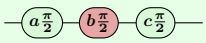
Quantum Processes and Computation

Assignment 5 – MT 2026

Unless otherwise noted, the following exercises are taken from *Picturing Quantum Software* (see github.com/zxcalc/book). There might be slight differences between the exercises on this problem sheet and how they appear in the book. **In that case, complete the version from this problem sheet and not the book version.** For some exercises, it may nevertheless be useful to locate the exercise in the book for additional results and explanation.

Exercise 5.1. A single-qubit Clifford circuit is constructed out of just Hadamard and S gates. Show that any single-qubit Clifford circuit can be rewritten to the form



for some integers a, b and c. *Hint: We know that a single S or Hadamard can be brought to this form. So you just need to show that when you compose this normal form with an additional S or Hadamard gate that the resulting circuit also be brought to this normal form. You probably will want to make a case distinction on the value of b.*

Exercise 5.2. Show that the following states are Clifford states, i.e. construct a Clifford circuit C that when applied to $|0\cdots0\rangle$ gives the desired state.

a) $|1\rangle$.

b) $|+\rangle$.

c) $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$.

d) $\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$.

e) $\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$.

Recall the following definition from Chapter 4 of *Picturing Quantum Software*.

Definition 4.2.4. We say a ZX-diagram is **two-coloured** when

1. no spiders of the same colour are connected to each other,
2. there are no self-loops,
3. there is at most one wire between each pair of spiders,
4. there are no Hadamards.

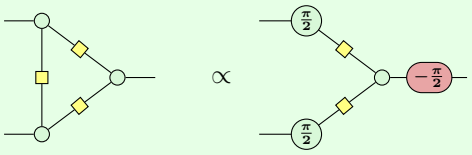
Exercise 5.3. Every two-coloured diagram can clearly be transformed into a graph-like diagram by changing all the X-spiders into Z-spiders using (***cc***), and then potentially introducing some more identities with (***id***) and (***hh***) to the input and output wires.

a) Show that the converse is not true: find a graph-like ZX-diagram where it is not possible to rewrite it back into a two-coloured diagram using just (***cc***).

b) Find a systematic way in which a graph-like diagram can be transformed into a two-coloured diagram.

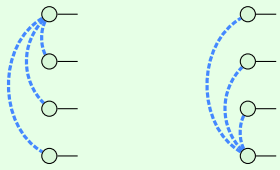
Exercise 5.5. A graph state has no internal spiders, but a general graph-like diagram does. Show that any graph-like diagram with no inputs can be written as a graph state followed by zero or more single-qubit post-selections (i.e. bras).

Exercise 5.6. Prove the local complementation rule in the special case of just two neighbours (without using (***lc***) of course!). That is, prove the following using the ZX-calculus:



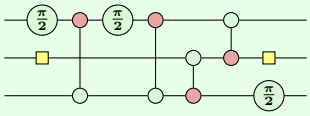
(5.1)

Exercise 5.7. We say two n-qubit quantum states $|\psi_1\rangle$ and $|\psi_2\rangle$ are **equivalent under local operations** when $U|\psi_1\rangle = |\psi_2\rangle$ for a local quantum circuit $U = U_1 \otimes U_2 \otimes \cdots \otimes U_n$ consisting of just single-qubit gates. Show that the following two graph states are equivalent under local operations.



Hint: Use the fact that a local complementation can be done using just local unitaries.

Exercise 5.10. Simplify the following circuit to a diagram that has no internal spiders with a $\pm\frac{\pi}{2}$ phase or pairs of internal spiders with a 0 or π phase.



Exercise 5.16. Reduce the following Clifford circuit to GSLC form:

