{17}\}, \\
& f_0 & \mapsto & \{f_6, f_7, f_{21}, f_{18}, f_{19}, f_{16}, f_{17}\}, \\
& f_0 & \mapsto & \{f_6, f_7, f_{22}, f_{18}, f_{19}, f_{16}, f_{17}\}, \\
& f_0 & \mapsto & \{f_6, f_7, f_{23}, f_{18}, f_{19}, f_{16}, f_{17}\}, \\
& f_0 & \mapsto & \{f_6, f_7, f_{24}, f_{18}, f_{19}, f_{16}, f_{17}\}, \\
& f_1 & \mapsto & \{\}, \\
& \vdots & \vdots & \vdots \\
& f_{17} & \mapsto & \{\}, \\
& f_{18} & \mapsto & \{f_{20}\}, \\
& f_{18} & \mapsto & \{f_{21}\}, \\
& f_{18} & \mapsto & \{f_{22}\}, \\
& f_{18} & \mapsto & \{f_{23}\}, \\
& f_{18} & \mapsto & \{f_{24}\}, \\
& f_{19} & \mapsto & \{\}, \\
& \vdots & \vdots & \vdots \\
& f_{24} & \mapsto & \{\}\}
\end{array}
\tag{12}
$$

The detailed procedure for managing acute ankle injuries is thus given by

$$\begin{aligned} T_{\mathrm{ANKLE}}{}^{\star}(\!| f_0 |\!) = \{ & \{f_1, f_2, f_3, f_4, f_5\}, \\ & \{f_6, f_7, f_8, f_9, f_{10}, f_{11}, f_3, f_5\}, \\ & \{f_6, f_7, f_8, f_9, f_{10}, f_{12}, f_{13}, f_{14}\}, \\ & \{f_6, f_7, f_8, f_9, f_{10}, f_{15}, f_{16}, f_{17}\}, \\ & \{f_6, f_7, f_{20}, f_{18}, f_{19}, f_{16}, f_{17}\}, \\ & \{f_6, f_7, f_{21}, f_{18}, f_{19}, f_{16}, f_{17}\}, \\ & \{f_6, f_7, f_{22}, f_{18}, f_{19}, f_{16}, f_{17}\}, \\ & \{f_6, f_7, f_{23}, f_{18}, f_{19}, f_{16}, f_{17}\}, \\ & \{f_6, f_7, f_{24}, f_{18}, f_{19}, f_{16}, f_{17}\}\} \end{aligned} \tag{13}$$

In practice, though, detailed procedures would be very much larger than this. For example, a protocol for the management of infertility based on these ideas had a total of 2310 paths recursing through a set of 32 flowcharts [Tod96]. Doubt then arises as to whether the theory contains inadvertent oversights and omissions. Confidence in the correctness of the theory can be improved by specifying necessary properties of the theory. These include, for example, observance of the indications and contraindications for particular investigations and treatments. A computer can be used to check whether the theory satisfies its specification, and if not, to identify inconsistencies.

Specifications

Properties required of each management pathway can be formulated as Boolean expressions over the appearance of particular facts in the pathway. For any expression E, let $[E]$ be the set of all paths with property E. A particularly simple requirement is that a given fact f necessarily appears on the pathway.

$$[f] \;\widehat{=}\; \{S : \mathbb{P}\,\mathrm{Fact} \mid f \in S\} \tag{14}$$

More complicated requirements are constructed using the usual Boolean connectives. Thus

$$[\neg E] \;\widehat{=}\; \overline{[E]} \tag{15}$$

where the overline symbol denotes complementation, and

$$[E \wedge F] \;\widehat{=}\; [E] \cap [F] \tag{16}$$

It then follows that

$$\begin{aligned} [\mathsf{true}] &= \mathsf{FREE} && (17) \\ [\mathsf{false}] &= \mathsf{STOP} && (18) \\ [E \vee F] &= [E] \cup [F] && (19) \\ [E \Rightarrow F] &= \overline{[E]} \cup [F] && (20) \end{aligned}$$

and so forth, for any E and F.

