

# Picturing Quantum Processes

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QTFT, Växjö 2015

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## Quantum Picturalism: what it **is**, what it **isn't**

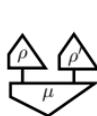
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## Quantum Pictorialism: what it **is**, what it **isn't**

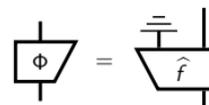
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- QP is not a *reconstruction*, but some ideas from operational reconstructions play a major role, e.g.



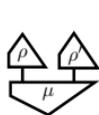
local/process tomography



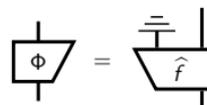
purification

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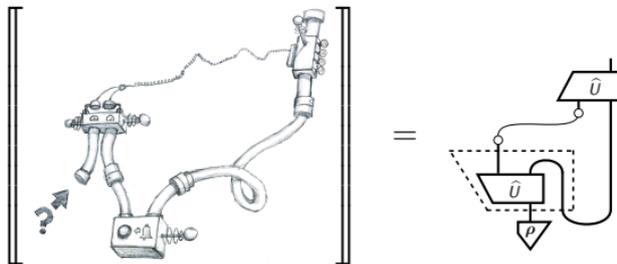


local/process tomography



purification

- ...and relationship between *operational setups* and *theoretical models*:





# Outline

**Picturing Quantum Processes**  
chapters 4-9 (roughly)

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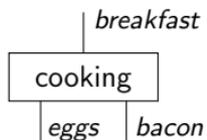
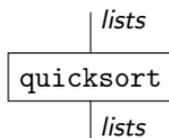
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5. Complementarity

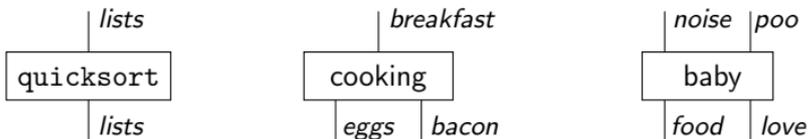
# Recap

- Wires represent *systems*, boxes represent *processes*

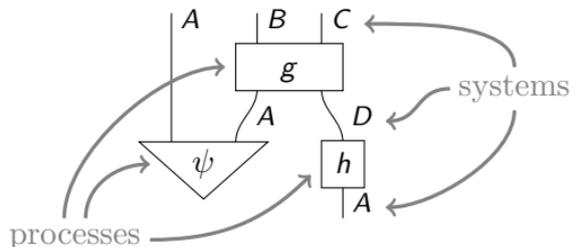


## Recap

- Wires represent *systems*, boxes represent *processes*



- The world is organised into *process theories*, collections of processes that make sense to combine into *diagrams*



## Recap

- Certain processes play a special role:

*states:* 

*effects:* 

*numbers:* 

## Recap

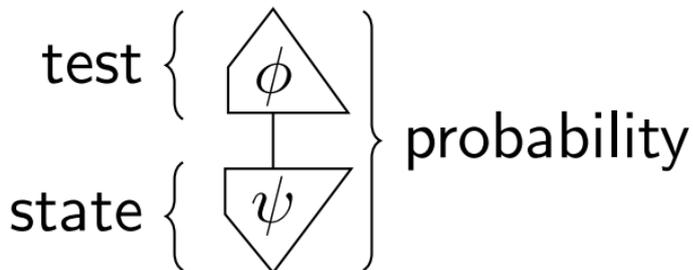
- Certain processes play a special role:

states: 

effects: 

numbers: 

- State + effect = number, interpreted as:



this is called the *Born rule*.

# linear maps

In the process theory of **linear maps**:

# linear maps

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(L1) Every type has a (finite) *basis*:

$$\left( \text{for all } \begin{array}{c} \downarrow \\ i \end{array} : \begin{array}{c} \boxed{f} \\ \downarrow \\ \begin{array}{c} \downarrow \\ i \end{array} \end{array} = \begin{array}{c} \boxed{g} \\ \downarrow \\ \begin{array}{c} \downarrow \\ i \end{array} \end{array} \right) \Rightarrow \begin{array}{c} \boxed{f} \\ \downarrow \end{array} = \begin{array}{c} \boxed{g} \\ \downarrow \end{array}$$

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(L2) Processes can be *summed*:

$$\sum_i \downarrow_i \boxed{f_i} \quad \text{where} \quad \left( \sum_i \begin{array}{c} \boxed{g} \\ \downarrow_i \end{array} \right) \boxed{f} = \sum_i \left( \begin{array}{c} \boxed{g} \\ \downarrow_i \end{array} \right)$$

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(L3) Numbers are the *complex numbers*:  $\diamond \lambda \in \mathbb{C}$

Bases  $\Leftrightarrow$  process tomography

## Theorem

$$\left( \text{for all } \begin{array}{c} \downarrow \\ i \end{array}, \begin{array}{c} \downarrow \\ j \end{array} : \begin{array}{c} \begin{array}{c} \uparrow \\ j \end{array} \\ \begin{array}{c} \text{f} \\ \text{trapezoid} \end{array} \\ \begin{array}{c} \downarrow \\ i \end{array} \end{array} = \begin{array}{c} \begin{array}{c} \uparrow \\ j \end{array} \\ \begin{array}{c} \text{g} \\ \text{trapezoid} \end{array} \\ \begin{array}{c} \downarrow \\ i \end{array} \end{array} \right) \Rightarrow \begin{array}{c} \downarrow \\ \text{f} \\ \text{trapezoid} \end{array} = \begin{array}{c} \downarrow \\ \text{g} \\ \text{trapezoid} \end{array}$$

