Irreversibility and non-classicality in a single system game

Lorenzo Catani

Joint work with L. Henaut, D. Browne, S. Mansfield and A. Pappa, PRA **98**, 060302 (2018)

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Contents

Contents

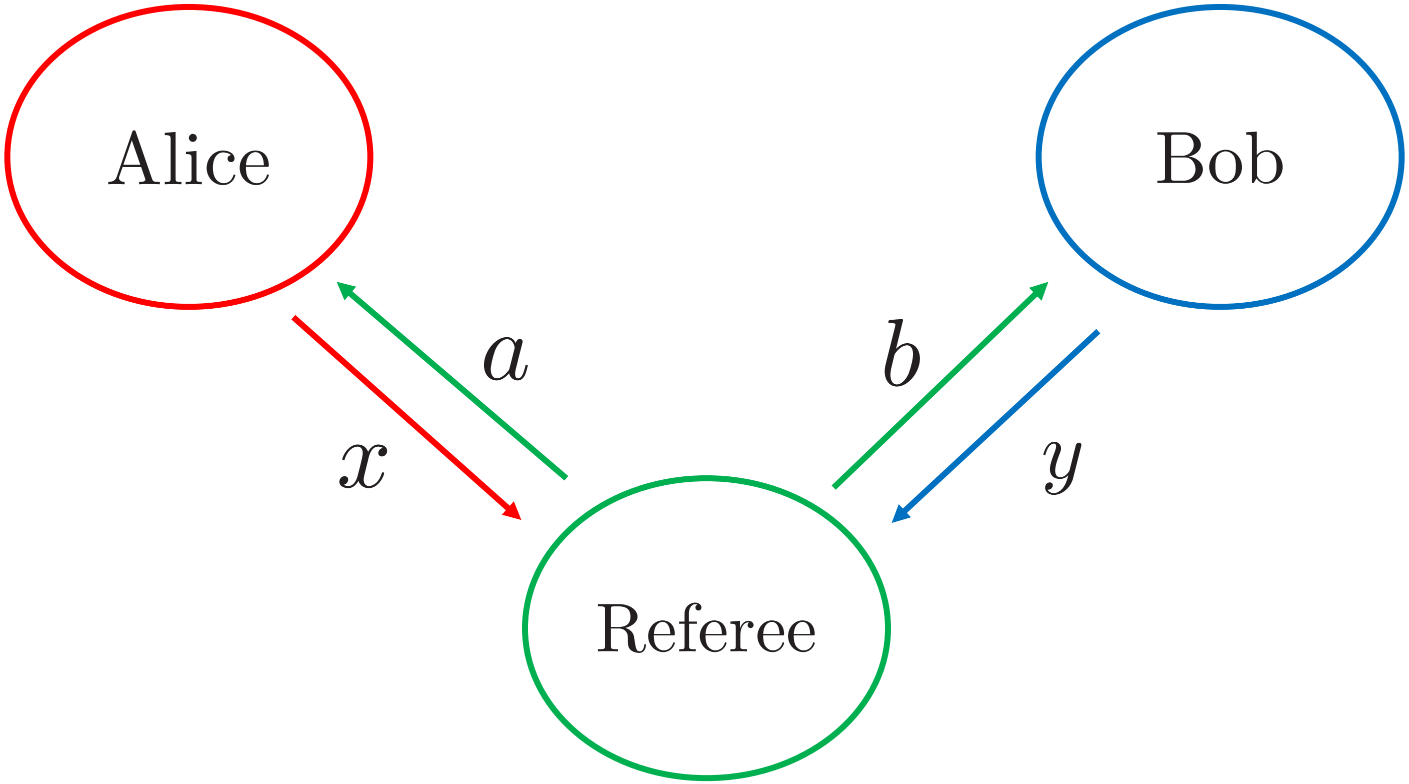
- Motivation
- The CHSH* game
  - Preliminaries
  - Description of the protocol and its settings
  - Sources of computational advantages
- Irreversibility and Contextuality
- Conclusion

Motivation

- Develop new protocols that show quantum computational advantages.
- Study the sources of the computational advantages.
- Study the relation between non-classicality and irreversibility.

Contents

CHSH game



Goal: $x + y = a \cdot b$.

CHSH game – optimal strategies

- Optimal classical strategy achieves $p_{suc}(x + y = a \cdot b) = 0.75.$

For example always output 0:

a	b	x+y	ab	
0	0	0	0	✓
0	1	0	0	✓
1	0	0	0	✓
1	1	0	1	✗

- Optimal quantum strategy achieves $p_{suc}(x + y = a \cdot b) \approx 0.85.$

For example Alice and Bob share a Bell state and perform

$$A_0 = X \qquad B_0 = T^\dagger X T$$
$$A_1 = Y \qquad B_1 = T X T^\dagger$$

$$T = R_z(\frac{\pi}{4})$$

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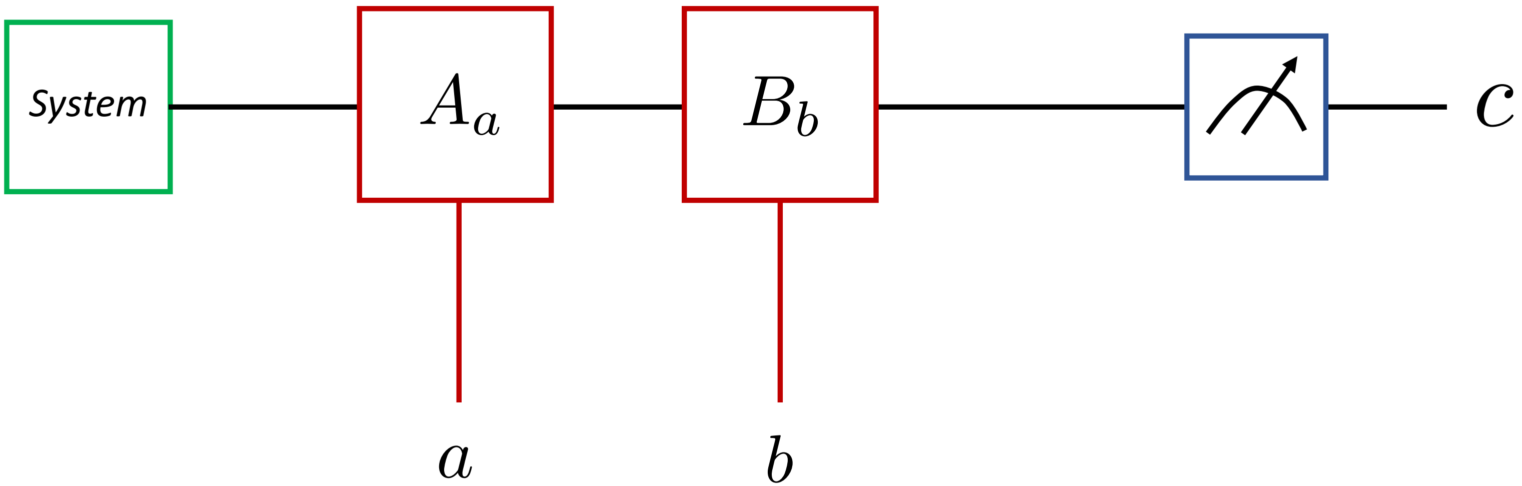
Description of the protocol

CHSH* game

L. Henaut, L. Catani, D.E. Browne, S. Mansfield, A. Pappa PRA **98**, 060302 (2018)



- Inputs $a, b \in \mathbb{Z}_2$.
- Goal: $\omega(CHSH^*) = \max_{all\ strategies} \frac{1}{4} \sum_{a,b \in \mathbb{Z}_2} p(c = a \cdot b | a, b).$

Description of the protocol

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Settings

Name of setting	System type(d=2)	Initial states	Transformations	Measurements	ω(CHSH*)

Description of the protocol

Settings

Name of setting	System type(d=2)	Initial states	Transformations	Measurements	ω(CHSH*)
Unitary	Quantum	Any	Unitary gates	Two-outcome PVM	

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Unitary	Quantum	Any	Unitary gates	Two-outcome PVM	
Clifford	Quantum	Stabilizer	Clifford gates	Pauli	

Description of the protocol

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Settings

Name of setting	System type(d=2)	Initial states	Transformations	Measurements	ω(CHSH*)
Unitary	Quantum	Any	Unitary gates	Two-outcome PVM	
Clifford	Quantum	Stabilizer	Clifford gates	Pauli	
Reversible Classical	Classical	Any	Reversible gates	n/a	

Description of the protocol

Settings

Name of setting	System type(d=2)	Initial states	Transformations	Measurements	ω(CHSH*)
Unitary	Quantum	Any	Unitary gates	Two-outcome PVM	
Clifford	Quantum	Stabilizer	Clifford gates	Pauli	
Reversible Classical	Classical	Any	Reversible gates	n/a	
Irreversible	Classical/ Quantum	Any	Any	Any	