A procedure P satisfies a requirement E precisely when all paths through the procedure have the necessary property ($P \subseteq [E]$). For example, regarding T_{ANKLE} we might wish to assert that if the patient is unable to bear weight (f_{20}) then the patient should be seen by the doctor with moderately high priority (f_9). By checking each pathway in (13) we can quickly verify that

$$T_{\mathrm{ANKLE}}{}^{\star}(\!| f_0 |\!) \subseteq [f_{20} \Rightarrow f_9] \tag{21}$$

However, we may equally wish to assert that a particular action is *not* indicated. This is weaker than saying that the action is contraindicated. For example, we may wish to assert that in the management of an acute ankle injury, an X-ray is not mandatory. This requires that there is at least one pathway in the procedure that does not entail taking an X-ray. This is weaker than asserting (incorrectly of course) that an X-ray is contraindicated. In order to formulate these kinds of statement, we identify a specification with the set of all procedures meeting the specification.

$$\mathrm{Spec} \ \widehat{=} \ \mathbb{P}\,\mathrm{Proc} \tag{22}$$

Each Boolean expression E over facts thus represents the specification $\mathbb{P}\,[E]$. We refer to any specification of this form as an *atomic property*. Other kinds of specification are constructed from atomic properties by using set combinators.

Thus, let S_{XRAY} be the specification that X-ray is not mandatory in every patient who presents with an acute ankle injury. This is simply the complement of the specification that X-ray *is* mandatory. The latter asserts that X-ray either of the tibia and fibula (f_4) or of the ankle (f_{10}) is always performed.

$$S_{\mathrm{XRAY}} \ \widehat{=} \ \overline{\mathbb{P}\,[f_4 \vee f_{10}]} \tag{23}$$

By using other set combinators such as union and intersection, we can construct more complicated specifications. For example, we may wish to assert that tenderness is not an indication for X-ray (specification S_{TENDER}). Thus, if tenderness logically implies X-ray, then the implication must hold only trivially because (as in this case) tenderness appears on no pathway.

$$S_{\mathrm{TENDER}} \ \widehat{=} \ \overline{\mathbb{P}\,[f_{25} \Rightarrow f_4 \vee f_{10}]} \cup \mathbb{P}\,[\neg f_{25}] \tag{24}$$

where f_{25} represents the fact that the patient has ankle tenderness. It is important to be able to refer to a fact in the specification even if the fact does not yet appear in the procedure because the fact may eventually be included at a later date as the procedure is refined and modified as practices change.

Some simple and useful lemmas regarding specifications follow quickly from these definitions. The power set operator distributes through conjunction of expressions.

Lemma 1

$$E, F : \mathrm{Expr} \ \vdash \ \mathbb{P}\,[E \wedge F] = \mathbb{P}\,[E] \cap \mathbb{P}\,[F]$$

where 'Expr' is the set of all Boolean expressions over facts. It is easier to satisfy the disjunction of two requirements than it is to satisfy either one or the other of them.

Lemma 2

$$E, F : \mathrm{Expr} \ \vdash \ \mathbb{P}\,[E \vee F] \supseteq \mathbb{P}\,[E] \cup \mathbb{P}\,[F]$$

However, is it harder to guarantee that no pathway possesses a given property than it is to ensure than not all pathways have the property.

Lemma 3

$$E : \mathrm{Expr} \ \vdash \ \mathbb{P}\,[\neg E] \subseteq \overline{\mathbb{P}\,[E]}$$

Theories

As a shorthand, we will write $T \ \underline{sat}_f \ s$ to mean that specification s is met by the procedure given by theory T for establishing fact f.

$$T \ \underline{sat}_f \ s \ \hat{=} \ (T^\star (\! | f | \!)) \in s \tag{25}$$

For example, by considering each path in turn we can verify that

$$T_{\text{ANKLE}} \ \underline{sat}_{f_0} \ S_{\text{XRAY}}$$

and

$$T_{\text{ANKLE}} \ \underline{sat}_{f_0} \ S_{\text{TENDER}}$$

Concealment

In general, the management of a patient is a co-operative activity involving many different persons: doctors, nurses, paramedics, physiotherapists *etc.* For example, P_{ANKLE} concerns not only the casualty officer but also the triage nurse who assigns priorities to cases. More appropriate guidelines for a particular member of staff can be extracted from P_{ANKLE} by removing any facts that are irrelevant to that person's rôle in the overall procedure. For any procedure P and set of facts F, let $P \, ! \, F$ be the procedure obtained by removing F from every pathway in P.

$$P \, ! \, F \ \ \hat{=} \ \ \{S : P \bullet S - F\} \tag{26}$$

For example, the triage nurse is concerned only with the immediate treatment and initial assignment of priority to cases. We can therefore extract a procedure P_{TRIAGE} from P_{ANKLE} by removing all other facts.

$$\begin{aligned} P_{\text{TRIAGE}} &= P_{\text{ANKLE}} \ ! \ \{f_3, f_4, f_5, f_{10}, f_{11}, f_{12}, f_{13}, f_{14}, f_{15}, f_{16}, f_{17}\} \\ &= \{\{f_1, f_2\}, \{f_6, f_7, f_8, f_9\}, \{f_6, f_7, f_{18}, f_{19}\}\} \end{aligned} \tag{27}$$

The resulting procedure is shown in the form of a flowchart in Figure 3. In the same way, a flowchart for the casualty officer can be obtained by removing the facts relating to triaged priority (f_9 and f_{19}) instead.

