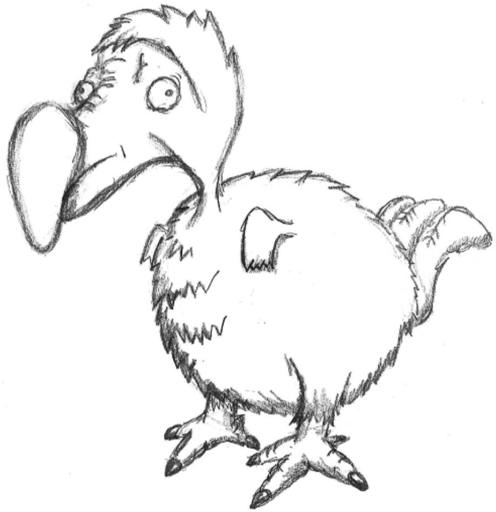
# Pictures of behaviours

Bob Coecke

University of Oxford



**From quantum foundations, via meaning in natural language , to a theory of everything**

**Our starting point** is the common structure of:

**Our starting point** is the common structure of:

- how quantum systems interact
- how meanings in natural language interact

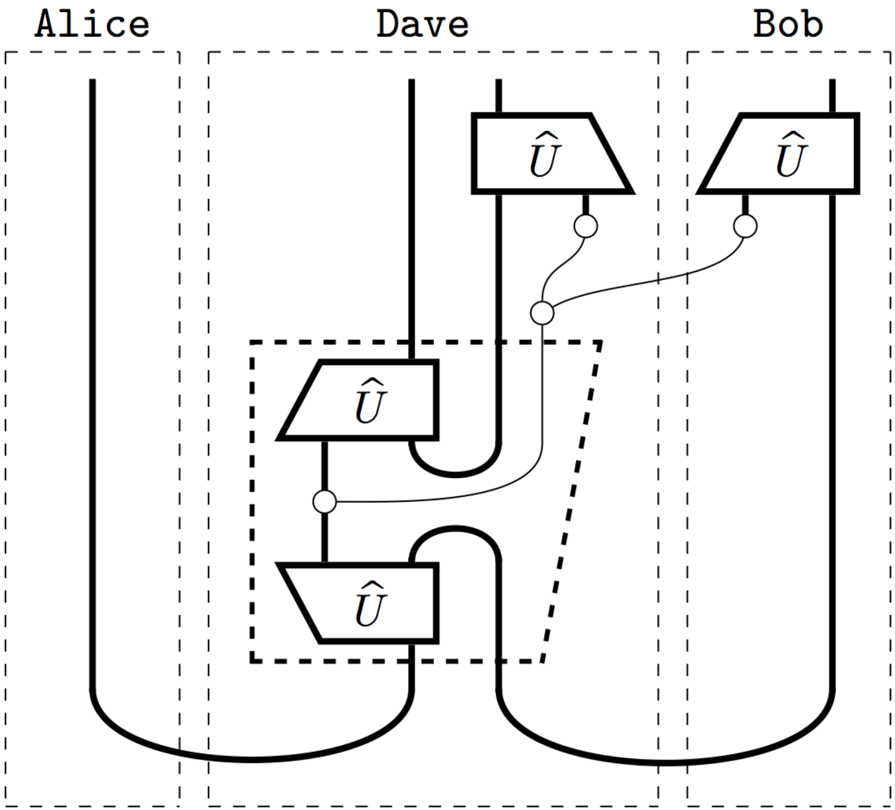
Initial papers:

- S. Abramsky & BC (2014) *A categorical semantics of quantum protocols*. LiCS'04. arXiv:quant-ph/0402130
- BC, M. Sadrzadeh & S. Clark (2010) *Mathematical foundations for a compositional distributional model of meaning*. arXiv:1003.4394

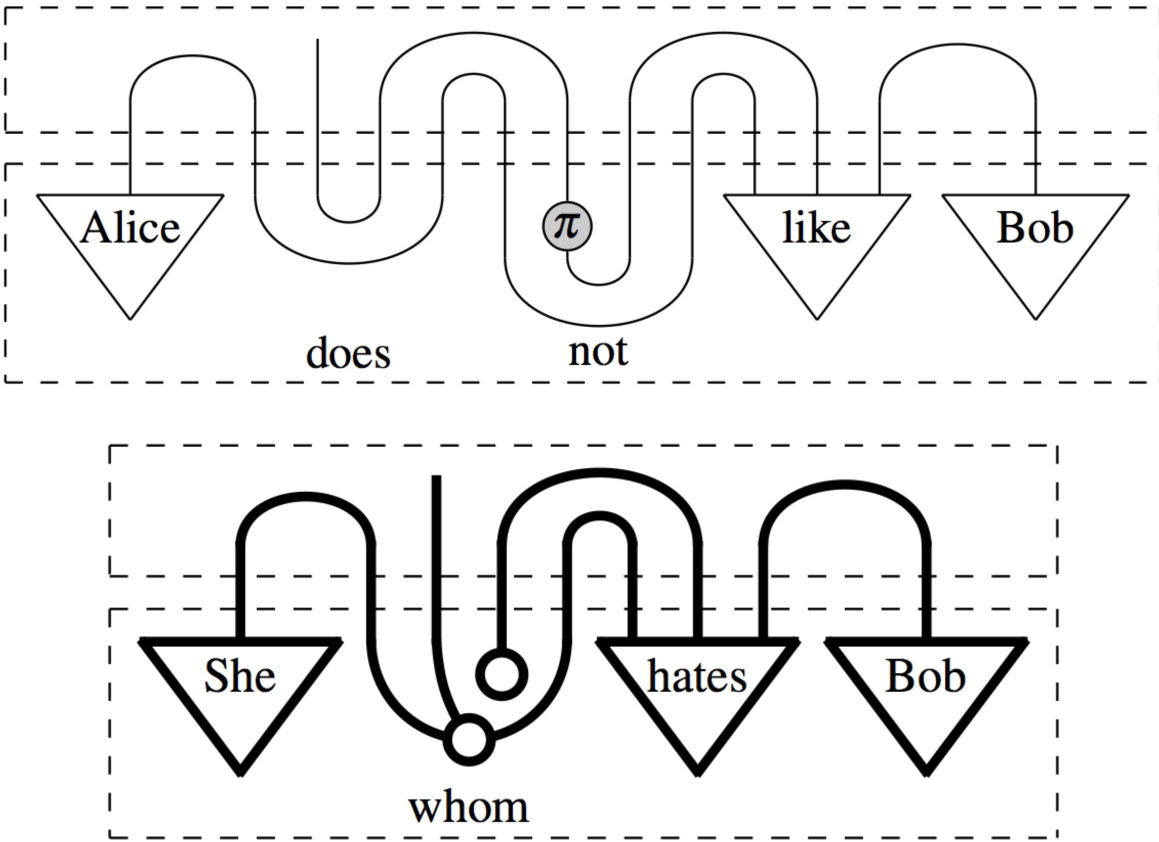
Focus on common structure:

- BC (2012) The logic of quantum mechanics - Take II. arXiv:1204.3458
- S. Clark, BC, E. Grefenstette, S. Pulman & M. Sadrzadeh (2013) *A quantum teleportation inspired algorithm produces sentence meaning from word meaning and grammatical structure*. arXiv:1305.0556

*– e.g. entanglement swapping –*



*– e.g. negation and relative pronouns –*



SCIENTIFIC AMERICAN™

Sign In / Register

Search ScientificAmerican.com

## Quantum Mechanical Words and Mathematical Organisms

By Joselle Kehoe | May 16, 2013 | 10

**FQXI ARTICLE** September 29, 2013

## Video Article: The Quantum Linguist

Bob Coecke has developed a new visual language that could be used to spell out a theory of quantum gravity—and help us understand human speech.

*by Sophie Hebden*

**QUANTUM LINGUISTICS** Leap forward for artificial intelligence

**NewScientist**

**WEEKLY** 11 December 2010

**Also in the scope of the framework:**

- **human/animal behaviour etc.**

Forthcoming paper:

- BC (2014) *From quantum foundations, via meaning in natural language, to a theory of everything*. In: The Incomputable, S. B. Cooper & S. Soskova, Eds. Springer.

**Initial question** (quant-ph/0510032):

**Initial question** (quant-ph/0510032):

**Can QM be formulated in pictures?**

**Initial question** (quant-ph/0510032):

**Can QM be formulated in pictures?**

Same question, put differently:

- **Can QM be formulated in terms of $\otimes$?**

**Initial question** (quant-ph/0510032):

**Can QM be formulated in pictures?**

Same question, put differently:

- **Can QM be formulated in terms of $\otimes$?**

  (contra: $\mathbb{C}$, +, matrices, ...)

**Initial question** (quant-ph/0510032):

**Can QM be formulated in pictures?**

Same question, put differently:

- **Can QM be formulated in terms of $\otimes$?**
  (contra: $\mathbb{C}$, +, matrices, ...)

- **Can QM be formulated in terms of processes?**

**Initial question** (quant-ph/0510032):

**Can QM be formulated in pictures?**

Same question, put differently:

- **Can QM be formulated in terms of $\otimes$?**
  (contra: $\mathbb{C}$, +, matrices, ...)

- **Can QM be formulated in terms of processes?**
  (contra: states, numbers)

**Initial question** (quant-ph/0510032):

**Can QM be formulated in pictures?**

Same question, put differently:

- **Can QM be formulated in terms of $\otimes$?**
  (contra: $\mathbb{C}$, +, matrices, ...)

- **Can QM be formulated in terms of processes?**
  (contra: states, numbers)

- **Does QM have logic features?**

**Initial question** (quant-ph/0510032):

**Can QM be formulated in pictures?**

Same question, put differently:

- **Can QM be formulated in terms of $\otimes$?**

  (contra: $\mathbb{C}$, +, matrices, ...)

- **Can QM be formulated in terms of processes?**

  (contra: states, numbers)

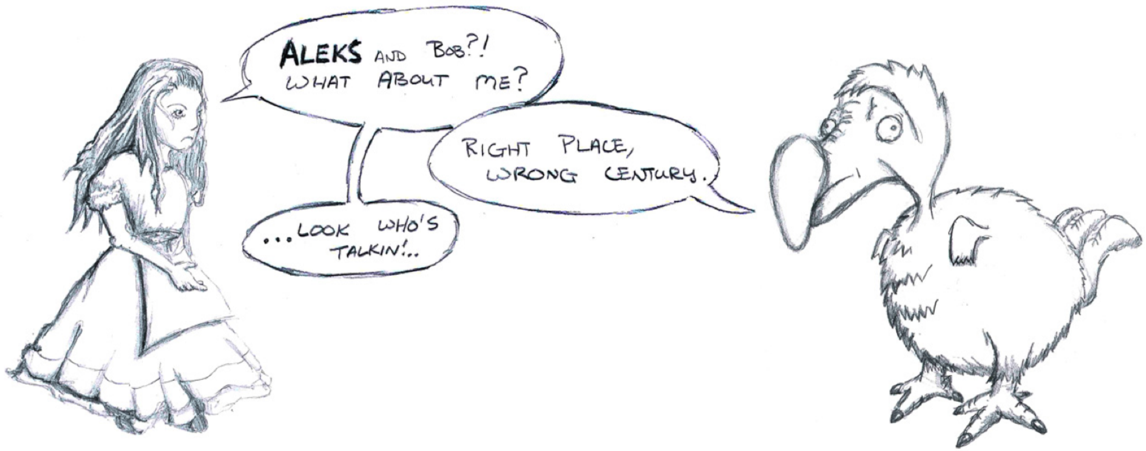
- **Does QM have logic features?**

  (contra: failures)

BC & Aleks Kissinger

*Picturing Quantum Processes*

Cambridge University Press, spring 2015.



