

# DESIGN AND ANALYSIS OF ALGORITHMS — HT 2022

## Problem Sheet 4

---

Answers for questions marked \*.

### DFS and connected components, cont'd

#### Shortest paths

##### Answer to question 3

- (a)  $n$ -vertex linear chain.  $(v_i, v_{i+1}), 0 \leq i < n - 1$
- (b)  $n$ -vertex star graph.  $(v_0, v_i), 0 \leq i < n$

#### Greedy algorithms

##### Answer to question 6

- (a) No. Take the series of denominations 40, 15, 1. The greedy algorithm yields change (1, 1, 5) for the amount of 60. A better solution is (0, 4, 0).
- (b) In the optimal solution, there cannot be more than  $p-1$  coins of any denomination except the largest denomination  $p^k$ , as any  $p$  coins of denomination  $p^j$  can be replaced by one  $p^{j+1}$  denomination, resulting in a better solution. The greedy algorithm chooses the maximum possible number of coins of the highest denomination  $p^k$ . Any optimal solution must do the same, since otherwise the solution would have to contain  $p$  or more coins of some smaller denomination. This follows from the fact that

$$(p-1)(p^{k-1} + \dots + p + 1) < p^k.$$

The remaining value for which change has to be given needs to be given using a subset of the remaining coins, and the optimal solution will contain optimal change for this subproblem.