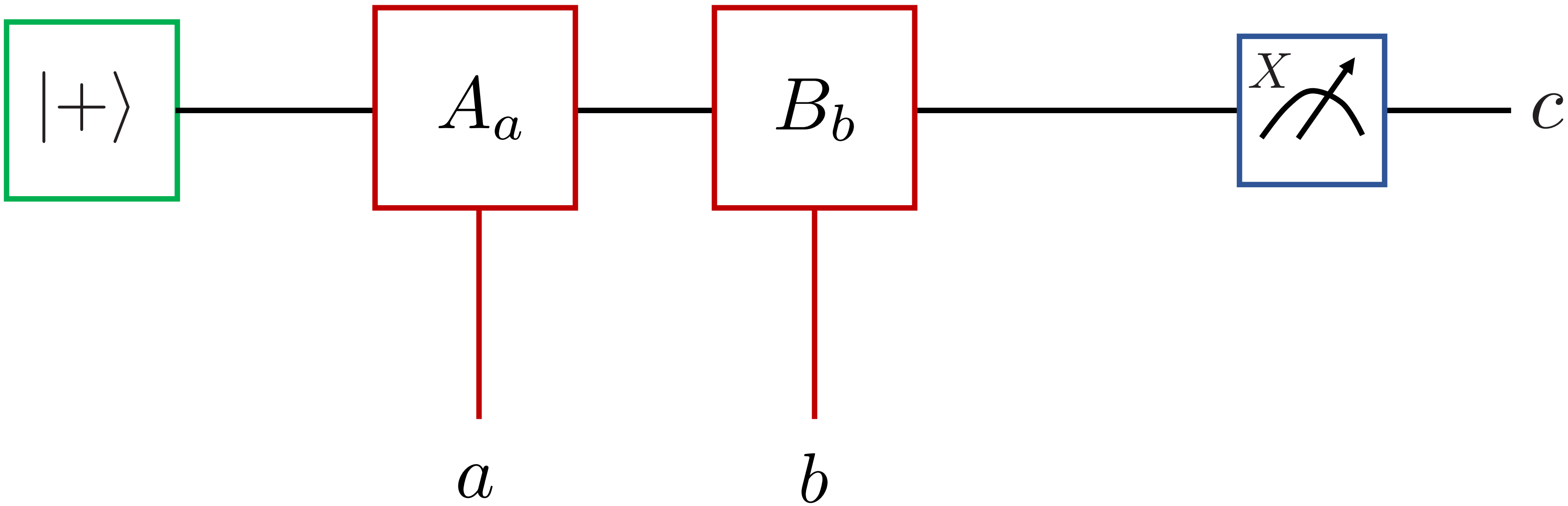
Description of the protocol

Unitary setting

<u>System</u>: *one qubit.*

<u>Measurement</u>: *Two outcome projective measurement.*

<u>Controlled gates</u>: *Unitary.*



Description of the protocol

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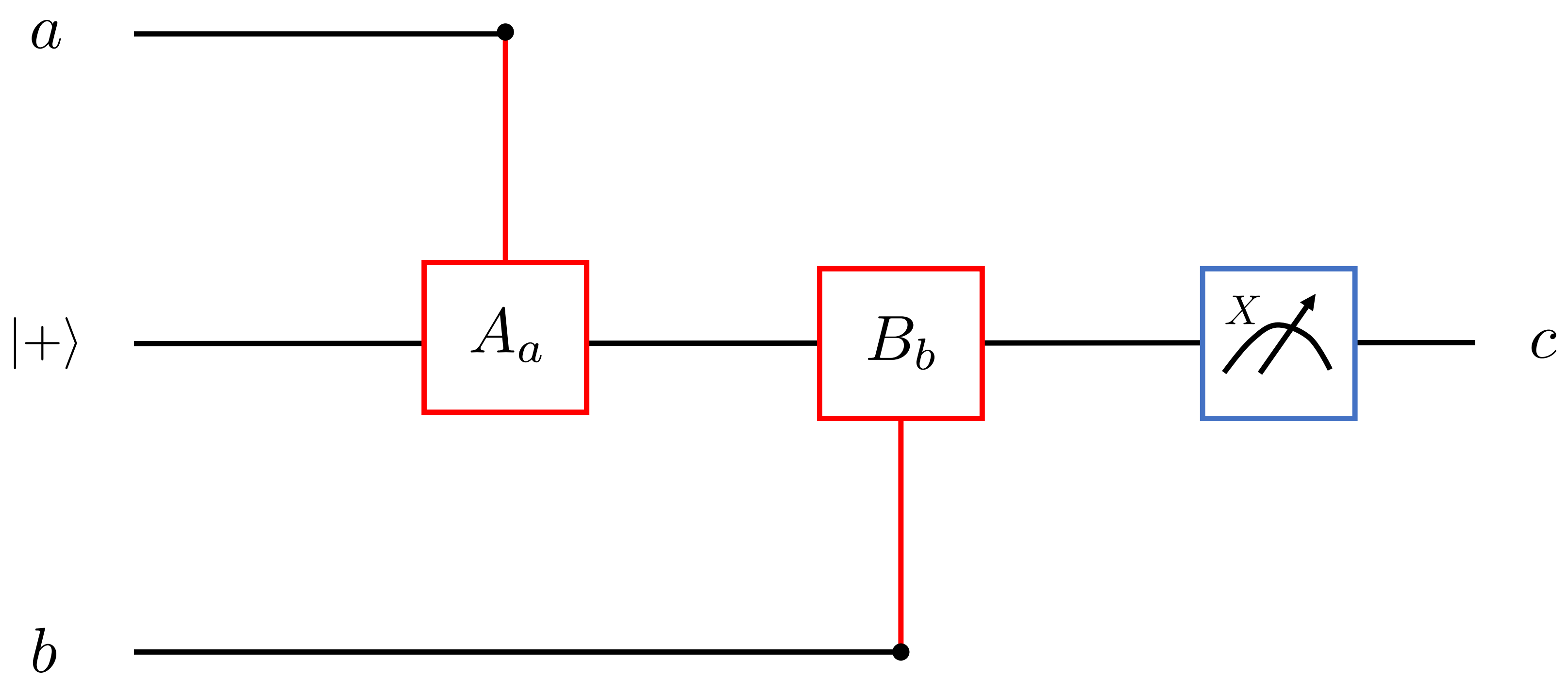
Theorem. *Any strategy in the CHSH* game can be mapped to a strategy in the CHSH game with the same success probability.*

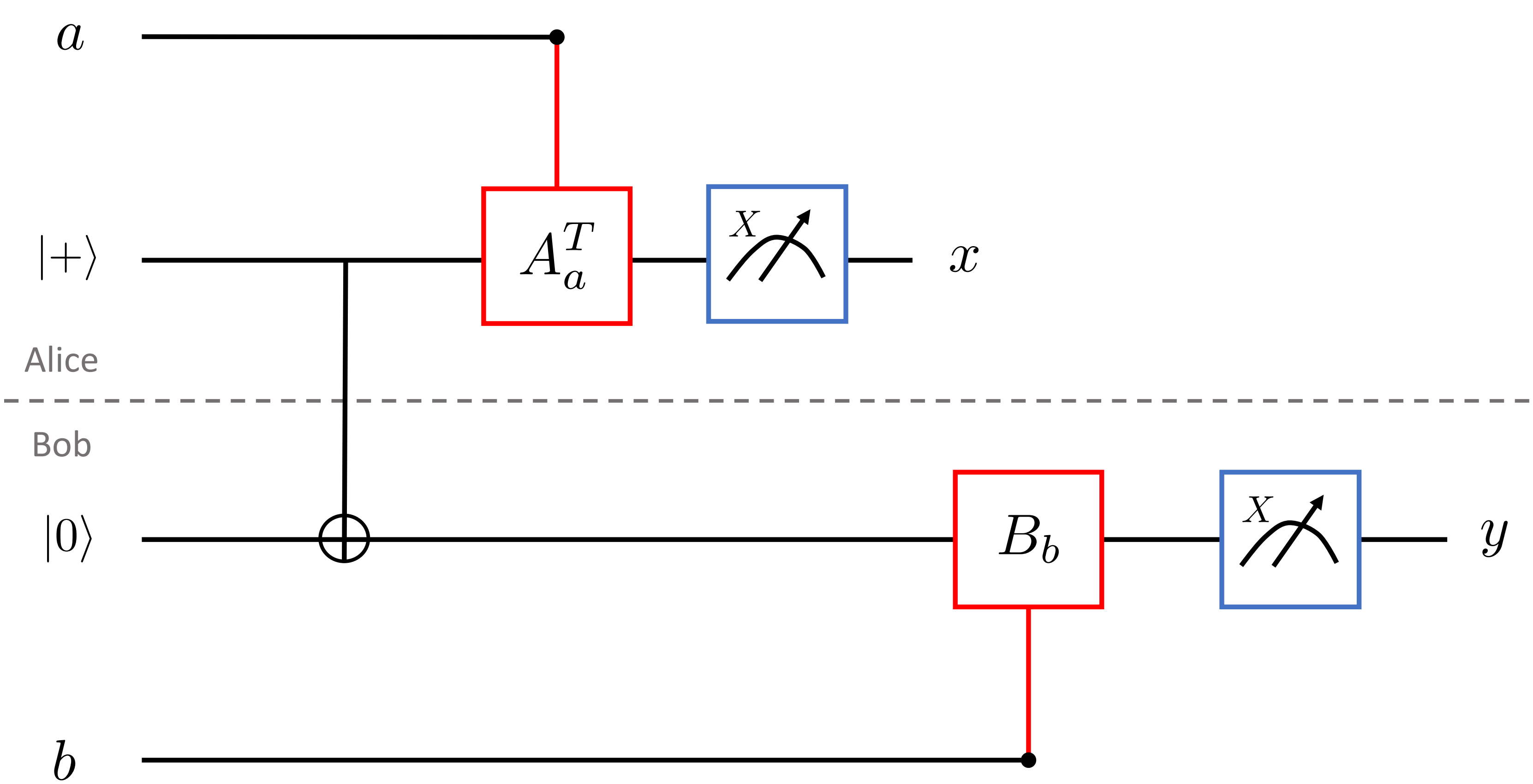
Proof sketch:

Description of the protocol

Unitary setting



Mapping to the CHSH game



Description of the protocol

Theorem. *Any strategy in the CHSH* game can be mapped to a strategy in the CHSH game with the same success probability.*

Proof sketch:

Teleportation stage works as $A_a^T \otimes \mathbb{I}(\frac{|00\rangle + |11\rangle}{\sqrt{2}}) = \mathbb{I} \otimes A_a(\frac{|00\rangle + |11\rangle}{\sqrt{2}})$.