Notice that the order in which facts are removed is immaterial.

Lemma 4

$$P : \text{Proc}; F, G : \mathbb{P}\,\text{Fact} \ \vdash \ (P \, ! \, F) \, ! \, G = P \, ! \, (F \cup G)$$

Furthermore, fact removal distributes through union of procedures.

Lemma 5

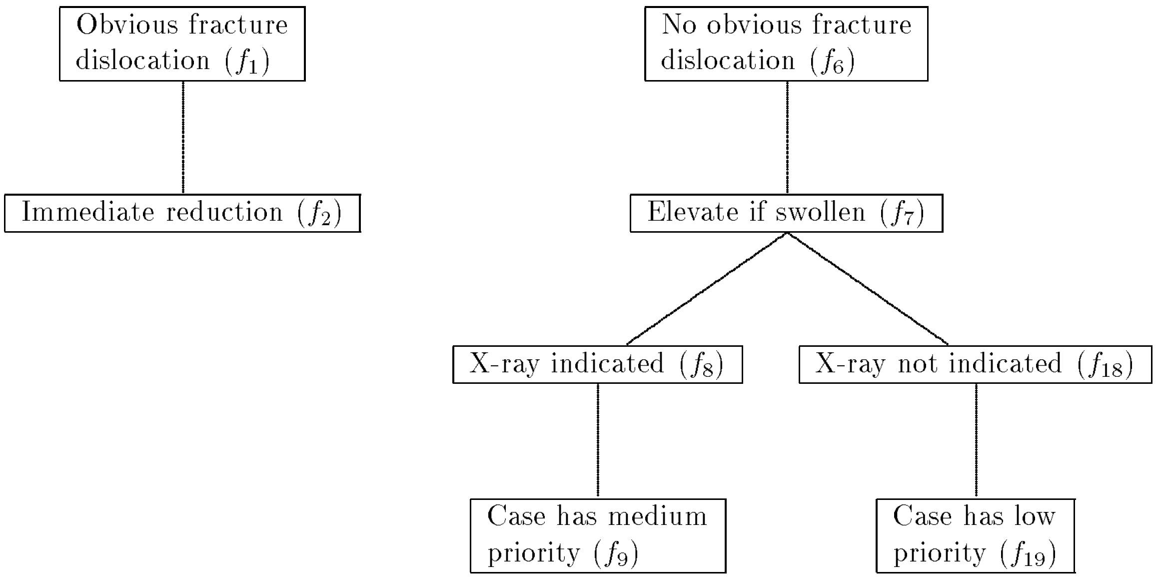
$$P, Q : \text{Proc}; F : \mathbb{P}\,\text{Fact} \ \vdash \ (P \cup Q) \, ! \, F = (P \, ! \, F) \cup (Q \, ! \, F)$$

However, in general, once facts are removed, a procedure no longer meets its specification. For example, let S_{DISLOC} be the specification that if the patient has an obvious fracture-dislocation (f_1) then the full length of the tibia and fibula should be X-rayed (f_4).

$$S_{\text{DISLOC}} \ \hat{=} \ \mathbb{P}\,[f_1 \Rightarrow f_4] \tag{28}$$

It is easily seen that $P_{\text{ANKLE}} \in S_{\text{DISLOC}}$, but $P_{\text{TRIAGE}} \notin S_{\text{DISLOC}}$ because f_4 has been removed from P_{TRIAGE}.

Figure 3: Flowchart for the triaging of a patient attending an accident and emergency department with an acute ankle injury.



Constraint Relaxation

When we remove facts from procedures we need to weaken the procedure's specification correspondingly so that all constraints regarding those facts are removed from the specification. If s is a specification, let $s \setminus F$ be the specification derived from s by relaxing any constraint as to whether facts F should or should not be established along any particular pathway. Thus no procedure P is prevented from meeting $s \setminus F$ purely on account of the presence or absence of any of the facts F along any pathway in P. Thus we define constraint relaxation for any s and F,

$$s \setminus F \;\; \widehat{=} \;\; \{P : \mathrm{Proc} \mid \exists Q : s \bullet P \,!\, F = Q \,!\, F\} \tag{29}$$

Notice that the order in which constraints are relaxed is irrelevant.