## Category-theoretic underpinning:

**Abramsky**, S., and Coecke, B. (2004) A categorical semantics of quantum protocols. LICS. arXiv:quant-ph/0402130.

**Selinger**, P. (2007) Dagger compact closed categories and completely positive maps. ENTCS.

Coecke, B., and **Pavlovic**, D. (2007) Quantum measurements without sums. In: Mathematics of Quantum Computing and Technology. Taylor and Francis. arXiv:quant-ph/0608035

Coecke, B., and **Duncan**, R. (2008) Interacting quantum observables. ICALP'08 & NJP'10. arXiv:quant- ph/09064725

Coecke, B., **Paquette**, E. O., and Pavlovic, D. (2010) Classical and quantum structuralism. In: Semantic Techniques in Quantum Computation. CUP. arXiv:0904.1997

**Chiribella**, G., **D'Ariano**, G. M., and **Perinotti**, P. (2010) Probabilistic theories with purification. Physical Review. arXiv:0908.1583

## . . . mainly borrowing from Australians:

**Kelly**, M. (1972) Many-variable functorial calculus I. LNM.

**Carboni**, A., and **Walters**, R. F. C. (1980) Cartesian bicategories I. JPAA.

**Joyal**, A., and **Street**, R. (1991) The geometry of tensor calculus I. AM.

**Lack**, S. (2004) Composing PROPs. TAC.

## New structural theorems:

**Selinger**, P. (2011) Finite dimensional Hilbert spaces are complete for dagger compact closed categories. ENTCS.

Coecke, B., **Pavlovic**, D., and **Vicary**, J. (2011) A new description of orthogonal bases. MSCS. arXiv:quant-ph/0810.1037

**Backens**, M. (2013) The ZX-calculus is complete for stabilizer quantum mechanics. arXiv:1307.7025.

**Kissinger**, A. (2014) Finite matrices are complete for (dagger-)multigraph categories. arXiv:1406.5942.

# — Ch. 1 – Processes as diagrams —

*Philosophy [i.e. physics] is written in this grand book—I mean the universe—which stands continually open to our gaze, but it cannot be understood unless one first learns to comprehend the language and interpret the characters in which it is written. It is written in the language of mathematics, and its characters are triangles, circles, and other geometrical figures, without which it is humanly impossible to understand a single word of it; without these, one is wandering around in a dark labyrinth.*

— Galileo Galilei, "Il Saggiatore", 1623.

---

Here we introduce:

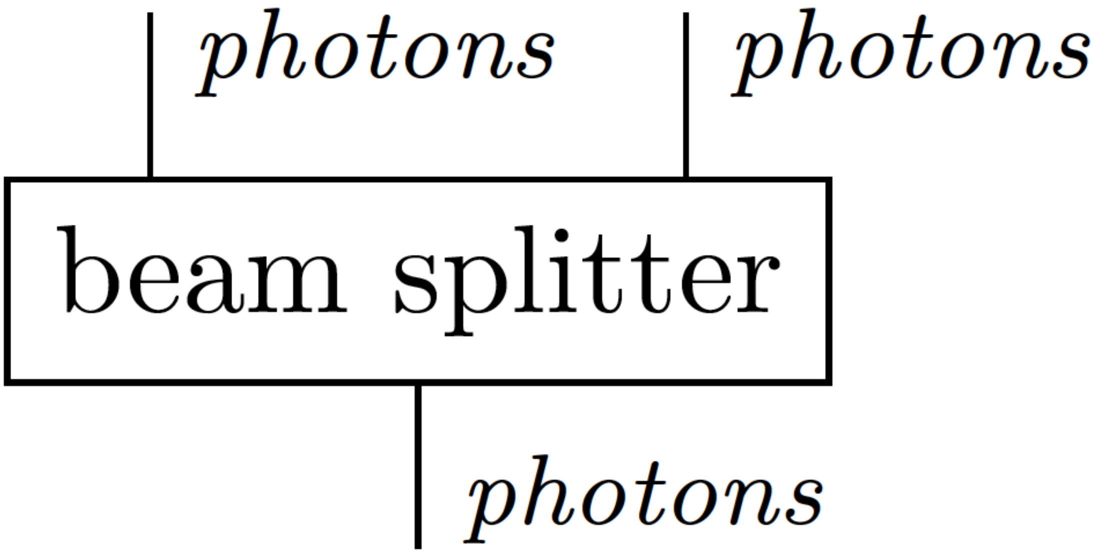
- diagrams
- process theories
- (boring) circuit diagrams

# — Ch. 1 – Processes as diagrams —

*– processes as boxes and systems as wires –*

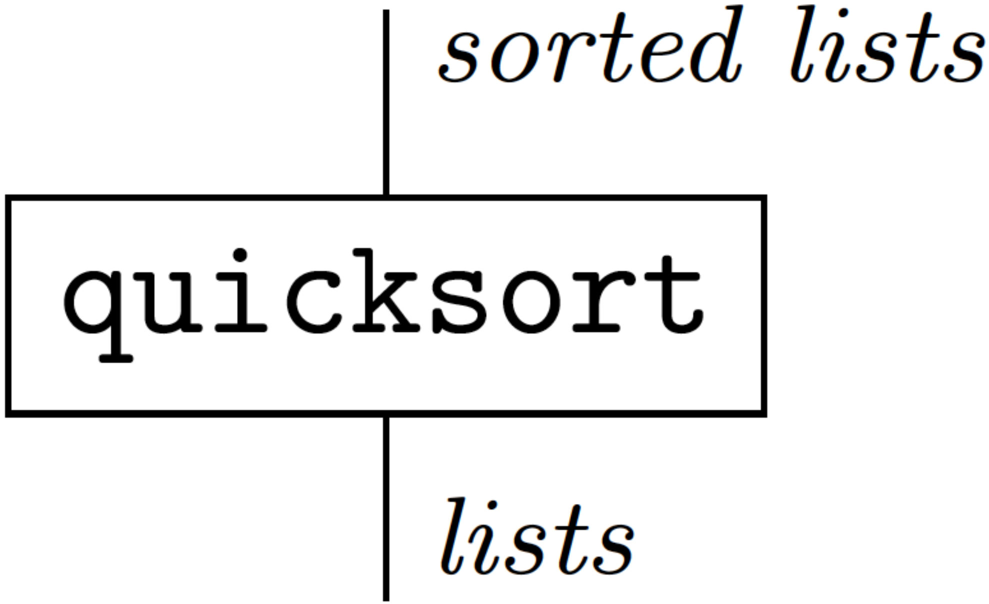
# — Ch. 1 – Processes as diagrams —

*– processes as boxes and systems as wires –*



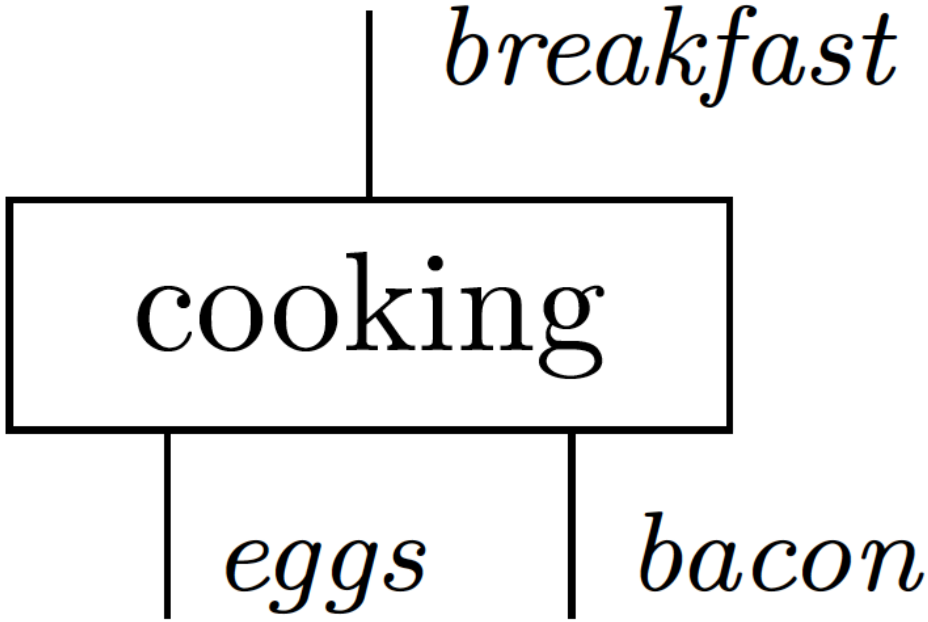
# — Ch. 1 – Processes as diagrams —

*– processes as boxes and systems as wires –*



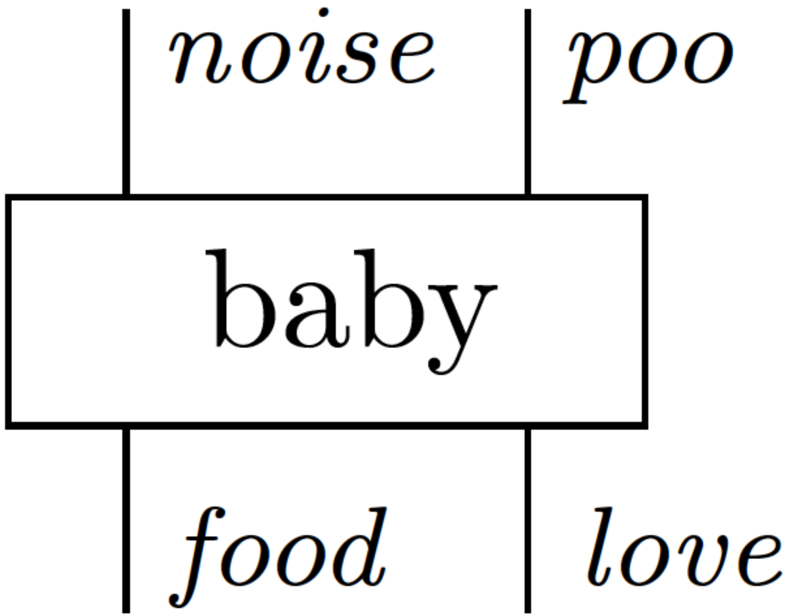
# — Ch. 1 – Processes as diagrams —

*– processes as boxes and systems as wires –*



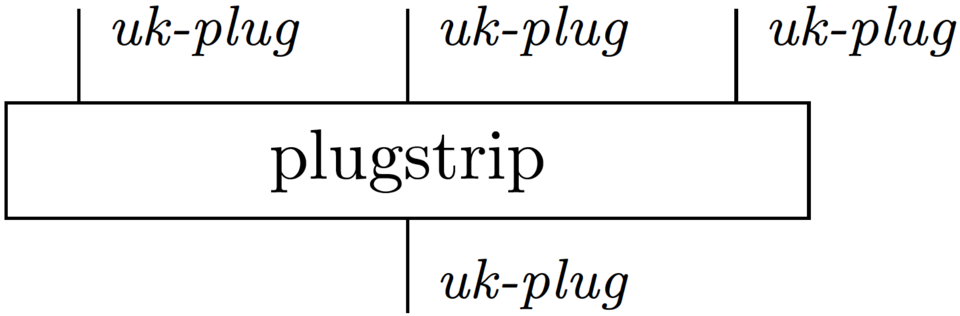
# — Ch. 1 – Processes as diagrams —

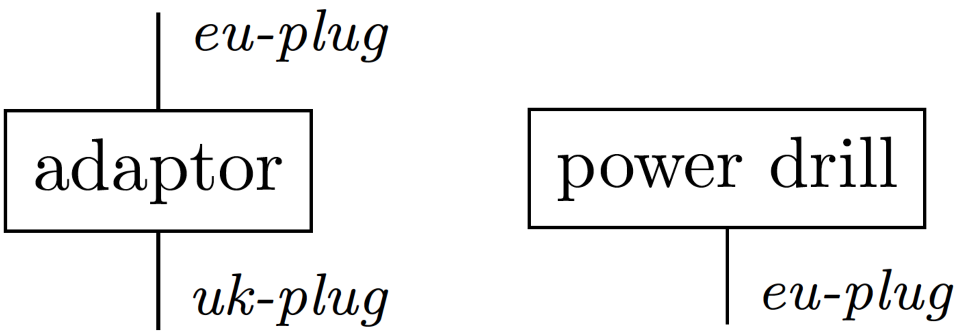
*– processes as boxes and systems as wires –*



