

Akitoshi Kawamura

Arriving: Friday, 10<sup>th</sup> March in the afternoon

Leaving: Thursday, 16<sup>th</sup> March in the morning

Title:

Computational complexity of differential equations

Abstract:

Many problems in mathematical analysis are naturally formulated as type-two problems (computation whose inputs and outputs are functions), because real numbers are infinite objects, of which one can access only partial information (approximation) at a time. I will introduce how the computational complexity of such problems can be formulated and proved. For example, the problem of solving an ordinary differential equation in a general form can be proved PSPACE-complete [1, 2] in the sense that naturally extends the standard notions of "polynomial space" and "completeness".

[1] <http://arxiv.org/abs/1004.4622>

[2] <http://arxiv.org/abs/1305.0453>

Amaury Pouly

Arriving: Friday, 10<sup>th</sup> March in the afternoon

Leaving: Friday, 17<sup>th</sup> March in the evening

Title:

On the decidability of point-to-point controllability in LTI systems

Abstract:

I will present some work in progress on the decidability of controlling a linear time invariant (LTI),

that is a system of the form  $x(n+1)=Ax(n)+u(n)$  where  $u(n)$  is a control. We are interested in the following question: given  $x(0)$  and  $y$ , can we find a finite number of controls  $u(0), u(1), \dots, u(k)$  such that  $x(k)=y$ . The problem is well-known to be decidable if  $u(k)$  is only required

to belong to a linear subspace (by reduction to the Orbit problem). In this talk, I will focus on more

interesting situation where  $u(k)$  must belong to a (bounded) polyhedron. Very little is known in this case

and we will see that there are interesting links with the Skolem/Positivity problems and self-affine fractals.

Borja Balle

Arriving: Friday, 10<sup>th</sup> March in the evening

Leaving: Friday, 17<sup>th</sup> March in the evening

2h slot

Title:

Approximation Algorithms for Weighted Automata

Abstract:

In this talk I will present two efficient approximation algorithms for weighted finite automata (WFA). The first algorithm relies on the singular value automaton (SVA) introduced by Balle, Panangaden and Precup (LICS 2015). The second algorithm exploits a result by Adamyan, Arov and Krein (AAK, 1971) about the singular vectors of Hankel operators on Hardy spaces. The SVA-based algorithm works for arbitrary alphabets, while the AAK-based algorithm provides stronger approximation guarantees but only works for one letter alphabets. I will conclude my talk with a discussion of interesting open problems in this area.

Carl Dettmann

Arriving: Saturday, 11<sup>th</sup> March in the evening

Leaving: Friday, 17<sup>th</sup> March in the afternoon

Title:

Escape rates and Diophantine approximation

Abstract:

One way to analyse a dynamical system is to consider a small subset, the "hole" through which orbits leak out of the

system, and for some natural measure of initial conditions, study the decay of the survival probability as a function of

time. For uniformly hyperbolic systems this decay is typically exponential, whilst for many others it is algebraic. In

both cases it is of interest to ask how the relevant escape rate depends on the size and location of the hole. Even for

very simple examples, the doubling map and semicircular billiard, which are expanding and integrable, respectively,

the escape rate depends sensitively on the hole parameters via their Diophantine properties.

Dick Lipton

Arriving: Sunday, 12<sup>th</sup> March in the morning

Leaving: ?

## The Skolem Conjecture: An Idea

RJL

Define the class of numbers  $N_c$  to be those positive numbers  $n$  of the form  $n = p^k l$  where  $p$  is a prime and both  $k$  and  $l$  are less than  $c$ .

**Theorem 0.1.** *Let  $\lambda_1, \dots, \lambda_m$  be algebraic numbers, let  $c > 0$ , and let  $a_1(n), \dots, a_m(n)$  be polynomials in  $n$  with integer coefficients. Then we can decide whether or not there is an  $n$  so that*

$$\sum_{k=1}^m a_k(n) \lambda_k^n = 0$$

for  $n$  in  $N_c$ .

The proof of this statement is something I worked on quite a while ago. So I have forgotten some of the details. Perhaps we can work together to figure out if there is some useful idea here.

Evgeniy Zorin

Arriving: Friday, 10<sup>th</sup> March in the evening

Leaving: Thursday, 16<sup>th</sup> March in the evening

Title:

Hartmanis, Stearns and Mahler

Abstract:

A famous conjecture by Hartmanis and Stearns, states that any number computable in the real time is either rational or transcendental. This conjecture stays open for more than 50 years, however, this kind of dichotomy, to be either rational or transcendental, has been established for some other large classes of numbers. One of such classes is formed by so called Mahler numbers, which have a particularly nice structure from the point of view of studying their Diophantine properties. Remarkably, one can quite easily check that Mahler numbers contain all automatic numbers, and this fact implies Hartmanis-Stearns conjecture for automatic numbers (a result proved for the first time by Adamczewski and Bugeaud with a use of Schmidt's subspace theorem).

I am going to present Mahler numbers, what is known on their Diophantine properties and their known relations to the classes of numbers defined via computational means.