Lemma 6

$$s : \mathrm{Spec}; F, G : \mathbb{P}\,\mathrm{Fact} \;\vdash\; (s \setminus F) \setminus G = s \setminus (F \cup G)$$

Furthermore, if a procedure P meets specification s, then $P \,!\, F$ meets $s \setminus F$, for any set of facts F,

Lemma 7

$$P : \mathrm{Proc}; s : \mathrm{Spec}; F : \mathbb{P}\,\mathrm{Fact} \mid P \in s \;\vdash\; (P \,!\, F) \in (s \setminus F)$$

as does the original procedure P.

Lemma 8

$$P : \mathrm{Proc}; s : \mathrm{Spec}; F : \mathbb{P}\,\mathrm{Fact} \mid (P \,!\, F) \in (s \setminus F) \;\vdash\; P \in (s \setminus F)$$

In general, a specification is presented in conjunctive normal form. Relaxation of constraints can be performed independently over each clause rather than over the entire specification because the result is weaker, and so satisfaction is preserved.

Lemma 9

$$s, t : \mathrm{Spec}; F : \mathbb{P}\,\mathrm{Fact} \;\vdash\; (s \cap t) \setminus F \subseteq (s \setminus F) \cap (t \setminus F)$$

Each clause consists of a disjunction of literals. Constraint relaxation distributes through union.

Lemma 10

$$s, t : \mathrm{Spec}; F : \mathbb{P}\,\mathrm{Fact} \;\vdash\; (s \cup t) \setminus F = (s \setminus F) \cup (t \setminus F)$$

Each literal is either an atomic property or the complement of an atomic property. It is easily shown by induction that relaxation of constraints over literals has a direct counterpart at the syntactic level.

Lemma 11

$$E : \mathrm{Expr}; f : \mathrm{Fact} \;\vdash\; (\mathbb{P}\,[E]) \setminus \{f\} = \mathbb{P}\,[E[f \setminus \mathsf{true}\,] \vee E[f \setminus \mathsf{false}\,]]$$

where $E[f \setminus \mathsf{true}\,]$ *and* $E[f \setminus \mathsf{false}\,]$ *are the expressions derived from* E *by substituting* true *and* false*, respectively, for all occurrences of* f*.*

Lemma 12

$$E : \mathrm{Expr}; f : \mathrm{Fact} \vdash (\overline{\mathbb{P}\,[E]}) \setminus \{f\} = \overline{\mathbb{P}\,[E[f \setminus \mathsf{true}\,] \wedge E[f \setminus \mathsf{false}\,]]}$$

Relaxation of the constraints over more than one fact in a literal can be achieved by using these results to relax those for each fact individually. The order in which we do this does not matter (Lemma 6).

For example, let S_{SPEED} be the specification that if the patient has no obvious fracture-dislocation (f_6), but the injury was sustained at high velocity (f_{24}), then the case has medium priority (f_9) and the ankle should be X-rayed (f_{10}).

$$S_{\mathrm{SPEED}} \;\widehat{=}\; \mathbb{P}\,[f_6 \wedge f_{24} \Rightarrow f_9 \wedge f_{10}] \tag{30}$$

If we relax the constraint regarding X-ray, we are left simply with

$$S_{\mathrm{SPEED}} \setminus \{f_{10}\} = \mathbb{P}\,[f_6 \wedge f_{24} \Rightarrow f_9] \tag{31}$$

Theories

In practice, of course, we represent complicated procedures only implicitly by means of a suitable theory. We therefore need an operator for removing facts from theories that is analogous to the '!' operator for procedures. Let '!!' denote the corresponding operator for theories. Thus for any theory T and set of facts F, $T \,!!\, F$ is the theory obtained by removing F from every path in T.

$$T \,!!\, F \;\widehat{=}\; \{f : \mathrm{Fact}; S : \mathbb{P}\,\mathrm{Fact} \mid (f, S) \in T \bullet f \mapsto (S - F)\} \tag{32}$$

For example, let T_{TRIAGE} be the theory relevant to triage.

