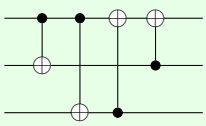
Quantum Processes and Computation

Assignment 4 – MT 2026

Unless otherwise noted, the following exercises are taken from *Picturing Quantum Software* (see github.com/zxcalc/book). There might be slight differences between the exercises on this problem sheet and how they appear in the book. **In that case, complete the version from this problem sheet and not the book version.** For some exercises, it may nevertheless be useful to locate the exercise in the book for additional results and explanation.

Exercise 4.1. Consider the following CNOT circuit:



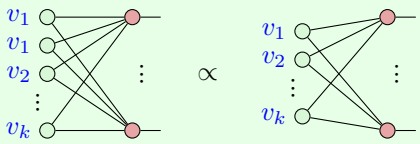
a) Calculate the parity matrix of this circuit.

b) Resynthesise a new equivalent CNOT circuit from this parity matrix by using Gaussian elimination.

Exercise 4.4. We need the assumption that the biadjacency matrix is invertible. Generate the parity normal form diagram of the following parity matrix and try to apply the strategy above. What goes wrong?

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

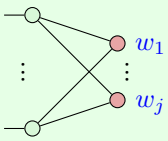
Exercise 4.7. Show, using the phase-free ZX calculus, that:



(4.1)

For the next question, read Section 4.3.2 of *Picturing Quantum Software* about X-Z normal forms and the perp $S^\perp$ of a linear subspace S.

Exercise 4.9. Show that the interpretation for X-Z normal forms given in Section 4.3.2 is true in general, namely that:



$$\propto \sum_{b \in S} \langle b| \qquad \text{where} \qquad S^\perp = \text{Span}\{w_1, \ldots, w_j\} \tag{4.2}$$

Hint: Plug in a computational basis state $|v_1, \ldots, v_n\rangle$ into the X-Z form, represented as a bunch of X-spiders, then use the fact that we can represent a parity of a subset of the variables

$v = (v_1, \ldots, v_n)$ given by the binary vector w_x as an inner product $w_x^T v$.

Exercise 4.12. Give Z-X and X-Z normal forms for the 4-qubit GHZ state $|\mathrm{GHZ}_4\rangle \propto |0000\rangle + |1111\rangle$.