## — Ch. 1 – Processes as diagrams —

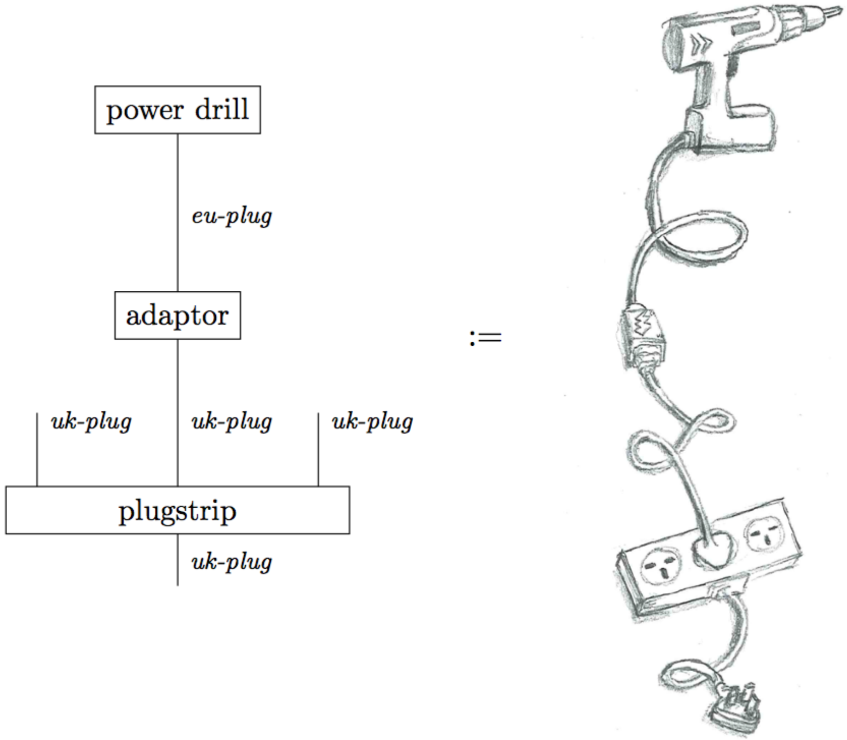
*– composing processes –*



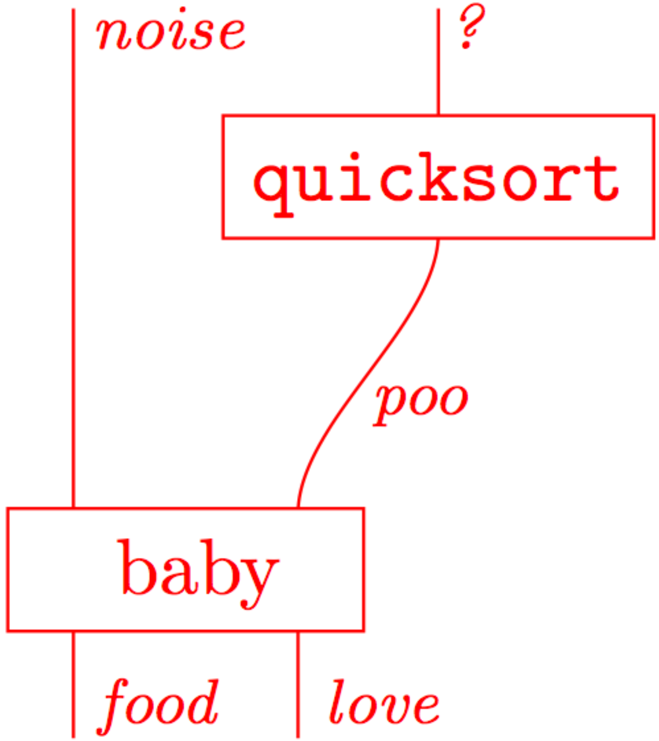
# — Ch. 1 – Processes as diagrams —

## *– composing processes –*



# — Ch. 1 – Processes as diagrams —

*– composing processes –*



## — Ch. 1 – Processes as diagrams —

*– process theory –*

... consists of:

- **set of systems** $S$
- **set of processes** $P$

which are:

- **closed under forming diagrams.**

## — Ch. 1 – Processes as diagrams —

*– process theory –*

... consists of:

- **set of systems** $S$
- **set of processes** $P$

which are:

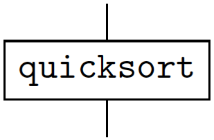
- **closed under forming diagrams.**

It tells us:

- how to *interpret* boxes and wires,
- and hence, when two diagrams are equal.

# — Ch. 1 – Processes as diagrams —

*– process theory –*

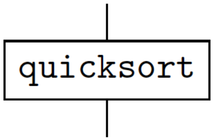



:=

```
qs [] = []
qs (x :: xs) =
  qs [y | y <- xs; y < x] ++ [x] ++
  qs [y | y <- xs; y >= x]
```

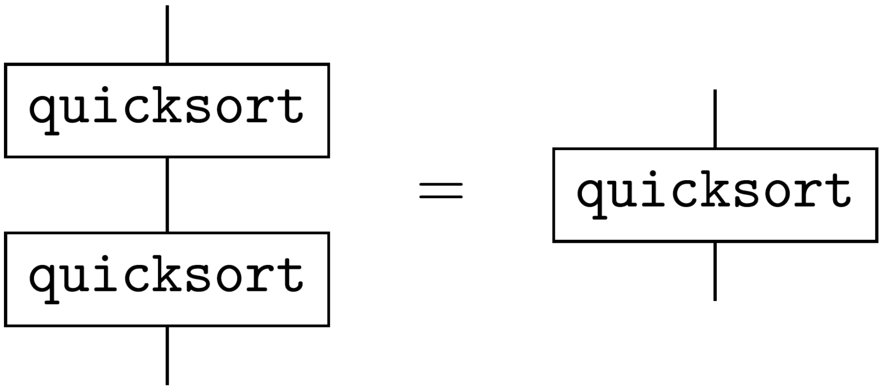
# — Ch. 1 – Processes as diagrams —

*– process theory –*




:=

```
qs [] = []
qs (x :: xs) =
   qs [y | y <- xs; y < x] ++ [x] ++
   qs [y | y <- xs; y >= x]
```



# Candidate systems:

**Candidate systems:**

- vector space with inner-product:
  - pure (or closed) quantum states (complex)
  - standard natural language processing (real)

**Candidate systems:**

- vector space with inner-product:
  - pure (or closed) quantum states (complex)
  - standard natural language processing (real)
- density matrices with trace:
  - mixed (or open) quantum states
  - neo natural language processing

**Candidate systems:**

- vector space with inner-product:
  - pure (or closed) quantum states (complex)
  - standard natural language processing (real)
- density matrices with trace:
  - mixed (or open) quantum states
  - neo natural language processing
- more abstract models and constructions

Vector space model of word meaning in NLP:

- vector space spanned by context words
- meaning vectors from relative occurrences
- similarity from inner product

Source: huge corpus

Pioneer:


- H. Schuetze (1998) *Automatic word sense discrimination*. Computational Linguistics, **24**, 97123.

Vector space model of social properties:

- vector space spanned by context words
- meaning vectors from relative occurrences
- similarity from inner product

Source: Facebook, personal page, ...

Pioneer:


- H. Schuetze (1998) *Automatic word sense discrimination*. Computational Linguistics, **24**, 97123.

# — Ch. 2 – String diagrams —

*When two systems, of which we know the states by their respective representatives, enter into temporary physical interaction due to known forces between them, and when after a time of mutual influence the systems separate again, then they can no longer be described in the same way as before, viz. by endowing each of them with a representative of its own.* ***I would <u>not</u> call that <u>one</u> <u>but</u> rather <u>the</u> characteristic trait of quantum mechanics****, the one that enforces its entire departure from classical lines of thought.*

— Erwin Schrödinger, 1935.

---

Here we introduce:

- string diagrams
- transposes and adjoints
- quantum phenomena in great generality

# — Ch. 2 – String diagrams —

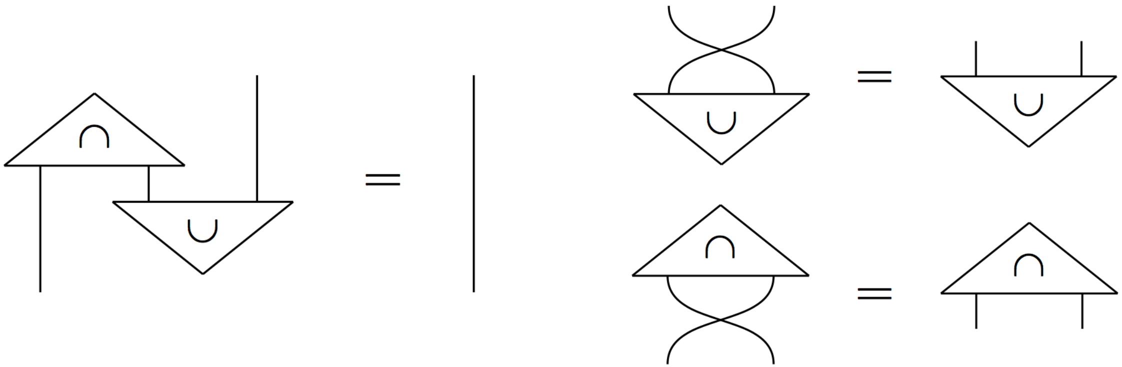
## – *TFAE* –

# — Ch. 2 – String diagrams —

## – *TFAE* –

**1.** ‘Circuits’ with cup-state and cup-effect:

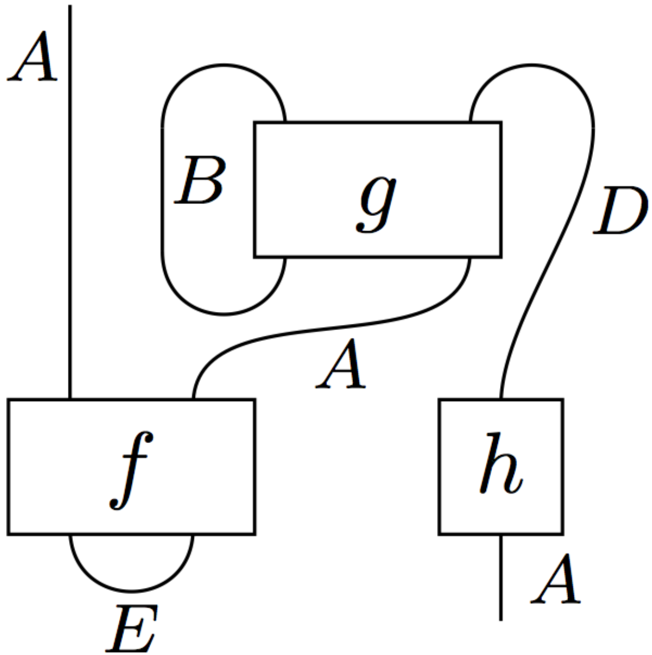
which satisfy:



# — Ch. 2 – String diagrams —

## – *TFAE* –

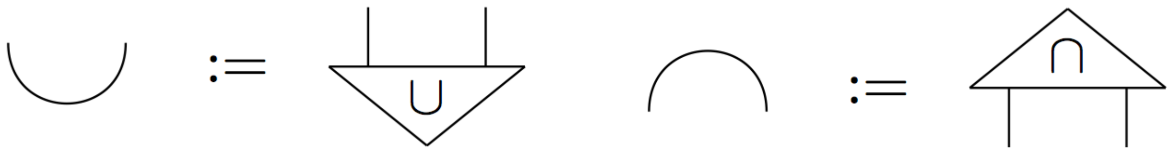
**2.** diagrams allowing in-in, out-out and out-in wiring:



# — Ch. 2 – String diagrams —

## – *TFAE* –

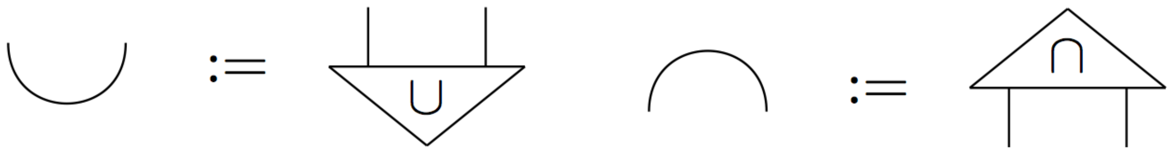
From **1.** to **2.**:



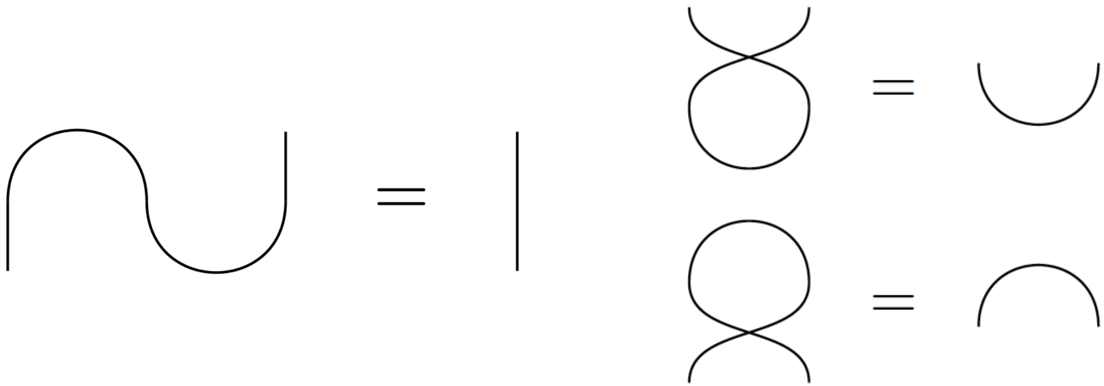
# — Ch. 2 – String diagrams —

## *– TFAE –*

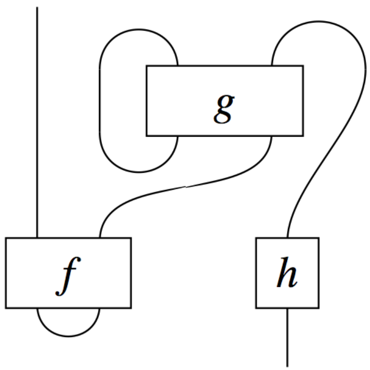
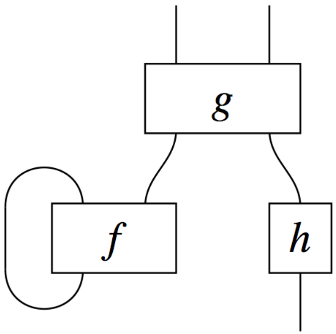
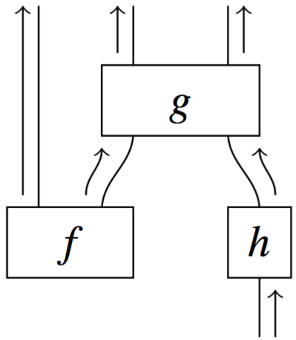
From **1.** to **2.**:



so that:



# Symmetric monoidal categories as diagrams:

| compact closed | traced | plain |
| --- | --- | --- |
|  |  |  |
| string diagrams | diagrams | cicuits |
| no ins/outs | outs to ins | causal structure |

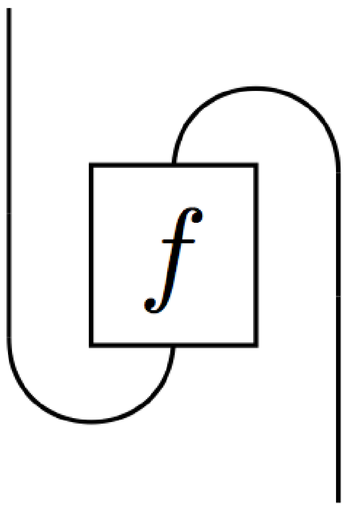
# — Ch. 2 – String diagrams —

*– transpose –*

# — Ch. 2 – String diagrams —

*– transpose –*

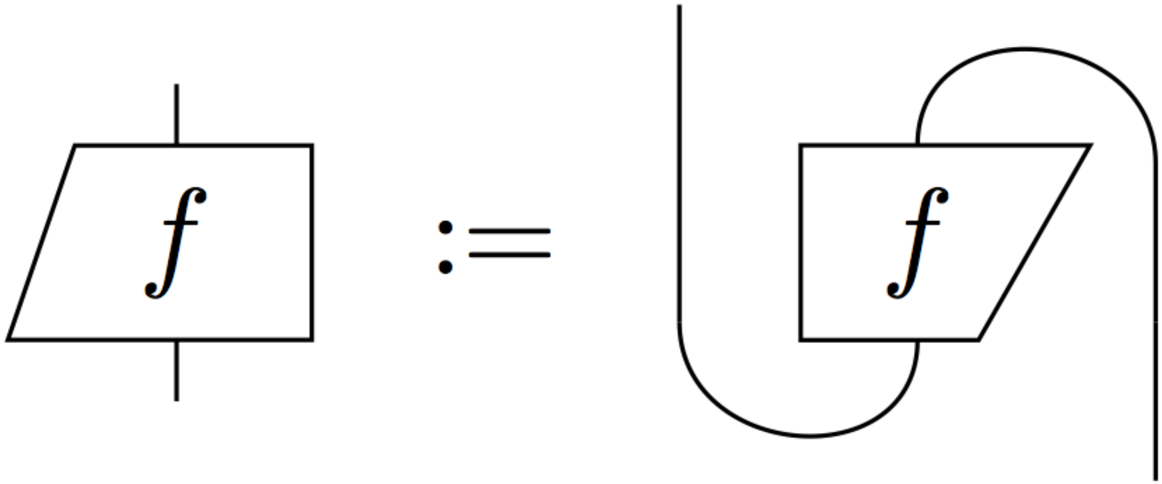
... :=



# — Ch. 2 – String diagrams —

*– transpose –*

Clever new notation:

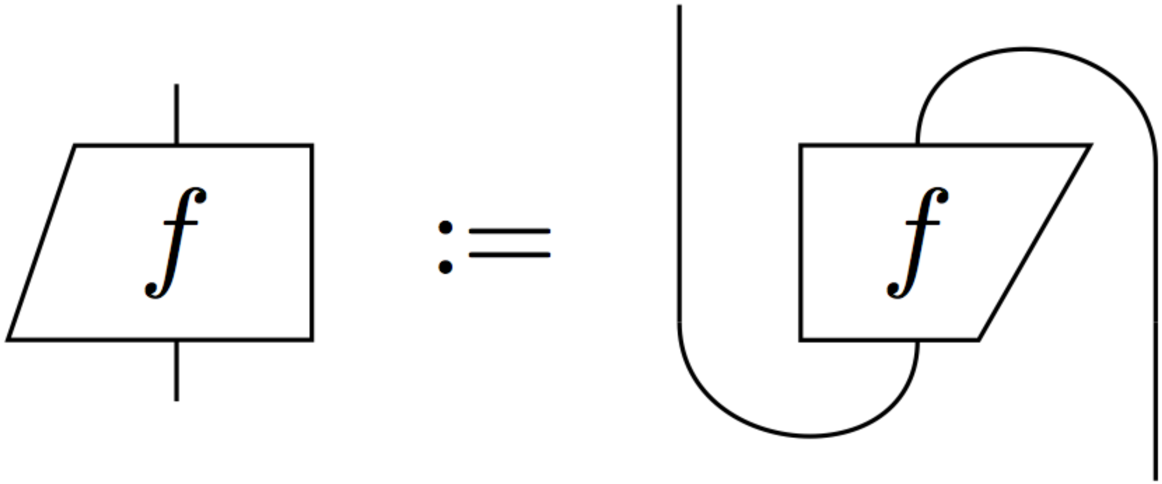



(intuition: )

# — Ch. 2 – String diagrams —

*– transpose –*

Clever new notation:

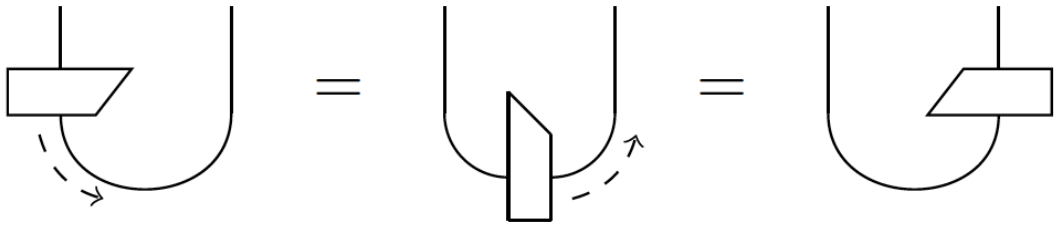



(intuition: again yanking the wire)

# — Ch. 2 – String diagrams —

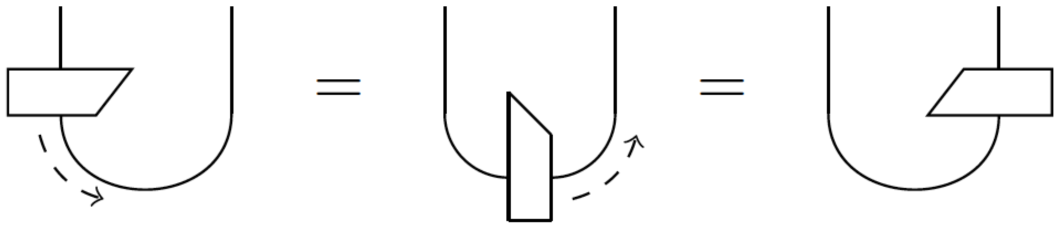
*– transpose –*

**Prop.** Sliding:

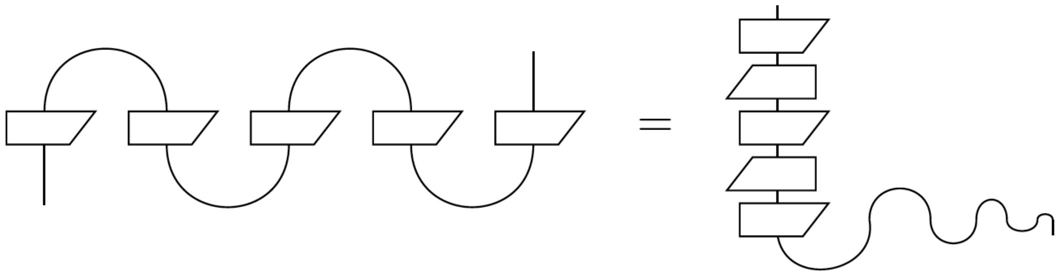


*– transpose –*

**Prop.** Sliding:



... so this is a mathematical equation:



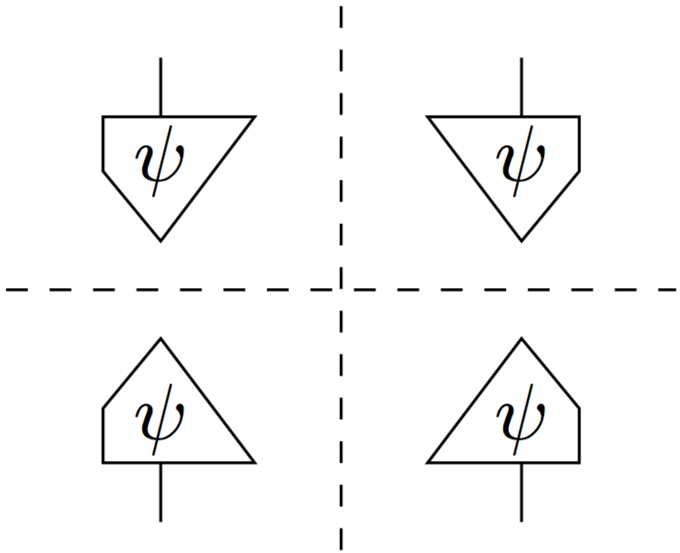
# — Ch. 2 – String diagrams —

*– adjoint & conjugate –*

# — Ch. 2 – String diagrams —

*– adjoint & conjugate –*

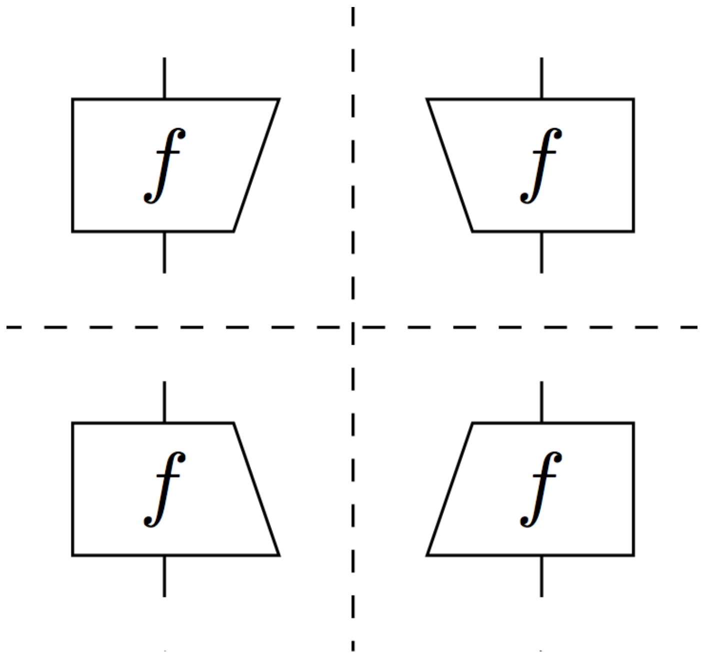
From a state to its test:



# — Ch. 2 – String diagrams —

## *– adjoint & conjugate –*

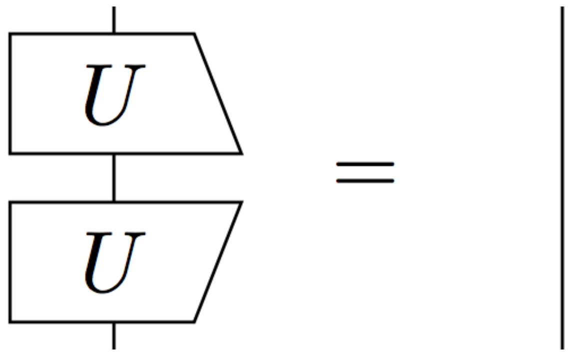
From a process to its adjoint:



# — Ch. 2 – String diagrams —

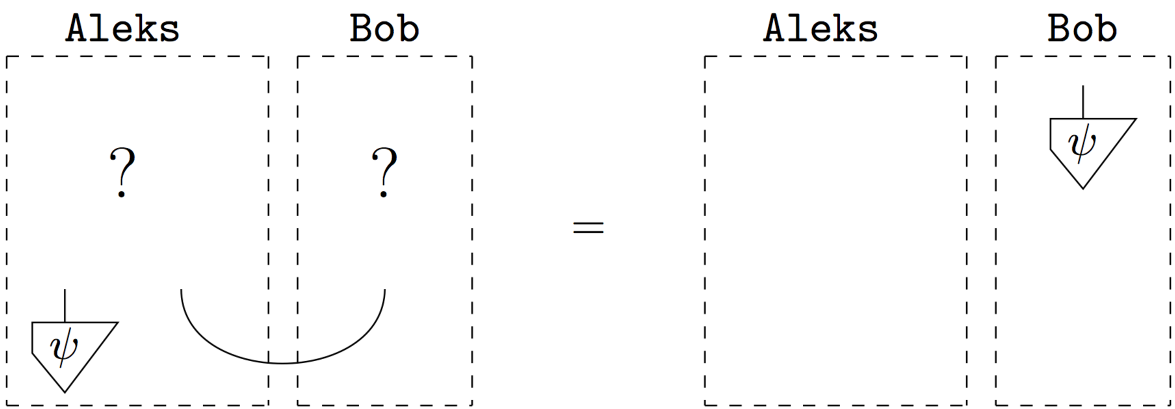
*– adjoint & conjugate –*

Unitarity/isometry :=



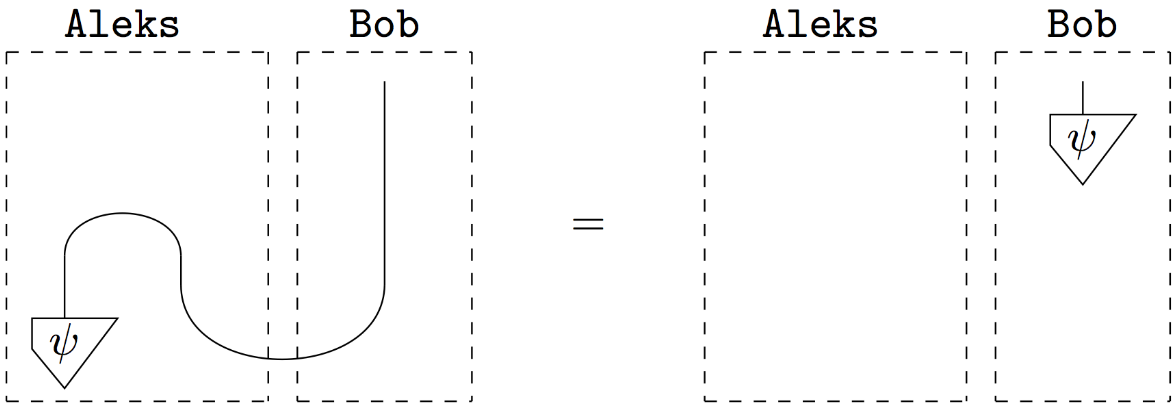
# — Ch. 2 – String diagrams —

## *– quantum teleportation –*



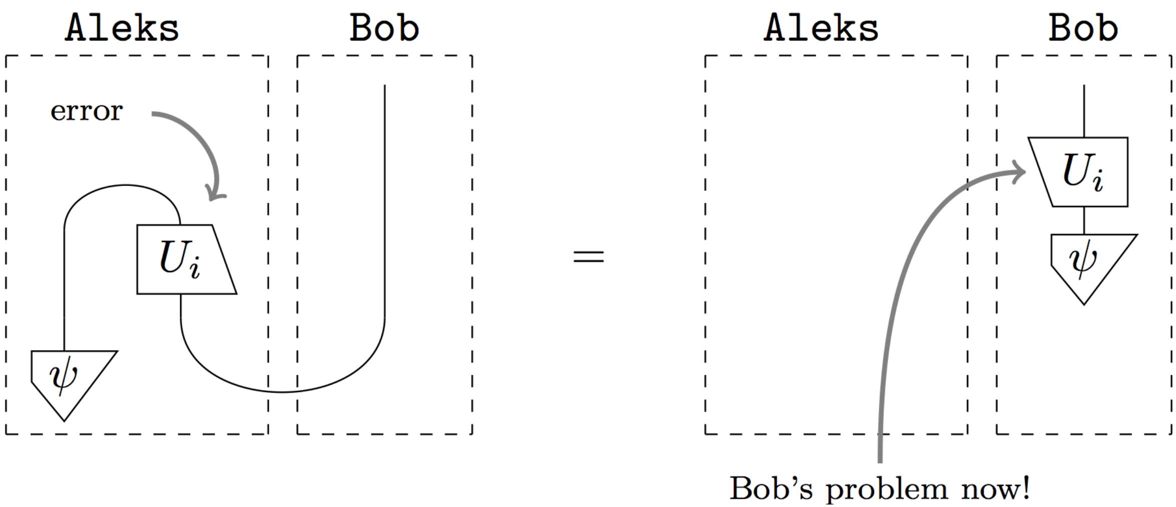
# — Ch. 2 – String diagrams —

*– quantum teleportation –*



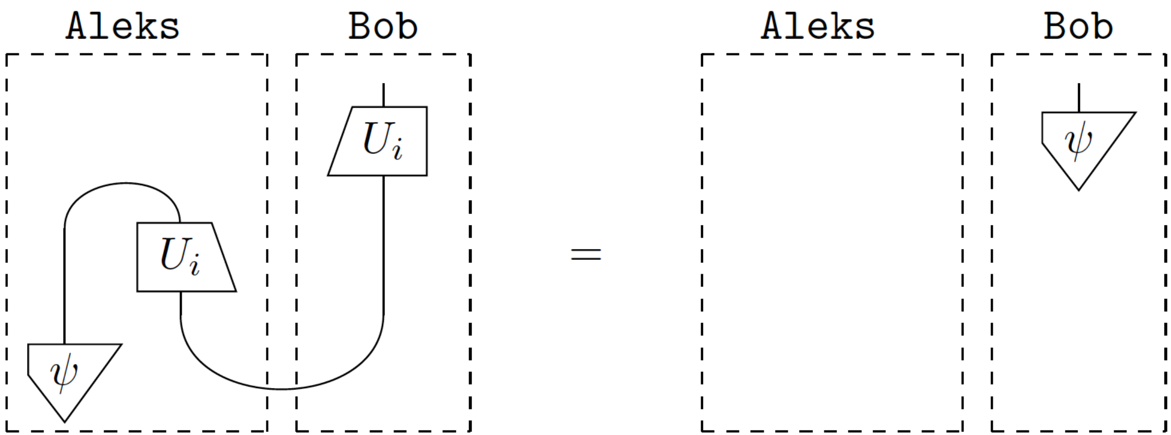
## — Ch. 2 – String diagrams —

*– quantum teleportation –*



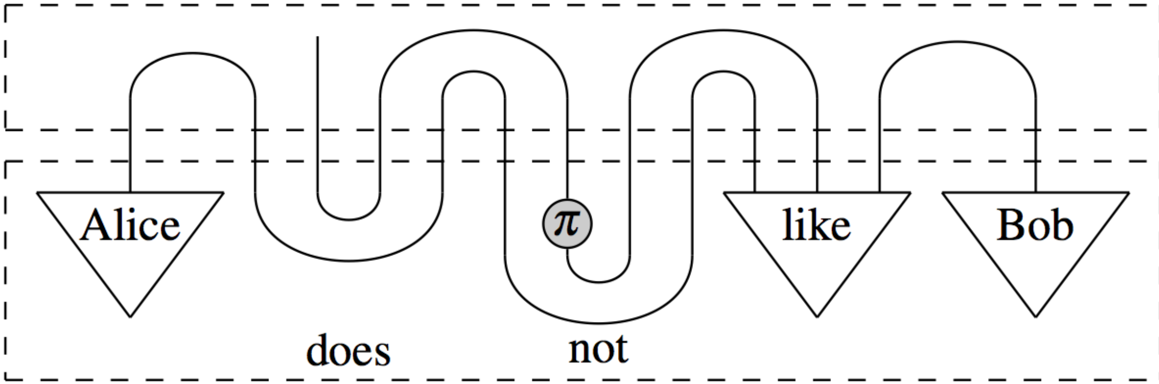
# — Ch. 2 – String diagrams —

*– quantum teleportation –*



# String diagrams for natural language meaning:

**String diagrams for natural language meaning:**




- Top part: grammar
- Bottom part: meaning vectors

**Lambek's Residuated monoids (1950's):**

$$b \leq a \multimap c \Leftrightarrow a \cdot b \leq c \Leftrightarrow a \leq c \multimapinv b$$

or equivalently,

$$a \cdot (a \multimap c) \leq c \leq a \multimap (a \cdot c)$$

$$(c \multimapinv b) \cdot b \leq c \leq (c \cdot b) \multimapinv b$$

**Lambek's Pregroups (2000's):**

$$a \cdot {}^{-1}a \leq 1 \leq {}^{-1}a \cdot a$$

$$b^{-1} \cdot b \leq 1 \leq b \cdot b^{-1}$$

**From grammar to meaning:**

For noun type $n$, verb type is ${}^{-1}n \cdot s \cdot n^{-1}$, so:

**From grammar to meaning:**

For noun type $n$, verb type is ${}^{-1}n \cdot s \cdot n^{-1}$, so:

$$n \cdot {}^{-1}n \cdot s \cdot n^{-1} \cdot n$$

**From grammar to meaning:**

For noun type $n$, verb type is ${}^{-1}n \cdot s \cdot n^{-1}$, so:

$$n \cdot {}^{-1}n \cdot s \cdot n^{-1} \cdot n \leq 1 \cdot s \cdot 1$$

**From grammar to meaning:**

For noun type $n$, verb type is ${}^{-1}n \cdot s \cdot n^{-1}$, so:

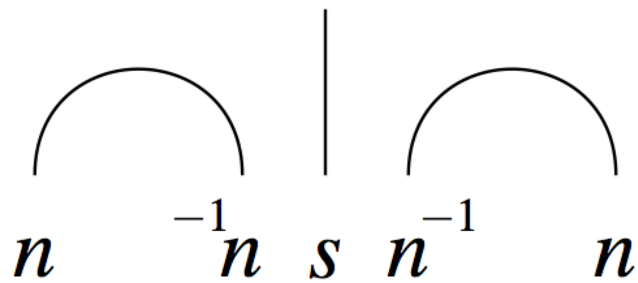
$$n \cdot {}^{-1}n \cdot s \cdot n^{-1} \cdot n \leq 1 \cdot s \cdot 1 \leq s$$

**From grammar to meaning:**

For noun type $n$, verb type is ${}^{-1}n \cdot s \cdot n^{-1}$, so:

$$n \cdot {}^{-1}n \cdot s \cdot n^{-1} \cdot n \leq 1 \cdot s \cdot 1 \leq s$$

**Diagrammatic type reduction:**

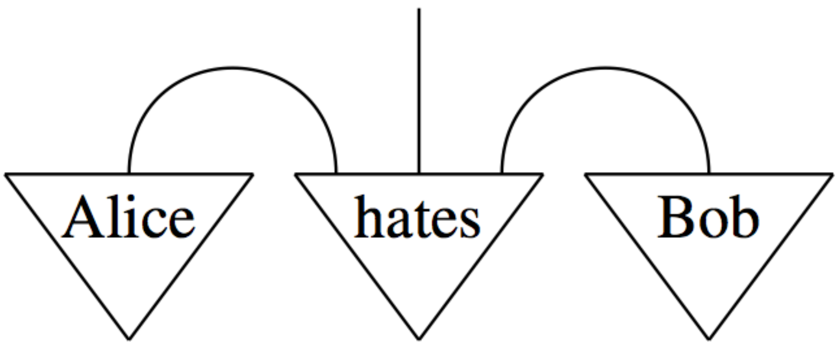


**From grammar to meaning:**

For noun type $n$, verb type is ${}^{-1}n \cdot s \cdot n^{-1}$, so:

$$n \cdot {}^{-1}n \cdot s \cdot n^{-1} \cdot n \leq 1 \cdot s \cdot 1 \leq s$$

**Diagrammatic type reduction:**



**Algorithm for meaning composition:**

1. Perform grammatical type reduction:

$$(\textit{word type } 1)\ldots(\textit{word type } n) \;\rightsquigarrow\; \textit{sentence type}$$

2. Interpret diagrammatic type reduction as linear map:

$f ::$  $\mapsto \left(\sum_i \langle ii|\right) \otimes \mathrm{id} \otimes \left(\sum_i \langle ii|\right)$

3. Apply this map to tensor of word meaning vectors:

$$f\left(\overrightarrow{v}_1 \otimes \ldots \otimes \overrightarrow{v}_n\right)$$

# Algorithm for meaning composition:

| Model | $\rho$ with cos | $\rho$ with Eucl. |
|---|---|---|
| Verbs only | 0.329 | 0.138 |
| Additive | 0.234 | 0.142 |
| Multiplicative | 0.095 | 0.024 |
| Relational | 0.400 | 0.149 |
| Rank-1 approx. of relational | 0.402 | 0.149 |
| Separable | 0.401 | 0.090 |
| Copy-subject | 0.379 | 0.115 |
| Copy-object | 0.381 | 0.094 |
| Frobenius additive | **0.405** | 0.125 |
| Frobenius multiplicative | 0.338 | 0.034 |
| Frobenius tensored | **0.415** | 0.010 |
| Human agreement | 0.60 | |

Dimitri Kartsaklis & Mehrnoosh Sadrzadeh (2013) *Prior Disambiguation of Word Tensors for Constructing Sentence Vectors*. In EMNLP'13.

**Algorithm for meaning composition:**

1. Perform grammatical type reduction:

$$(\textit{word type } 1)\ldots(\textit{word type } n) \ \rightsquigarrow \ \textit{sentence type}$$

2. Interpret diagrammatic type reduction as linear map:

$f ::$  $\mapsto \left(\sum_i \langle ii|\right) \otimes \mathrm{id} \otimes \left(\sum_i \langle ii|\right)$

3. Apply this map to tensor of word meaning vectors:

$$f\left(\vec{v}_1 \otimes \ldots \otimes \vec{v}_n\right)$$

**Algorithm for social behaviour composition:**

1. Perform social type reduction:

$$(person\ type\ 1)\ldots(person\ type\ n) \rightsquigarrow group\ type$$

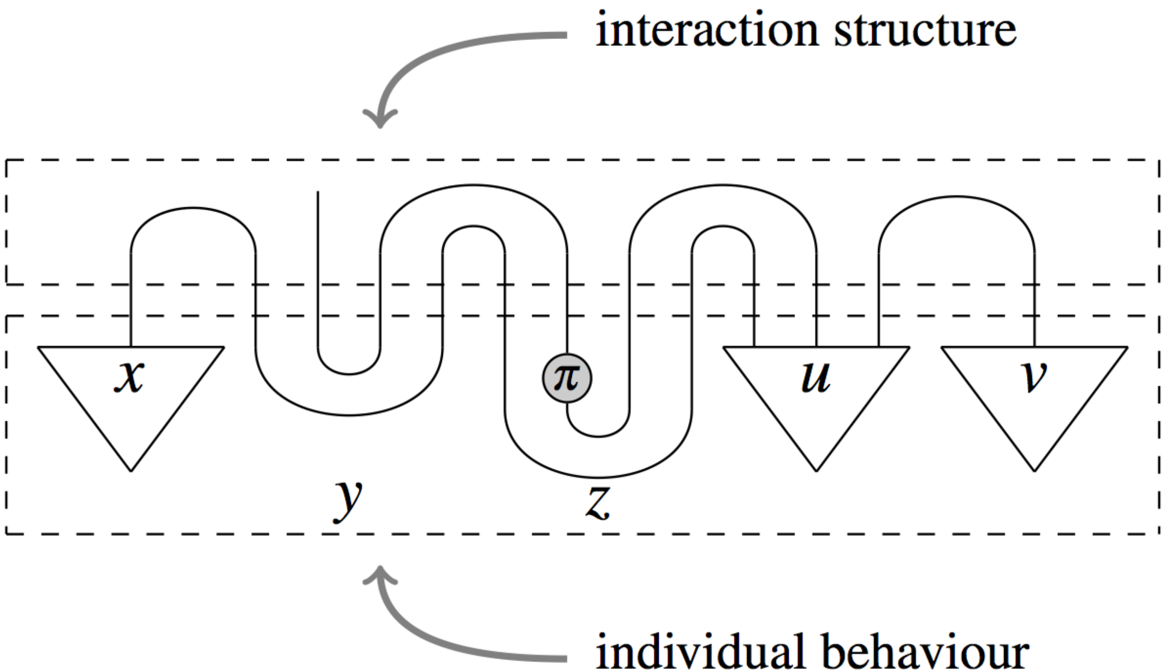
2. Interpret diagrammatic type reduction as linear map:

$f ::$  $\mapsto \left(\sum_i \langle ii|\right) \otimes \mathrm{id} \otimes \left(\sum_i \langle ii|\right)$

3. Apply this map to tensor of word meaning vectors:

$$f\left(\overrightarrow{v}_1 \otimes \ldots \otimes \overrightarrow{v}_n\right)$$

**speaking about ≡ reasoning about**



# — Ch. 3 – Hilbert space from diagrams —

*I would like to make a confession which may seem immoral: **I do not believe absolutely in Hilbert space any more.***

— John von Neumann, letter to Garrett Birkhoff, 1935.

---

Here we define for string diagrams:

- ONBs, matrices and sums
- (multi-)linear maps & Hilbert spaces

## — Ch. 3 – Hilbert space from diagrams —

*– completeness –*

**THM.** (Selinger, 2008)

An equation between string diagrams holds, if and only if it holds for Hilbert spaces and linear maps.

I.e. defining Hilbert spaces and linear maps in this manner is a ‘conservative extension’ of string diagrams.

# — Ch. 4 – Quantum processes —

*The art of progress is to preserve order amid change, and to preserve change amid order.*

— Alfred North Whitehead, Process and Reality, 1929.

---

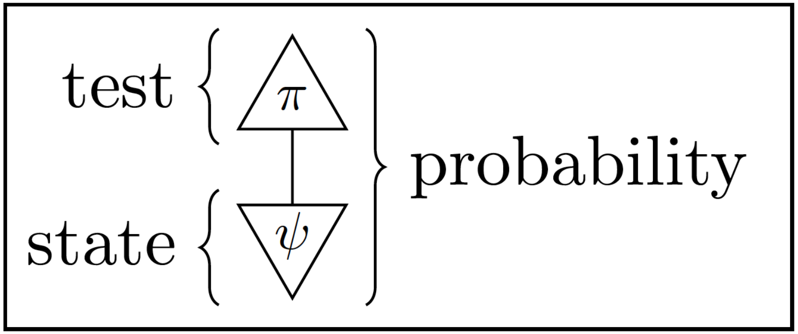
Here we introduce:

- pure quantum maps
- general quantum maps
- causality, no-signalling & Stinespring dilation

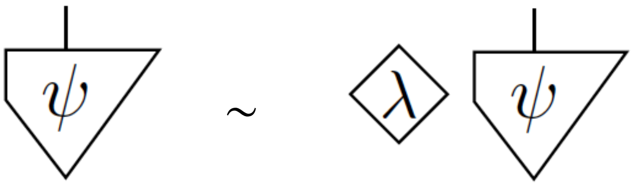
# — Ch. 4 – Quantum processes —

*– pure quantum maps –*

Goal 1:



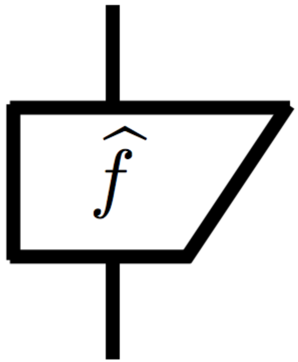

Goal 2:



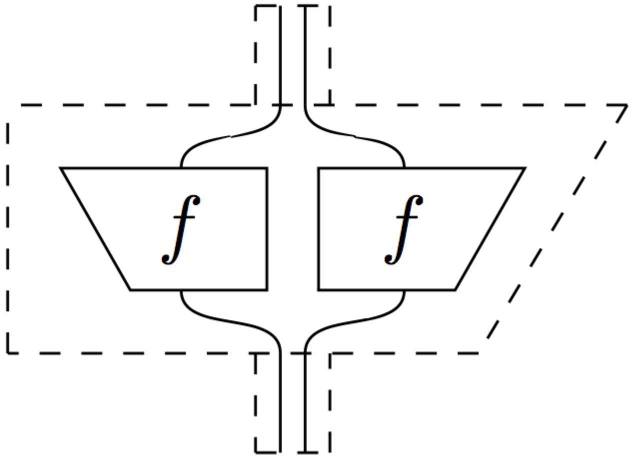
# — Ch. 4 – Quantum processes —

*– pure quantum maps –*

... :=



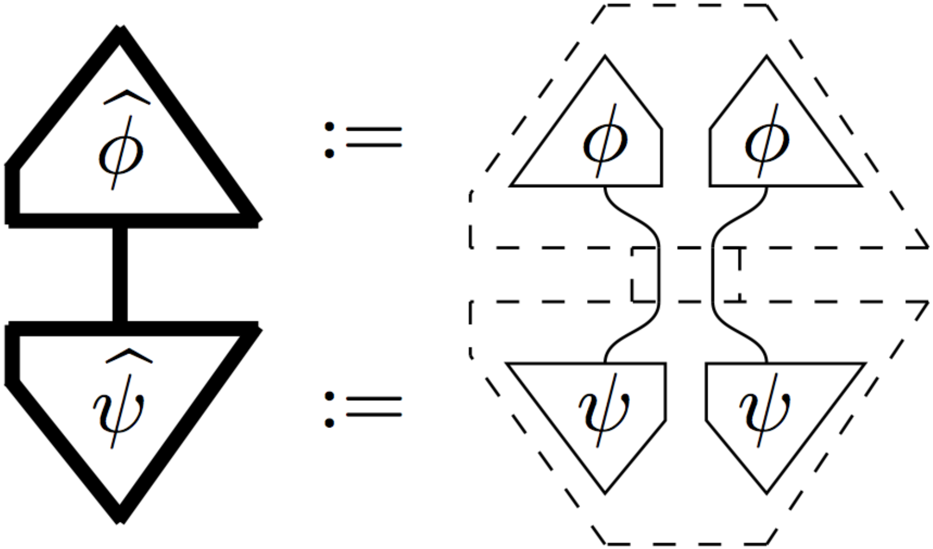
$:=$



## — Ch. 4 – Quantum processes —

*– pure quantum maps –*

Born-rule :=



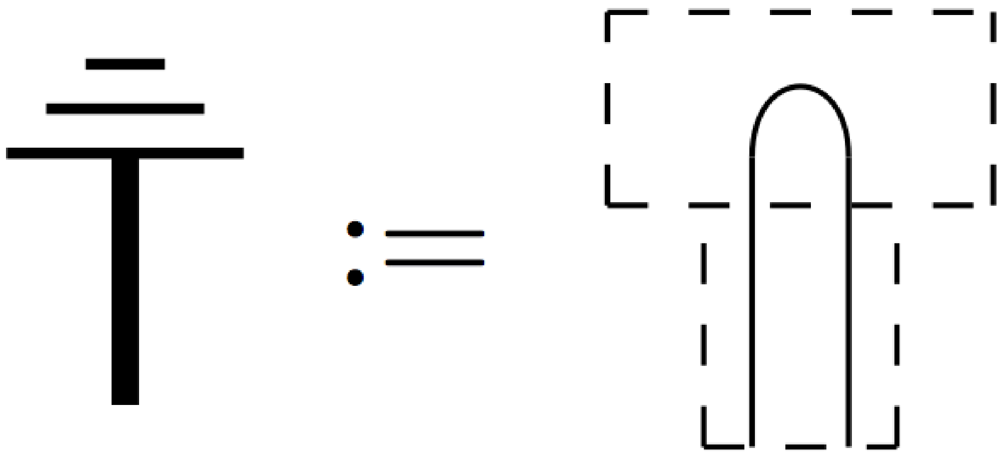
# — Ch. 4 – Quantum processes —

*– quantum maps –*

# — Ch. 4 – Quantum processes —

*– quantum maps –*

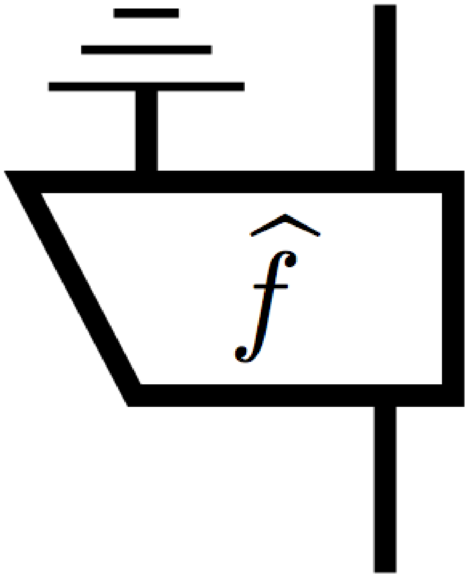
Discarding :=



## — Ch. 4 – Quantum processes —

*– quantum maps –*

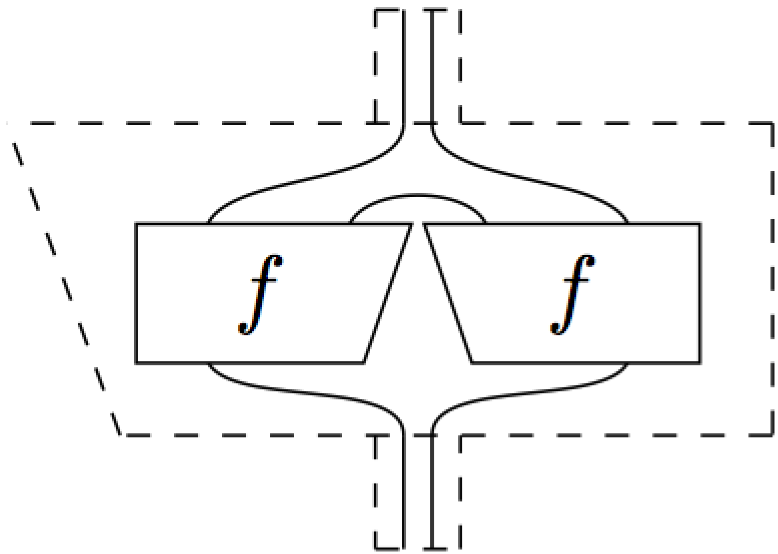
... := pure quantum maps + discarding



# — Ch. 4 – Quantum processes —

*– quantum maps –*

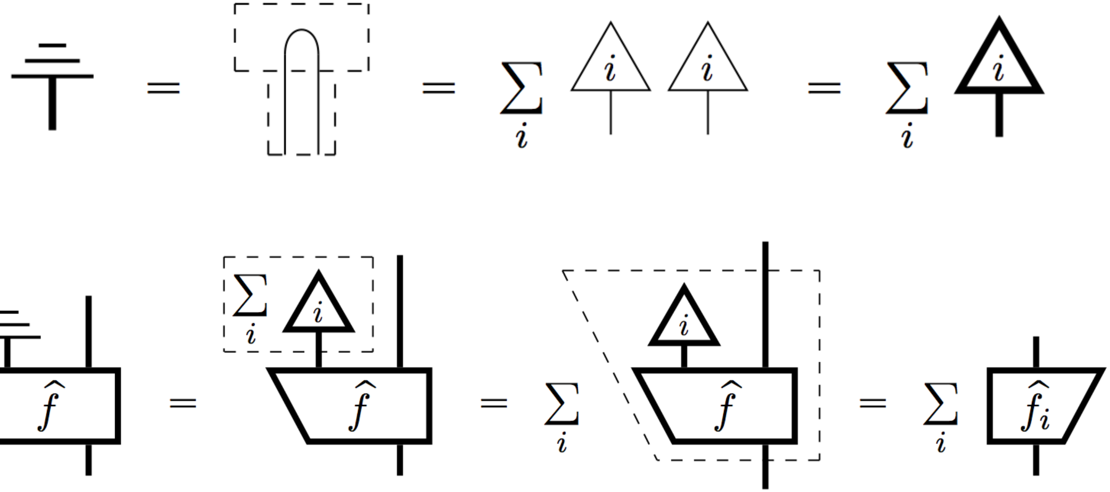
... := pure quantum maps + discarding



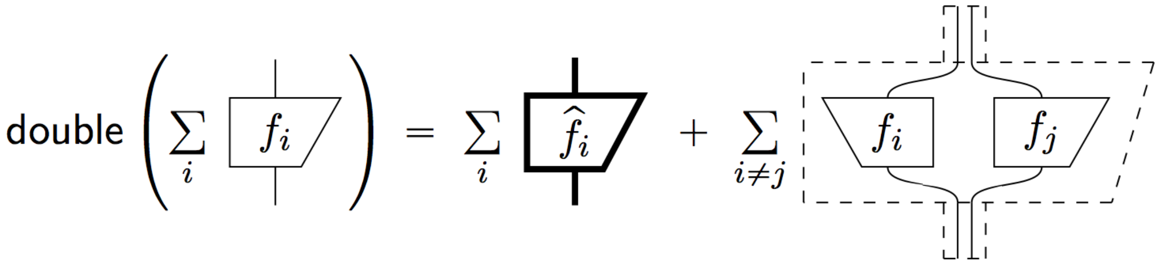
*Candidate systems:*

- *vector space with inner-product*:
  - *pure (or closed) quantum states (complex)*
  - *standard natural language processing (real)*
- **density matrices with trace:**
  - **mixed (or open) quantum states**
  - **neo natural language processing**
- *more abstract models and constructions*

# Mixing:



# Two distinct sums:



Advantages over vector spaces of meaning:

- Ambiguity:
  - Robin Piedeleu's MSc thesis (2014)
  - Dimitri Kartsaklis's PhD thesis (2014)

Advantages over vector spaces of meaning:

- Ambiguity:
  - Robin Piedeleu's MSc thesis (2014)
  - Dimitri Kartsaklis's PhD thesis (2014)
- Information/propositional content:
  - Esma Balkir's MSc thesis (2014)

Advantages over vector spaces of meaning:

- Ambiguity:
  - Robin Piedeleu's MSc thesis (2014)
  - Dimitri Kartsaklis's PhD thesis (2014)
- Information/propositional content:
  - Esma Balkir's MSc thesis (2014)
- Construction can be iterated

## — Ch. 6 – Picturing classical processes —

*Damn it! I knew she was a monster! John! Amy! Listen! Guard your buttholes.*

— David Wong, This Book Is Full of Spiders, 2012.

