Categories of Matroids

Vaia Patta, Chris Heunen

June 8, 2015

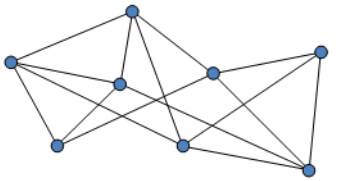
Matroid Theory
Concept and usage
Definitions
Commonly used matroid constructions
Strong maps
Categorical perspective on matroids

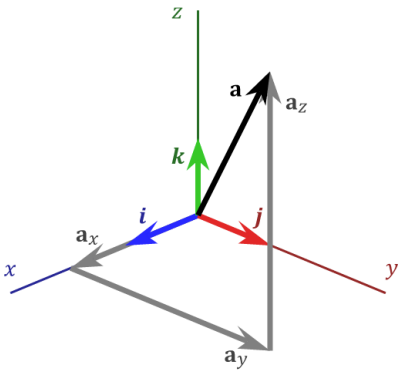
Matroid Theory

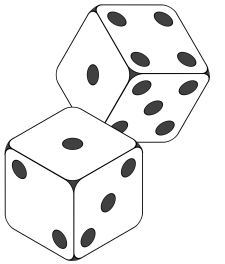
Concept and usage

Matroid Theory
Concept and usage
Definitions
Commonly used matroid constructions
Strong maps
Categorical perspective on matroids

A matroid describes *dependence*.







$$x = ax_1 + bx_2 \qquad \text{...etc}$$

Extensively used in combinatorial optimisation problems.

Definitions

Matroid Theory
Concept and usage
Definitions
Commonly used matroid constructions
Strong maps
Categorical perspective on matroids

Ground set

- Independent sets
- Maximal independent sets (bases)
- Minimal dependent sets (circuits)
- Closed sets (flats)
- "Maximal" closed sets (hyperplanes)
- Spanning sets
- Rank function
- Closure function
- ...etc

Commonly used matroid constructions

Matroid Theory
Concept and usage
Definitions
Commonly used matroid constructions
Strong maps
Categorical perspective on matroids

Deletion of an element
Contraction of an element

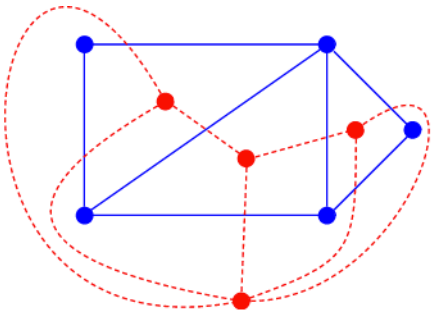
Minors

Commonly used matroid constructions

Matroid Theory
Concept and usage
Definitions
Commonly used matroid constructions
Strong maps
Categorical perspective on matroids

Deletion of an element
Contraction of an element

Minors



Dual matroid
Quotients, Lifts, Extensions, Coextensions
Matroid union, Matroid direct sum
Other matroid sums and products

Strong maps

Matroid Theory
Concept and usage
Definitions
Commonly used matroid constructions
Strong maps
Categorical perspective on matroids

Functions between the elements of two matroids such that the preimage of a closed set is closed.

- Closest thing to linear transforms
- Applications in graph theory

Categorical perspective on matroids

The category

Matroid Theory
Categorical perspective on matroids
The category
Matroid applications as functors
General properties
Application to common contstructions

Objects: Matroids

- All matroids (Matr)
- Loopless (LMatr)
- No dependent sets (FMatr)
- Every rank-1 flat has 1 element (SMatr)
- Representable over k (Matr_k)

Morphisms: Strong maps

Matroid applications as functors

Matroid Theory

Categorical perspective on matroids

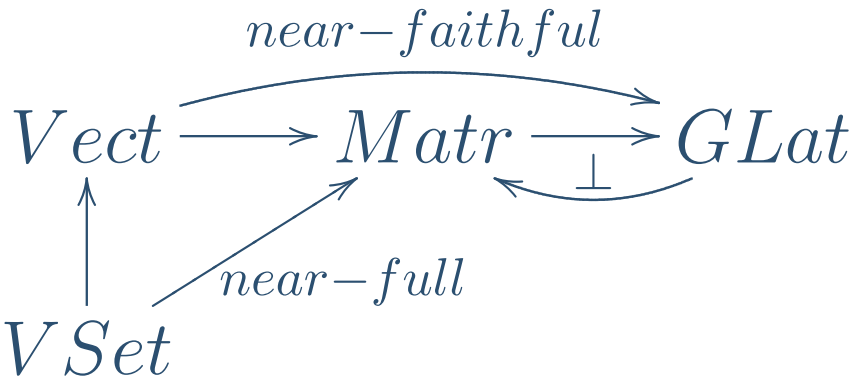
The category

Matroid applications as functors

General properties

Application to common contstructions

$Graph \to Matr$



General properties

Matroid Theory
Categorical perspective on matroids
The category
Matroid applications as functors
General properties
Application to common contstructions

- $Matr \underset{\perp}{\overset{\longrightarrow}{\longleftarrow}} Set$

General properties

Matroid Theory
Categorical perspective on matroids
The category
Matroid applications as functors
General properties
Application to common contstructions

- $Matr \underset{\perp}{\overset{\longrightarrow}{\longleftarrow}} Set$
- Not many limits and colimits

General properties

Matroid Theory

Categorical perspective on matroids

The category

Matroid applications as functors

General properties

Application to common contstructions

- $Matr \underset{\bot}{\overset{\longrightarrow}{\longleftarrow}} Set$
- Not many limits and colimits
- Various free and cofree constructions

$$\begin{array}{ccc} Matr & \hookleftarrow & LMatr \\ \uparrow & \nearrow & \uparrow \\ SMatr & \hookleftarrow & FMatr \end{array}$$

General properties

Matroid Theory

Categorical perspective on matroids

The category

Matroid applications as functors

General properties

Application to common contstructions

- $Matr \underset{\perp}{\overset{\longrightarrow}{\longleftarrow}} Set$
- Not many limits and colimits
- Various free and cofree constructions

$$\begin{array}{ccc} Matr & \hookleftarrow & LMatr \\ \uparrow & \nearrow & \uparrow \\ SMatr & \hookleftarrow & FMatr \end{array}$$

- $GLat \xrightarrow{Matroid} Sub$

Application to common contstructions

Matroid Theory
Categorical perspective on matroids
The category
Matroid applications as functors
General properties
Application to common contstructions

Minors: Functorial
Duals: Quasi-functorial
Lifts, extensions, etc: Work in progress
Monoidal structures based on combinations?