Quantum Processes and Computation

Assignment 2, Monday 28 Oct 2024

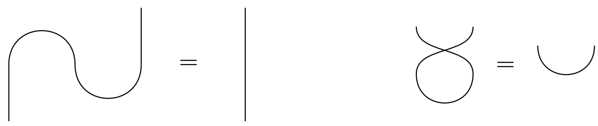
Deadline: Class in week 4 (Check Minerva for weekly marking deadline.)

Goals: After completing these exercises you should know how to do concrete calculations involving string diagrams and linear maps. Material covered in book: Chapter 4 and 5.

Note: Many of these exercises also appear in *Picturing Quantum Processes*, but sometimes they have been slightly modified for the problem sheet. The corresponding exercise number from the book is shown in brackets.

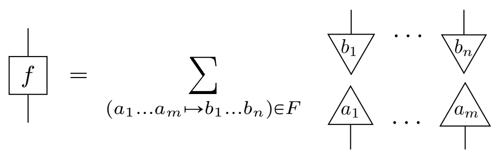
Exercise 1: We can write the cup/cap for any dimension as a sum over ONB elements:

(i) Using this definition (and not the matrix form) verify the yanking equations.



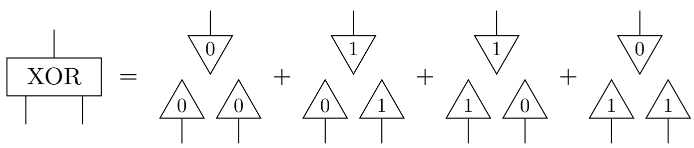
(ii) Compute the matrices for the cup and cap in 3 dimensions.

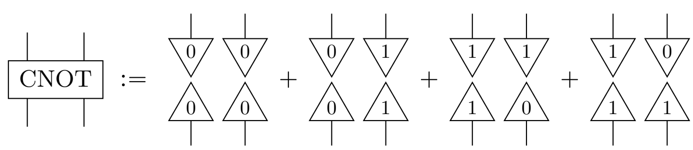
Exercise 2: This excercise is about encoding classical functions as linear maps using ONB states and effects, as explained in Section 5.3.4. For a function $F : \{0,1\}^m \to \{0,1\}^n$, we can define an associated linear map f as follows:

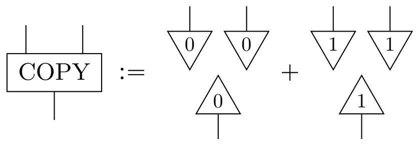


where the notation $(a_1...a_m \mapsto b_1...b_n) \in F$ means we are summing over the *graph of* F, i.e. the set of bitstrings $\{(a_1, \ldots, a_m, b_1, \ldots b_n) \mid F(a_1, \ldots, a_m) = (b_1, \ldots b_n)\}$.

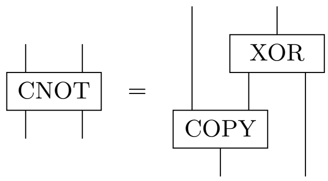
Using this encoding, define:





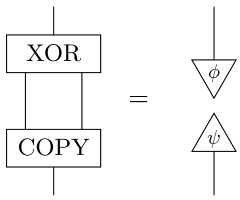


Exercise 3 (5.86): Show that



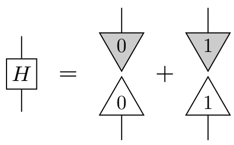
(Hint: try comparing the LHS to the RHS on all basis states, rather than writing out a big sum.)

Next, find ψ and ϕ such that the following equation holds:



Although it might not look like much now, this equation will turn out to lie at the heart of the notion of *complementarity* which is an important part of the ZX-calculus.

Exercise 4: Let the *Hadamard gate*, which sends the Z-basis to the X-basis be defined as follows:



where

Compute the matrix of H. Show that $H = H^\dagger = H^T$. Using this fact (or otherwise) show that H also sends the X-basis back to the Z-basis.

Exercise 5: Write the following diagrams as tensor contractions, i.e. as sums over products of matrix elements f_{ij}^{kl}, etc.