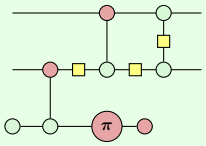
Quantum Processes and Computation

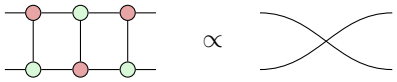
Assignment 3 – MT 2026

Unless otherwise noted, the following exercises are taken from *Picturing Quantum Software* (see github.com/zxcalc/book). There might be slight differences between the exercises on this problem sheet and how they appear in the book. **In that case, complete the version from this problem sheet and not the book version.** For some exercises, it may nevertheless be useful to locate the exercise in the book for additional results and explanation.

Exercise 3.7. Help the trapped π phase find its way to the exit.



That is, without changing the rest of the diagram, use the rules of the ZX-calculus to move/copy π-phased spider around until (Z or X) spiders with π phases only appear on the outputs.



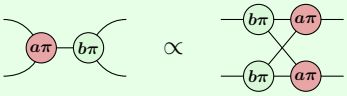
(3.1)



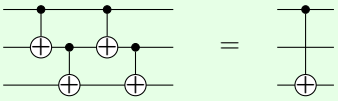
(3.2)

Exercise 3.8. We already proved Eq. (3.1) by applying (3.2) in the reverse direction (replacing the RHS with the LHS). Show that you can also prove it with a single use of the (3.2) in forward direction (replacing the LHS with the RHS), possibly followed by some spider fusions and applications of the complementarity rule.

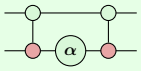
Exercise 3.10. Prove this equation using the rules of the ZX-calculus:



Exercise 3.11. Prove the following circuit identity by translating them to ZX-diagrams and rewriting the left-hand side into the right-hand side.

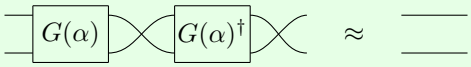


Exercise 3.12. Consider the following parametrised unitary:

$$G(\alpha) \;=\; \boxed{G(\alpha)} \;:=$$ 

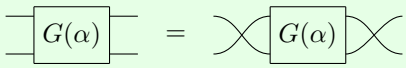
a) Using the ZX-calculus show that $G(\beta) \circ G(\alpha) = G(\alpha + \beta)$ and that $G(0) = I$. Note that this equation in particular implies that $G(\alpha)^\dagger = G(-\alpha)$.

b) Using the ZX-calculus show that:



(3.3)

c) This previous equation implies that:

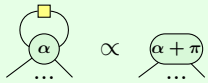


(3.4)

Show that this indeed holds by calculating the matrix of both sides.

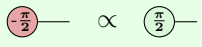
d) Show that Eq. (3.4) holds by using the ZX-calculus to reduce the left-hand side to a diagram that is clearly symmetric under interchanging the two qubits.

Exercise 3.14. Prove using the ZX-calculus that we can get rid of Hadamard self-loops at the cost of acquiring a phase:



(3.5)

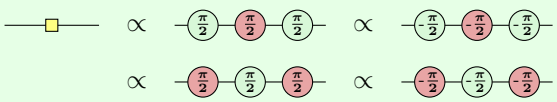
Exercise 3.15. There are two ways in which we can write the eigenstate $|i\rangle := \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$ of the Pauli operator Y (up to global phase). Namely, as $|i\rangle = R_X(-\frac{\pi}{2})|0\rangle$ or as $|i\rangle = R_Z(\frac{\pi}{2})|+\rangle$. Prove this equivalence using the ZX-calculus:



(3.6)

An analogous equation holds for $|-i\rangle := \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$, which boils down to flipping the signs of the phases in the spiders.

Exercise 3.16. Prove that the Hadamard gate has all of these equivalent decompositions using the rules of the ZX-calculus:



Exercise 3.18. Show that this equation follows from the rules of the ZX calculus:

