The existence of an information unit as a postulate of quantum theory

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Computation is dynamics when the physical substrate is ignored.

Quantum information perspective

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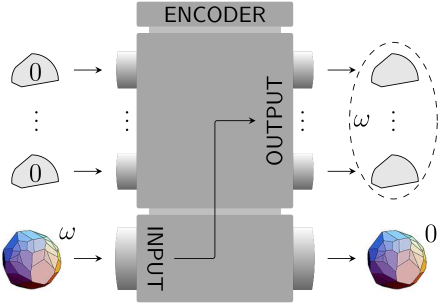
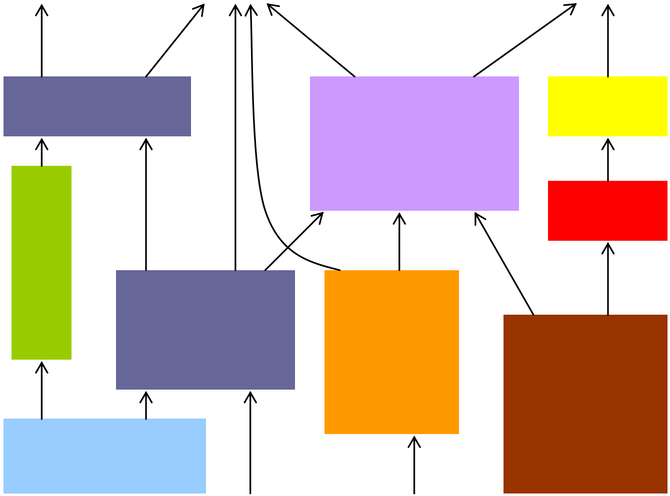


Universe as a circuit



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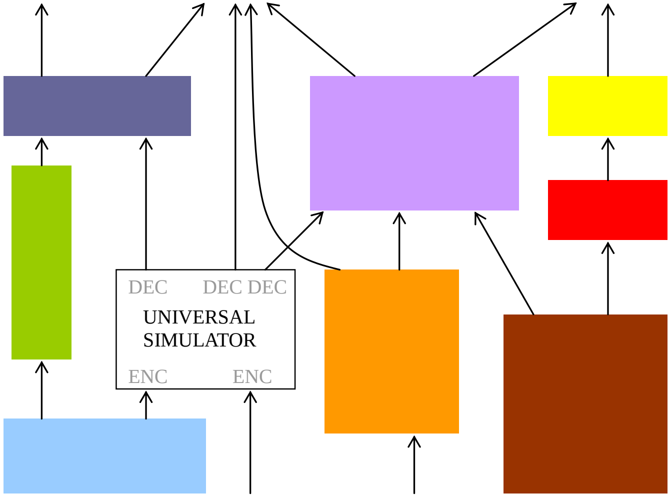
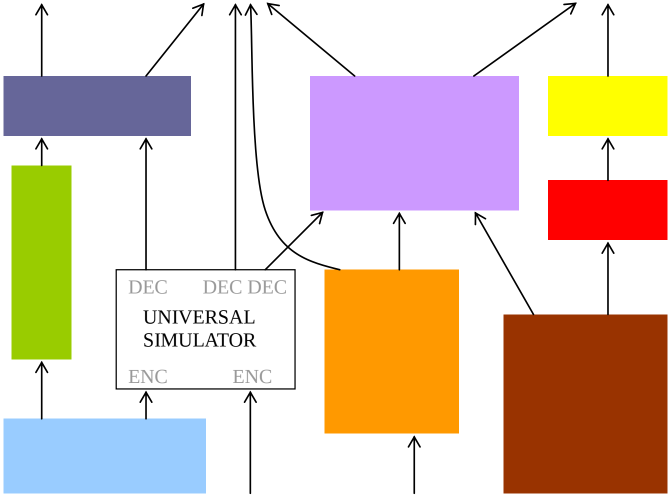
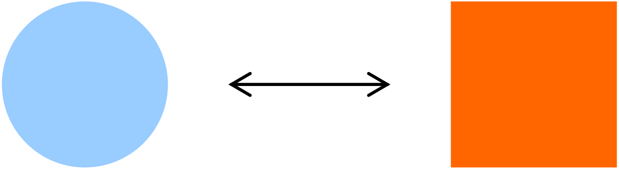


Universe as a circuit – Pancomputationalism?



Coding is in general not possible



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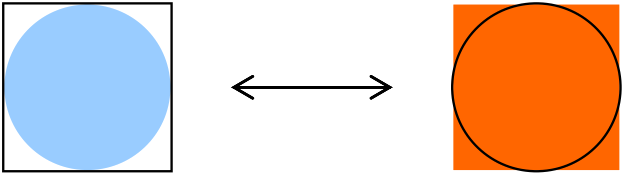
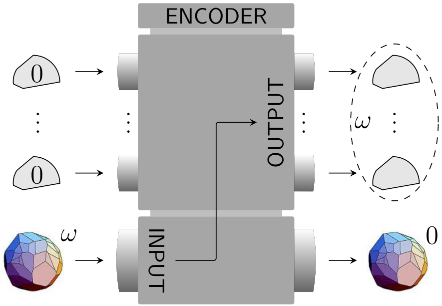


Postulate *Existence of an Information Unit*



Otuline

1. A new axiomatization of QT
   - The standard postulates of QT
   - Generalized probability theory
   - The new postulates
2. The central theorem
3. DIY - construct your own axiomatization

The standard postulates of QT

Postulate 1: states are density matrices

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Postulate 2: tomographic locality

Postulate 3: closed systems evolve reversibly and continuously in time

Postulate 4: the immediate repetition of a projective measurement always gives the same outcome

Goal

To break down “states are density matrices” into meaningful physical principles.

Generalized probability theories

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Birkhoff and von Neumann generalized the formalism of classical probability theory to include incompatible measurements.

Generalized probability theories

Generic features of GPT:

1. Bell-inequality violation
2. no-cloning
3. monogamy of correlations
4. Heisenberg-type uncertainty relations
5. measurement-disturbance tradeoffs
6. secret key distribution
7. Inexistence of an Information Unit

Generalized probability theories

Why nature seems to be quantum instead of classical?

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Why nature seems to be quantum instead of classical?

Why QT instead of any other GPT?

Generalized probability theories - states

The state of a system is represented by the probabilities of some pre-established measurement outcomes $x_1, \dots x_k$ called fiducial:

$$\omega = \begin{bmatrix} p(x_1) \\ \vdots \\ p(x_k) \end{bmatrix} \in \mathcal{S} \subset \mathbb{R}^k$$

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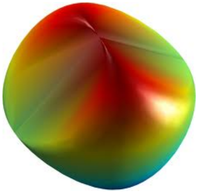
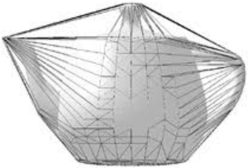
Pure states are the extreme points of the convex set $\mathcal{S}$.

Generalized probability theories

Every compact convex set is the state space $\mathcal{S}$ of an imaginary type of system.


Generalized probability theories - measurements

The probability of a measurement outcome x is given by a function $E_x : \mathcal{S} \to [0,1]$ which has to be linear.

$$E_x(q\omega_1 + (1-q)\omega_2) = qE_x(\omega_1) + (1-q)E_x(\omega_2)$$

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In classical probability theory and QT, all such linear functions correspond to outcomes of measurements, but this need not be the case in general.

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The set of reversible transformations generated by time-continuous dynamics forms a compact connected Lie group $\mathcal{G}$. The elements of the corresponding Lie algebra are the hamiltonians of the theory.

New postulates for QT

1. Continuous Reversibility
2. Tomographic Locality
3. Existence of an Information Unit

New postulates for QT

Continuous Reversibility: for every pair of pure states in $\mathcal{S}$ there is a continuous reversible dynamics which brings one state to the other.

New postulates for QT

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$$\begin{aligned}\mathrm{dim}\mathcal{S}_{AB} &= \mathrm{dim}\mathcal{S}_A \times \mathrm{dim}\mathcal{S}_B \\ p(x,y) &= (E_x \otimes E_y)(\omega_{AB})\end{aligned}$$

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3. Gbits can interact pair-wise.
4. No Simultaneous Encoding: when a gbit is being used to perfectly encode a classical bit, it cannot simultaneously encode any other information.

New postulates for QT – No Parallel Encoding

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$a, a' \in \{0, 1\}$

Bob
$a =?$ or $a' =?$

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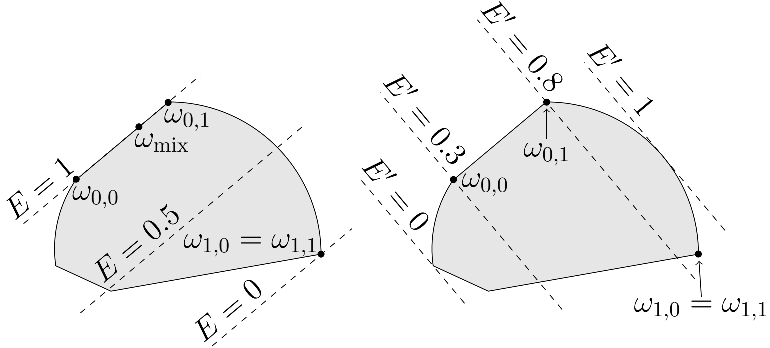
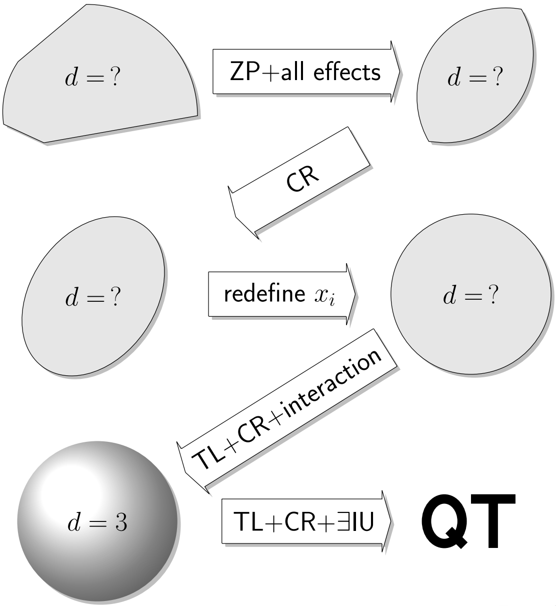
$$P(a_{\text{guess}}|a, a') = \delta^{a}_{a_{\text{guess}}} \qquad \Rightarrow \qquad P(a'_{\text{guess}}|a, a') = P(a'_{\text{guess}}|a)$$

New postulates for QT – No Parallel Encoding



New postulates for QT – proof sketch



Summary

The following postulates single out QT:

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This allows us to go beyond QT in new ways (e.g. removing "interaction").

Comparison with Hardy's last axiomatization

- **A maximal set of distinguishable states** for a system is any set of states containing the maximum number of states for which there exists some measurement, called a **maximal measurement**, which can identify which state from the set we have in a single shot.
- **An informational subset of states** is the full set of states which only give rise to some given subset of outcomes of a given maximal measurement (and give probability zero for the other outcomes).
- **Non-flat sets of states.** A set of states is non-flat if it is a spanning subset of some informational subset of states.
- **A filter** is a transformation that passes unchanged those states which would give rise only to the given subset of outcomes of the given maximal measurement and block states which would give rise only to the complement set of outcomes.
- **Sturdiness Postulate:** Filters are non-flattening.

Comparison with Chiribella's et al axiomatization

- A state is **completely mixed** if every state in the state space can stay in one of its convex decomposition.
- **Perfect distinguishability Axiom.** Every state that is not completely mixed can be perfectly distinguished from some other state.
- **Purification Postulate.** Every state has a purification. For fixed purifying system, every two purifications of the same state are connected by a reversible transformation on the purifying system.

The central theorem - 2 qubits

$$\rho = \frac{1}{4}\left(I \otimes I + a^i \sigma_i \otimes I + b^j I \otimes \sigma_j + c^{ij} \sigma_i \otimes \sigma_j\right)$$

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$$p(x, y) = \frac{1}{4}\left(1 + x \cdot a + y \cdot b + (x \otimes y) \cdot c\right)$$

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The central theorem

Let $G_1 \leq \mathrm{SO}(d)$ be transitive on the unit sphere in $\mathbb{R}^d$, and let $G_2 \leq \mathrm{GL}(\mathbb{R}^{2d+d^2})$ be compact, connected and satisfy

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and

$$\frac{1}{4} + \frac{1}{4} \begin{bmatrix} u \\ u \\ u \otimes u \end{bmatrix} \cdot g \begin{bmatrix} u \\ u \\ u \otimes u \end{bmatrix} \in [0, 1], \quad \forall g \in G_2,$$

then...

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or $d = 3$ and G_2 is the adjoint action of SU(4).

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- Continuous Reversibility
- Tomographic Locality

and supplement them with another postulate(s) implying

- gbits: $d < \infty$
- gbits: no mixed states in the boundary
- gbits: interact
- the n-gbit state spaces suffice to characterize the theory

DIY - construct your own axiomatization

"No mixed states in the boundary" is a consequence of:

- Spectrality
- No information gain implies no disturbance
- The second law of thermodynamics?

References and Collaborators

- Lluís Masanes, Markus P. Müller, David Pérez-García, Remigiusz Augusiak, *The existence of an information unit as a postulate of quantum theory*, arXiv:1208.0493
- Lluís Masanes, Markus P. Müller, David Pérez-García, Remigiusz Augusiak; *Entanglement and the three-dimensionality of the Bloch ball*, arXiv:1110.5482
- Gonzalo de la Torre, Lluís Masanes, Anthony J. Short, Markus P. Müller, *Deriving quantum theory from its local structure and reversibility*, Phys. Rev. Lett. 119, 090403 (2012). [arXiv:1110.5482]