State on Bob before the measurement: $B_b A_a Z^x |+\rangle$.

Explicitly compute the probabilities and show that

$$p_{suc}(c = a \cdot b) = p_{suc}(x + y = a \cdot b).$$

Therefore the Tsirelson bound holds also in the CHSH* game.

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Optimal quantum strategies

CHSH optimal strategy was,

$$A_0 = X \qquad B_0 = T^\dagger X T$$
$$A_1 = Y \qquad B_1 = T X T^\dagger$$

$$T = R_z(\frac{\pi}{4})$$

↓

<u>System</u>: *one qubit in state* $|+\rangle$.

<u>Measurement</u>: *Pauli* X.

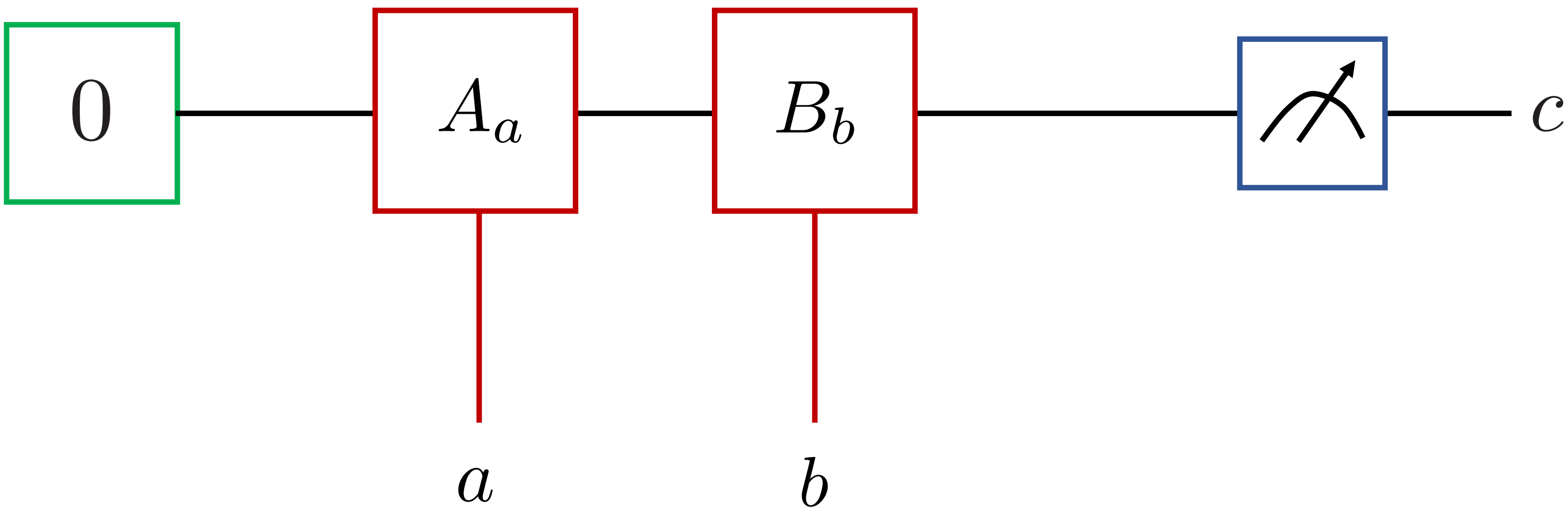
<u>Controlled gates</u>: *gates that map between above observables,*

$$A_0 = \mathbb{I} \qquad B_0 = T^\dagger$$
$$A_1 = S \qquad B_1 = T$$

$$S = R_z(\frac{\pi}{2}) \qquad T = R_z(\frac{\pi}{4})$$

Classical reversible setting

- System: *one bit.*
- Controlled gates: ***reversible** classical gates.*



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Classical reversible setting

- System: *one bit.*
- Controlled gates: ***reversible*** *classical gates.*

- The value of the game is $\omega(CHSH^*) = 0.75.$

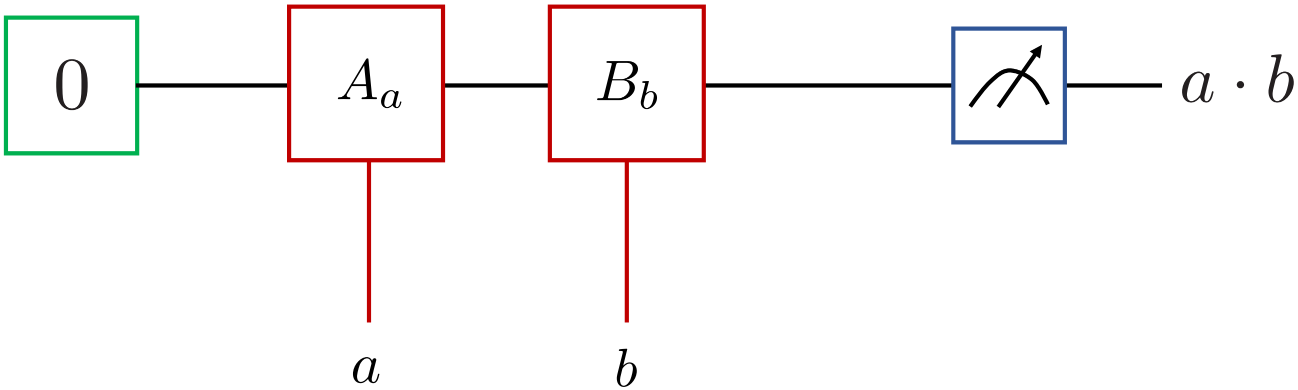
e.g. Input system in state 0 and $A_0 = A_1 = B_0 = B_1 = \mathbb{I}.$

Irreversible setting

In our protocol an irreversible transformation allows us to win the game with certainty.

$$A_0 = B_1 = \mathbb{I}, A_1 = NOT, B_0 = ERASE.$$

$$\boxed{\omega(CHSH^*) = 1.}$$



Settings

Name of setting	System type(d=2)	Initial states	Transformations	Measurements	ω(CHSH*)
Unitary	Quantum	Any	Unitary gates	Two-outcome PVM	$\cos^2(\frac{\pi}{8})$
Clifford	Quantum	Stabilizer	Clifford gates	Pauli	0.75
Reversible Classical	Classical	Any	Reversible gates	n/a	0.75
Irreversible	Classical/Quantum	Any	Any	Any	1

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Sources of computational advantages

Irreversibility

Quantum mechanics

Sources of computational advantages

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Sources of computational advantages

Irreversibility

Quantum mechanics

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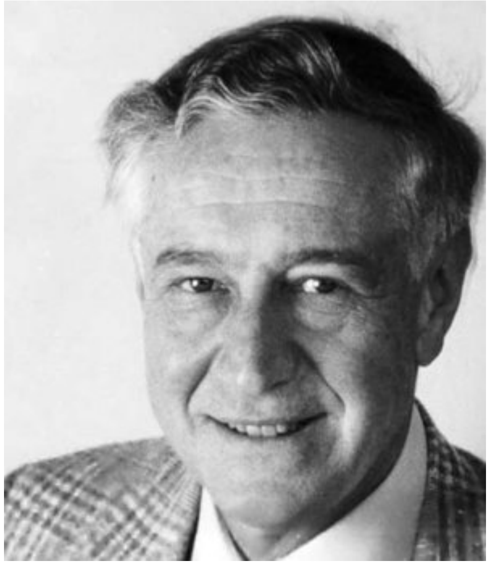
Landauer’s principle



Rolf Landauer (1927-1999)

Every irreversible classical operation on logical bits must be accompanied by an entropic cost in the physical bits or the environment.

e.g. irreversible computation on one bit =

reversible computation on two bits, where one is erased ($\Delta S = k \log 2$).

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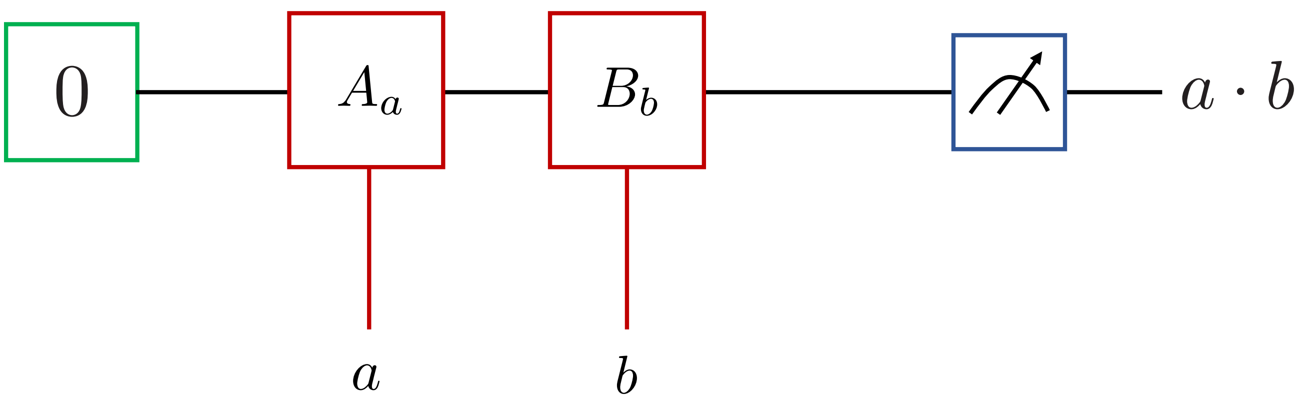
Irreversible Setting

In our protocol an irreversible transformation allows us to win the game with certainty.

$$A_0 = B_1 = \mathbb{I}, A_1 = NOT, B_0 = ERASE.$$

$$\omega(CHSH^*) = 1.$$



Notice that only when $a = 1$ and $b = 0$ the erasure is actually needed.