$$T_{\mathrm{TRIAGE}} = T_{\mathrm{ANKLE}} \;!!\; \{f_3, f_4, f_5, f_{10}, f_{11}, f_{12}, f_{13}, f_{14}, f_{15}, f_{16}, f_{17}\} \tag{33}$$

$$\begin{array}{lll} = \{f_0 & \mapsto & \{f_1, f_2\}, \\ \quad f_0 & \mapsto & \{f_6, f_7, f_8, f_9\}, \\ \quad f_0 & \mapsto & \{f_6, f_7, f_{18}, f_{19}\}, \\ \quad f_1 & \mapsto & \{\}, \\ \quad f_2 & \mapsto & \{\}, \\ \quad \vdots & \vdots & \vdots \\ \quad f_{17} & \mapsto & \{\}, \\ \quad f_{18} & \mapsto & \{f_{20}\}, \\ \quad f_{18} & \mapsto & \{f_{21}\}, \\ \quad f_{18} & \mapsto & \{f_{22}\}, \\ \quad f_{18} & \mapsto & \{f_{23}\}, \\ \quad f_{18} & \mapsto & \{f_{24}\}, \\ \quad f_{19} & \mapsto & \{\}, \\ \quad \vdots & \vdots & \vdots \\ \quad f_{24} & \mapsto & \{\}\} \end{array}$$

This is equivalent to removing the facts from each explicit procedure (flowchart) within the theory.

Lemma 13

$$f : \mathrm{Fact}; F : \mathbb{P}\,\mathrm{Fact}; T : \mathrm{Theory} \vdash (T \,!!\, F)(\!| f |\!) = (T(\!| f |\!))\,!\,F$$

Thus,

$$\begin{array}{ll} T_{\mathrm{TRIAGE}}(\!| f_0 |\!) & = P_{\mathrm{TRIAGE}} \\ T_{\mathrm{TRIAGE}}(\!| f_1 |\!) & = \mathsf{SKIP} \\ \vdots & \\ T_{\mathrm{TRIAGE}}(\!| f_{17} |\!) & = \mathsf{SKIP} \\ T_{\mathrm{TRIAGE}}(\!| f_{18} |\!) & = P_{\mathrm{XRAY}} \\ T_{\mathrm{TRIAGE}}(\!| f_{19} |\!) & = \mathsf{SKIP} \\ \vdots & \\ T_{\mathrm{TRIAGE}}(\!| f_{24} |\!) & = \mathsf{SKIP} \end{array} \tag{34}$$

Unfortunately, removal of facts F from a theory does not in general distribute through expansion of the theory. This is because removal of a fact from a pathway removes all expansions of the fact too. However, in the case that every expansion of a fact in F consists only of facts in F, then the distributive law holds.

Lemma 14

$$F : \mathbb{P}\,\mathrm{Fact}; T : \mathrm{Theory} \mid T(\!| F |\!) \subseteq \mathbb{P}\,F \vdash (T \,!!\, F)^\star = T^\star \,!!\, F$$

The same precondition is therefore required when extending Lemma 7 to theories.

Lemma 15

$$T : \mathrm{Theory}; F : \mathbb{P}\,\mathrm{Fact}; f : \mathrm{Fact}; s : \mathrm{Spec} \mid T(\!| F |\!) \subseteq \mathbb{P}\,F \wedge T \;\underline{sat}_f\; s \vdash (T \,!!\, F) \;\underline{sat}_f\; (s \setminus F)$$

More generally, we can simply relax any constraints both over facts F that we remove from T, and over facts that are reachable through T from any in F.

Lemma 16

$$T : \mathrm{Theory}; F, G : \mathbb{P}\,\mathrm{Fact}; f : \mathrm{Fact}; s : \mathrm{Spec} \mid G = \bigcup(T^\star(\!| F |\!)) \wedge T \;\underline{sat}_f\; s \vdash (T \,!!\, F) \;\underline{sat}_f\; (s \setminus G)$$

Moreover, if after removal of facts a theory satisfies a correspondingly relaxed specification, then so too does the original theory.

Lemma 17

$$T : \mathrm{Theory}; F, G : \mathbb{P}\,\mathrm{Fact}; f : \mathrm{Fact}; s : \mathrm{Spec} \mid G = \bigcup(T^\star(\!| F |\!)) \wedge (T \,!!\, F) \;\underline{sat}_f\; (s \setminus G) \vdash T \;\underline{sat}_f\; (s \setminus G)$$

This last lemma suggests an algorithm for validating theories against their specifications. For each clause s, let F be the set of all facts that appear syntactically nowhere in s and from which no fact in s can be reached through the expanded theory. Thus no fact in G appears in s, and so $s \setminus G = s$. Therefore T meets s if and only if $T \,!!\, F$ meets s (Lemmas 16 and 17). Typically $T \,!!\, F$ is much smaller that T, and so it is very much quicker to enumerate all paths in $(T \,!!\, F)^\star$ than it is to enumerate all paths in $T^\star$.

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