---

Here we fully diagrammatically describe:

- classical-quantum processes
- classical data as spiders
- fully diagrammatic protocols

## — Ch. 6 – Picturing classical processes —

*– classical-quantum maps –*

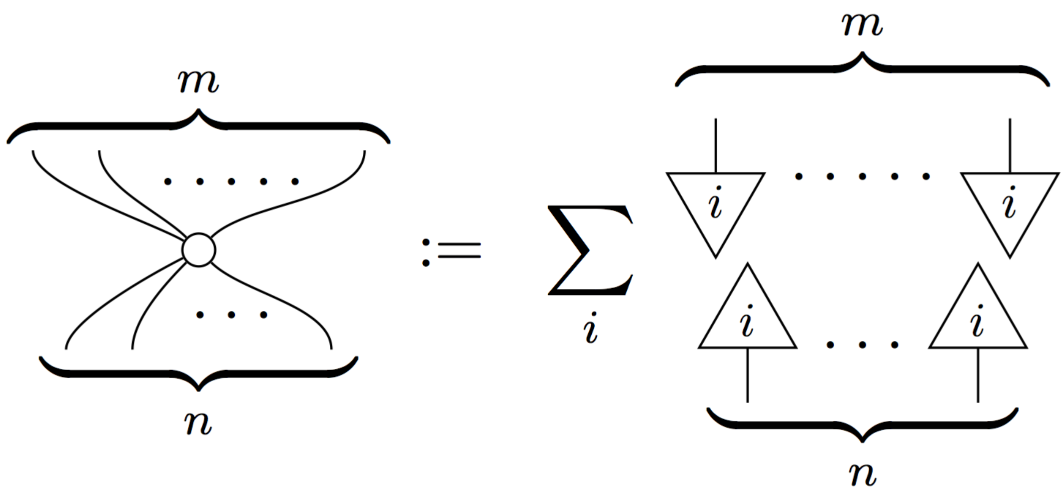
Main idea:

$$\frac{\text{classical system}}{\textbf{quantum system}} = \frac{\text{single wire}}{\textbf{double wire}}$$

# — Ch. 6 – Picturing classical processes —

*– spiders –*

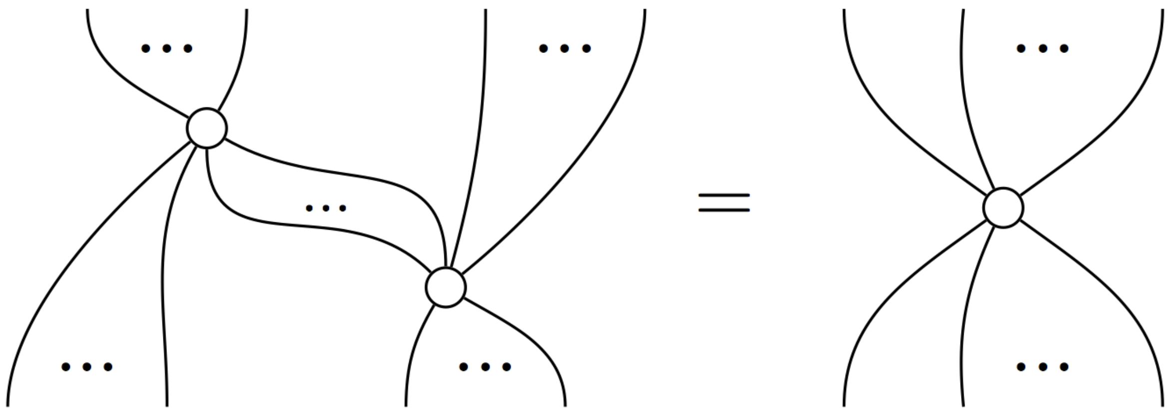
... :=



# — Ch. 6 – Picturing classical processes —

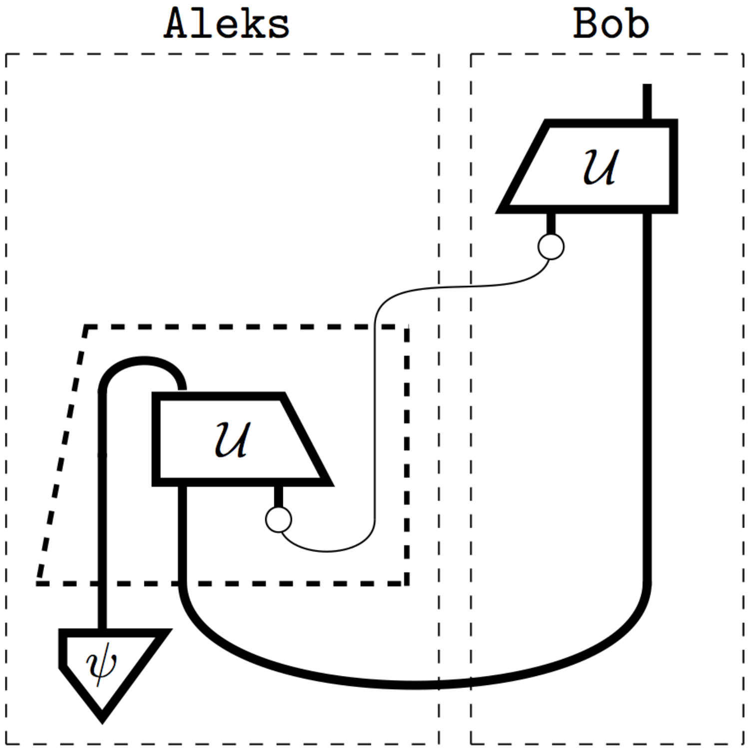
*– spiders –*

**Prop.**



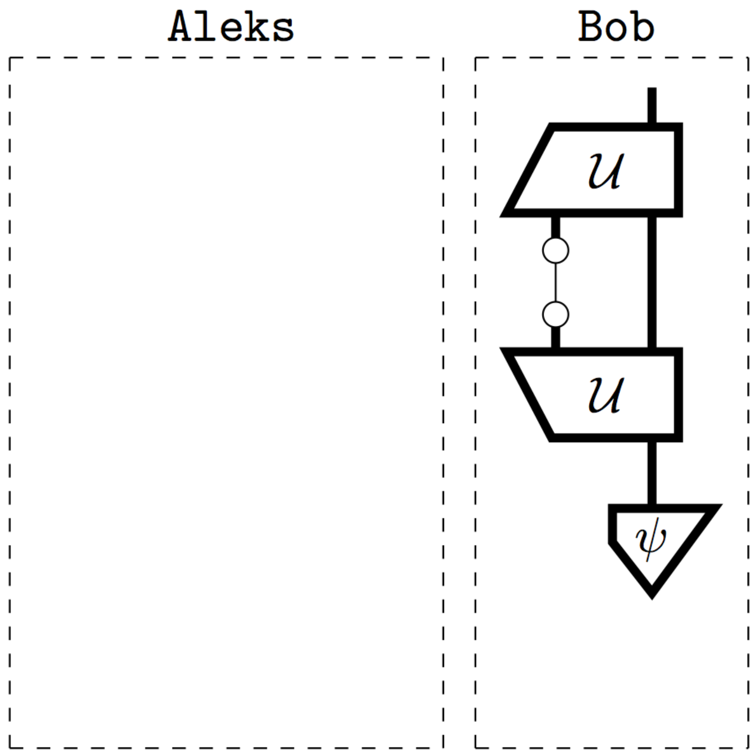
# — Ch. 6 – Picturing classical processes —

*– teleportation diagrammatically –*



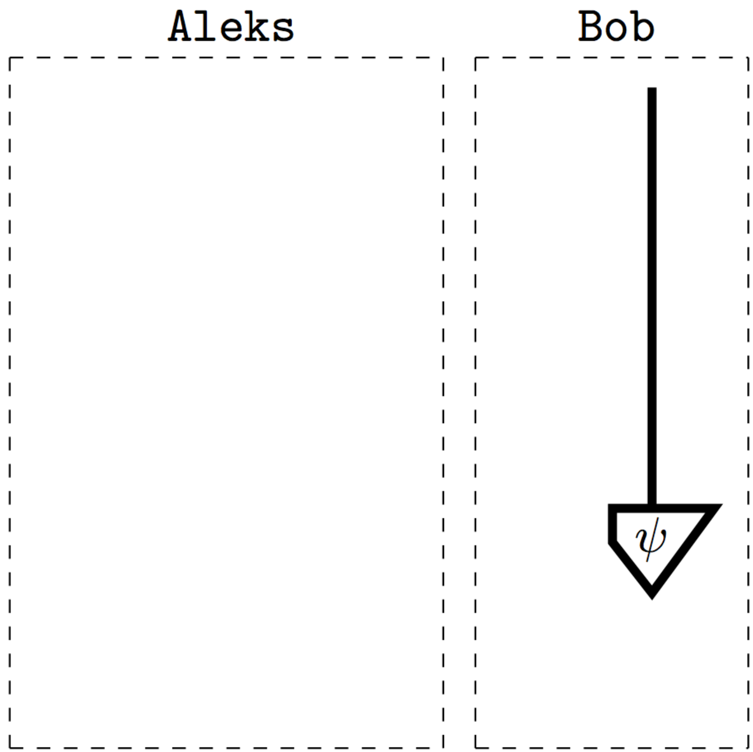
# — Ch. 6 – Picturing classical processes —

*– teleportation diagrammatically –*



# — Ch. 6 – Picturing classical processes —

*– teleportation diagrammatically –*



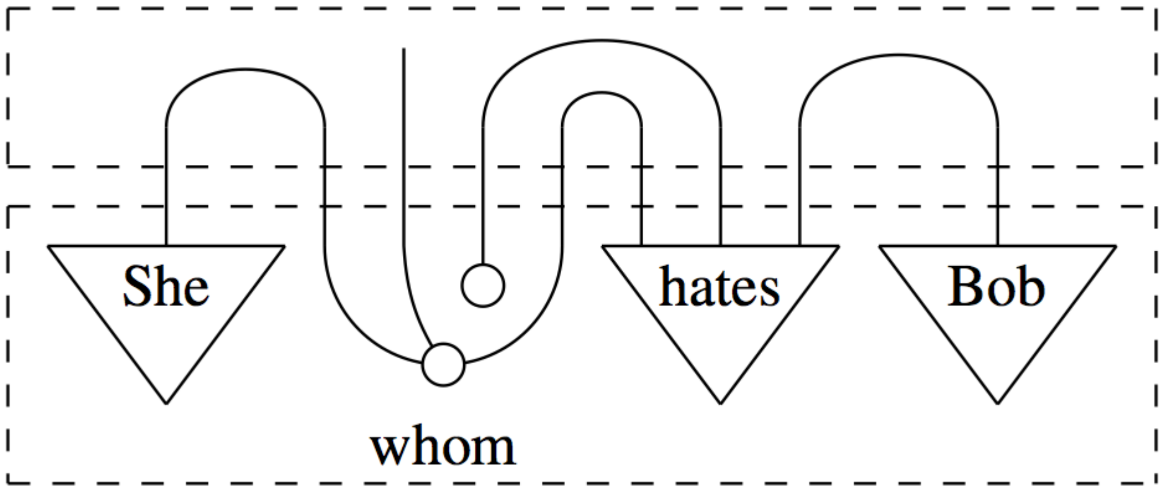
# — Ch. 3 – Hilbert space from diagrams —

*– completeness –*

**THM.** (Kissinger, 2014)

An equation between dot diagrams holds, if and only if it holds for Hilbert spaces with a fixed basis and linear maps, that is, for matrices of complex numbers.

## Dot diagrams for natural language meaning:




- Top part: grammar
- Bottom part: meaning vectors