Sources of computational advantages

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- We can imagine a quantum strategy ($p_{suc} > 0.75$) as the optimal classical reversible strategy + partial erasure.

- We define the *Landauer's Erasure* (**LE**) as the erasure needed to obtain a success probability greater than the Bell bound.

More formally :

- Distance on functions:

$$\text{Given } \; f, g : (\mathbb{Z}_2)^n \to \mathbb{Z}_2, \quad d(f,g) = 2^{-n} \left| \{ i | f(i) \neq g(i) \} \right|.$$

- Non-linearity of a function $f : (\mathbb{Z}_2)^n \to \mathbb{Z}_2$

$$\nu(f) = \min\{ d(f,g) | \; g : (\mathbb{Z}_2)^n \to \mathbb{Z}_2 \;\; \text{linear} \}.$$

- Landauer's Erasure (LE) of a strategy for the CHSH* game with p_{suc} :

$$LE \geq \frac{p_{suc} - p_{suc}^{rev}}{\nu(f)}$$

i.e. $p_{suc} = p_{suc}^{rev} + \nu(f = a \cdot b) LE = \frac{3}{4} + \frac{1}{4} LE \approx 0.85 \to LE \approx 0.41.$

$$p_{fail} \geq (1 - LE)\nu(f)$$

Sources of computational advantages

Sources of computational advantages

Irreversibility

Quantum mechanics

Quantum advantages

Which features of quantum mechanics are responsible for the computational advantage?

- Kochen- Specker Contextuality. ✖
  Contexts (*different sets of commuting projectors*) are not present.

- Generalized Contextuality. ✖
  Contexts (*different operationally equivalent decompositions of a quantum channel, state or measurement element*) are not present.

- Non-locality. ✖
  Just one party.

- …?

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Sequential transformation noncontextuality

S. Mansfield, E. Kashefi, PRL **121**, 230401 (2018)

- Context = sequence of transformations, $C = \{U_i\}_{i=1}^{t}$.

- Same gate in different contexts has the same ontological representation:

$$\Gamma_{U(C)} = \Gamma_{U(C')}.$$

- *Theorem.*
  A sequential transformation noncontextual *l2-ontological model* cannot in general represent the CHSH* protocol in the unitary setting ($p_{suc} > 0.75$).

Contextuality (with the assumption of *l2*-ontology) is a resource for the game.

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Contextual fraction

S. Abramsky, R. S. Barbosa, S. Mansfield PRL **119**, 050504 (2017)

- Empirical model: for every context a distribution over possible outcomes.
- Given an empirical model e, compute $\max \lambda$ over all decompositions
  $e = \lambda e^{NC} + (1-\lambda)e'.$
  Contextual fraction: $CF = 1 - \max \lambda.$
- *Theorem*.
  Given $f : (\mathbb{Z}_2)^n \rightarrow \mathbb{Z}_2$ and a *l2-TBQC protocol* (e.g. CHSH* protocol) that uses the empirical model e to compute f with average success probability p_{suc} and corresponding failure probability $p_{fail} = 1 - p_{suc}$.
  Then
  $$p_{fail} \geq (1 - CF)\nu(f).$$
- Same relation for CF and LE!!

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Sources of computational advantages