Igor Potapov

Arriving: Friday, 10<sup>th</sup> March in the evening

Leaving: Friday, 17<sup>th</sup> March in the evening

Title:

Reachability problems in low-dimensional matrices and maps

Abstract:

Reachability for piecewise affine maps (PAMs) is known to be undecidable, starting from dimension two. One-dimensional piecewise affine maps are frequently used as a reference model to show the openness of the reachability questions in many hybrid systems. The reachability problem for one-dimensional PAM is still open even if we define it with only two intervals. The solution of this problem has close connections with analysis of topological properties of orbits, representation of rational numbers in a rational base system and reachability questions in  $2 \times 2$  matrix semigroups. In this talk I will discuss the decidability and the complexity of reachability problems for PAMs and other tightly connected problems for matrices and maps.

James Worrell

Arriving: Friday, 10<sup>th</sup> March in the afternoon

Leaving: Friday, 17<sup>th</sup> March in the evening

Title:

On the Skolem Problem for Continuous Linear Dynamical Systems

Abstract:

The Continuous Skolem Problem asks whether a real-valued function satisfying a linear differential equation has a zero in a given interval of real numbers. Decidability of the problem is currently open---indeed decidability is open even for the sub-problem in which a zero is sought in a bounded interval. In this paper we show decidability of the bounded problem subject to Schanuel's Conjecture, a unifying conjecture in transcendental number theory. We furthermore analyse the unbounded problem in terms of the frequencies of the differential equation, that is, the imaginary parts of the characteristic roots. We show that the unbounded problem can be reduced to the bounded problem if there is at most one rationally linearly independent frequency, or if there are two rationally linearly independent frequencies and all characteristic roots are simple. We complete the picture by showing that decidability of the unbounded problem in the case of two (or more) rationally linearly independent frequencies would entail a major new effectiveness result in Diophantine approximation, namely computability of the Diophantine-approximation types of all real algebraic numbers.

This is joint work with Ventsislav Chonev and Joel Ouaknine.



Joel Ouaknine

Arriving: 10<sup>th</sup> March in the afternoon

Leaving: 17<sup>th</sup> March in the evening

Title:

Positivity Problems for Simple Linear Recurrence Sequences

Abstract:

Given a linear recurrence sequence (LRS) over the integers, the Positivity Problem asks whether all terms of the sequence are positive, and the Ultimate Positivity Problem asks whether all but finitely many terms are positive. I'll examine the question of decidability of both problems for simple LRS (those whose characteristic polynomial has no repeated roots) using tools from number theory, Diophantine geometry, and real algebraic geometry, and discuss some open problems.

This is joint work with James Worrell.

Nathanael Fijalkow

Arriving: Friday, 10<sup>th</sup> March in the afternoon

Leaving: Friday, 17<sup>th</sup> March in the evening

Title:

Hidden Markov Models of Bounded Ambiguity

Abstract:

Hidden Markov Models - or, almost equivalently for this talk, probabilistic automata - are simple stochastic machines defining a distribution over strings, and are broadly used for instance in computational biology, machine learning and linguistics.

Hidden Markov Models can be tricky to analyse algorithmically. An appealing restriction is that of ambiguity: a Hidden Markov Model is  $k$ -ambiguous if each string is generated by at most  $k$  runs.

I will show some results and many open problems related to the algorithmic analysis of Hidden Markov Models of bounded ambiguity.

Raphael Jungers

Arriving: Sunday, 12<sup>th</sup> March in the afternoon

Leaving: Friday, 17<sup>th</sup> March in the evening

Title:

Path-Complete Lyapunov Techniques: when Algebra and Combinatorics meet in Control

Abstract:

I will present an overview of 'Path-complete techniques' which are algorithms that have been developed to analyze stability properties of dynamical systems. These tools rely on concepts from Mathematics, Computer Science, and Optimization. Starting with the pioneering works of Bliman, Ferrari-Trecate, Lee, Dullerud, Daafouz, and others, the nature of path-complete Lyapunov functions has been progressively understood in the last 20 years, and their range of application is still widening. These Algebro-combinatorial techniques allow in favorable cases to obtain provably efficient algorithms (e.g. for switching systems stability analysis), and are promising for tackling more general problems in Cyber-Physical Systems control. I will finish by presenting recent works, where we use these tools to analyze constrained switching systems, and to generalize the notion of monotone systems.

As a motivation for constrained switching systems, I will describe a simple model of Linear Dynamical System controlled by a wireless communication-equipped feedback loop. I will show how the controllability problem can be solved on this model thanks to the celebrated Skolem Theorem.

Valerie Berthe

Arriving: Saturday, 11<sup>th</sup> March in the evening

Leaving Bellairs: Thursday, 16<sup>th</sup> March in the morning

Title:

Numeration dynamics and carry propagation

Abstract:

Arithmetic dynamics provides explicit expansions of real numbers (or of vectors) which have a dynamical meaning in order to produce symbolic codings of dynamical systems preserving their arithmetic structure. It is a classical topic in arithmetic dynamics to study numeration dynamical systems and digital expansions.

We study in this context the carry propagation. Given the representation of an integer  $n$  in a numeration system, the successor function provides the representation of  $n+1$  in this system. We provide estimates for the average length of the carry propagation when applying the successor map, and we relate the study of finite orbits with generic orbits produced by the odometer (also called “adding machine”) considered as a dynamical system. We consider various well-known classes of numeration systems, namely, beta-numerations, abstract numeration systems, and rational base numerations.

This a joint work with C. Frougny, M. Rigo, J. Sakarovitch