Irreversibility

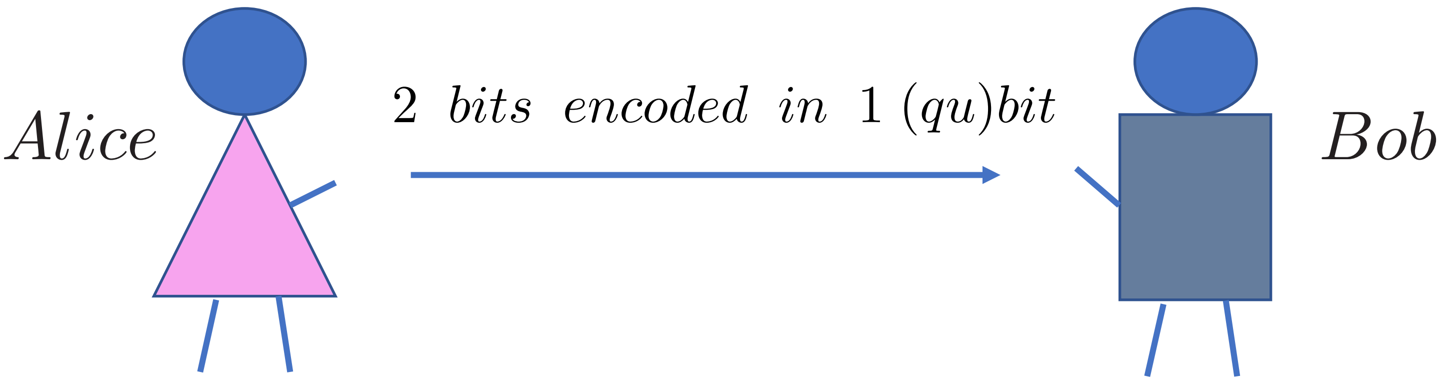
$$p_{fail} \geq (1 - LE)\nu(f)$$

Quantum mechanics

$$p_{fail} \geq (1 - CF)\nu(f).$$

2-bits parity oblivious multiplexing

R. W. Spekkens et al. PRL **102**, 010401 (2009)



- Bob needs to know one of 2 bits. Alice does not know which one.
- Goal: maximize the least probability of success.
- Constraint: Alice cannot communicate the parity of the 2 bits.
- Optimal strategy with bits achieves $p_{suc} = 0.75.$ With qubits $p_{suc} \approx 0.85.$

Preparation noncontextuality

R.W. Spekkens, Phys. Rev. A **71**, 052108 (2005)

- Contexts = possible preparations P of a mixed state ρ.

- Operationally equivalent preparations are represented by the same ontological stochastic map:

$$\mu_P(\lambda) = \mu_\rho(\lambda) \qquad \forall\, P, \ \ \forall\, \lambda \in \Omega.$$

$P =$ a preparation of the state ρ.

$\Omega =$ ontic space.

Two preparations P, P' are operationally equivalent if

$$p(k|P,T,M) = p(k|P',T,M) \ \ \forall\, T, M.$$

- Preparation contextuality necessary for $p_{suc} > 0.75.$

2-bits parity oblivious multiplexing

- We can perform the same analysis of the CHSH* game.
- Landauer’s erasure (LE) = Fraction of parity communicated.
- Contextual fraction (CF) now associated to a decomposition of the ontological model.
- Again CF and LE obey analogous relations.

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Sources of computational advantages

Parity Communicated

$$p_{fail} \geq (1 - LE)\nu(f)$$

Quantum mechanics

$$p_{fail} \geq (1 - CF)\nu(f).$$

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Erasure of information and contextuality

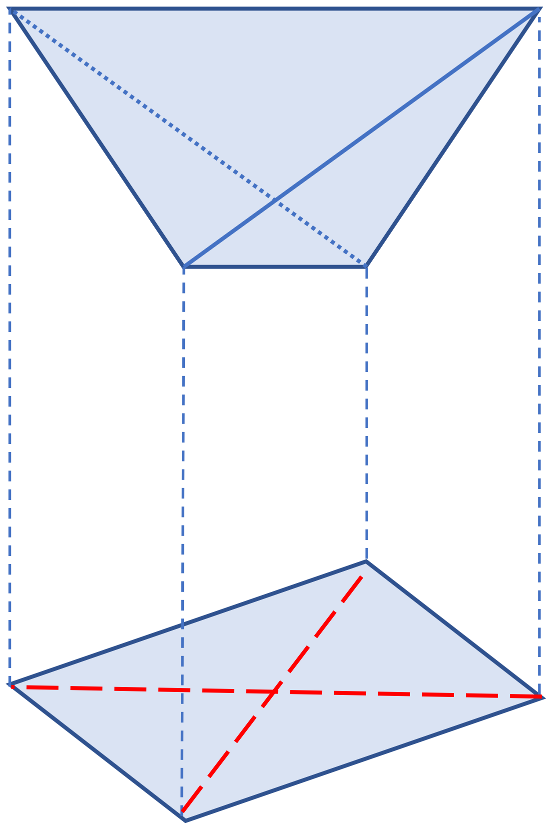
Contextuality as compression of information in all its formulations:

- Generalized notion: Information loss from the ontological to the operational level.

- GPT approach: contextual theories seen as non-contextual theories in higher dimensions with restrictions on measurements.

Irreversibility and contextuality

e.g. a 3 dimensional **simplex** with 4 extremals (**non-contextual state space**) can be projected down to the 2 dimensional **gbit** (**contextual state space**) by restrictions on allowed effects.



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Erasure of information and contextuality

Contextuality as compression of information in all its formulations:

- Generalized notion: Information loss from the ontological to the operational level.

- GPT approach: contextual theories seen as non-contextual theories in higher dimensions with restrictions on measurements.

- Kochen-Specker: Performing a context erases the information associated to the other context. Memory cost of contextuality.

- Sheaf theoretic approach: Local consistency but global inconsistency in the family of data because of lack of information?!

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Erasure of information and contextuality

Contextuality as compression of information in all its formulations.

↓

Can it be explained from a physical mechanism of erasure of information (proper to QT)?

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Conclusion

- Novel single system protocol (CHSH* game) showing the Bell and Tsirelson bounds.

- Computational advantages from irreversibility (LE) and sequential transformation contextuality (CF).
  CF and LE obey the same relations.

Future challenges

- Further study the relation between irreversibility and contextuality.

- Extend the results to the CHSH* game in arithmetic modulo $q > 2$.