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$$\left( \text{for all } \begin{array}{c} | \\ \hline i \\ \hline \end{array}, \begin{array}{c} | \\ \hline j \\ \hline \end{array} : \begin{array}{c} \begin{array}{c} \triangle j \\ \hline \end{array} \\ \begin{array}{c} \hline \\ \hline f \\ \hline \end{array} \\ \begin{array}{c} \hline \\ \hline i \\ \hline \end{array} \end{array} = \begin{array}{c} \begin{array}{c} \triangle j \\ \hline \end{array} \\ \begin{array}{c} \hline \\ \hline g \\ \hline \end{array} \\ \begin{array}{c} \hline \\ \hline i \\ \hline \end{array} \end{array} \right) \Rightarrow \begin{array}{c} | \\ \hline \\ \hline f \\ \hline \end{array} = \begin{array}{c} | \\ \hline \\ \hline g \\ \hline \end{array}$$

## Proof.

$$\begin{array}{c} \begin{array}{c} \triangle j \\ \hline \end{array} \\ \begin{array}{c} \hline \\ \hline f \\ \hline \end{array} \\ \begin{array}{c} \hline \\ \hline i \\ \hline \end{array} \end{array} = \begin{array}{c} \begin{array}{c} \triangle j \\ \hline \end{array} \\ \begin{array}{c} \hline \\ \hline g \\ \hline \end{array} \\ \begin{array}{c} \hline \\ \hline i \\ \hline \end{array} \end{array}$$

□

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$$\left( \text{for all } \begin{array}{c} | \\ \textit{i} \\ \nabla \end{array}, \begin{array}{c} | \\ \textit{j} \\ \nabla \end{array} : \begin{array}{c} \textit{j} \\ \triangle \\ \textit{f} \\ \textit{i} \\ \nabla \end{array} = \begin{array}{c} \textit{j} \\ \triangle \\ \textit{g} \\ \textit{i} \\ \nabla \end{array} \right) \Rightarrow \begin{array}{c} | \\ \textit{f} \\ \textit{f} \\ | \end{array} = \begin{array}{c} | \\ \textit{g} \\ \textit{g} \\ | \end{array}$$

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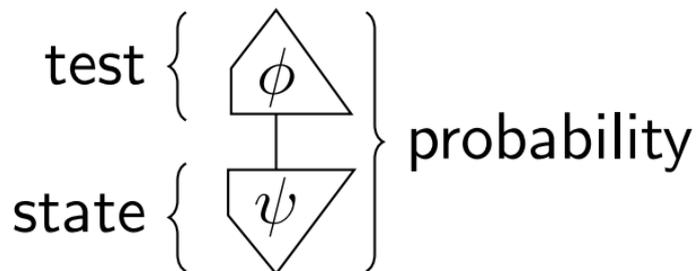
- In other words,  $f$  is uniquely fixed by its *matrix*:

$$\begin{pmatrix} f_1^1 & f_2^1 & \cdots & f_m^1 \\ f_1^2 & f_2^2 & \cdots & f_m^2 \\ \vdots & \vdots & \ddots & \vdots \\ f_1^n & f_2^n & \cdots & f_m^n \end{pmatrix}$$

where

$$f_i^j := \begin{array}{c} \begin{array}{c} \triangleup \\ j \end{array} \\ \begin{array}{c} \square \\ f \end{array} \\ \begin{array}{c} \triangle \\ i \end{array} \end{array}$$

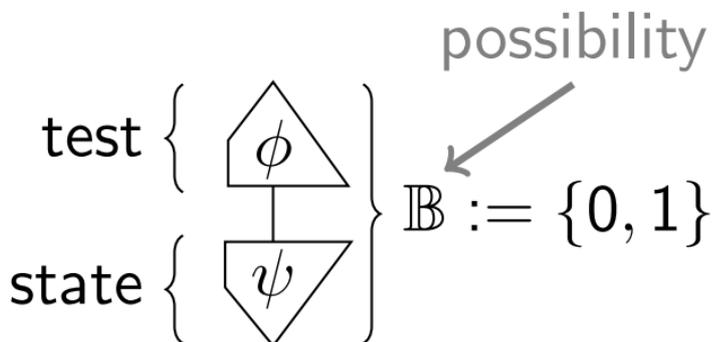
## What about the Born rule?



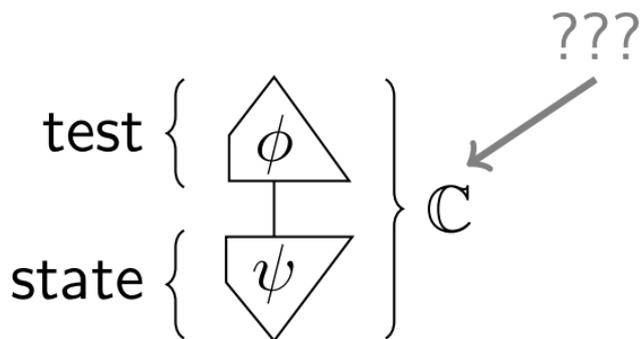
## The Born rule for **relations**

$$\left. \begin{array}{l} \text{test} \\ \text{state} \end{array} \right\} \left\{ \begin{array}{c} \text{◇} \\ \phi \\ | \\ \psi \\ \text{◇} \end{array} \right\} \mathbb{B} := \{0, 1\}$$

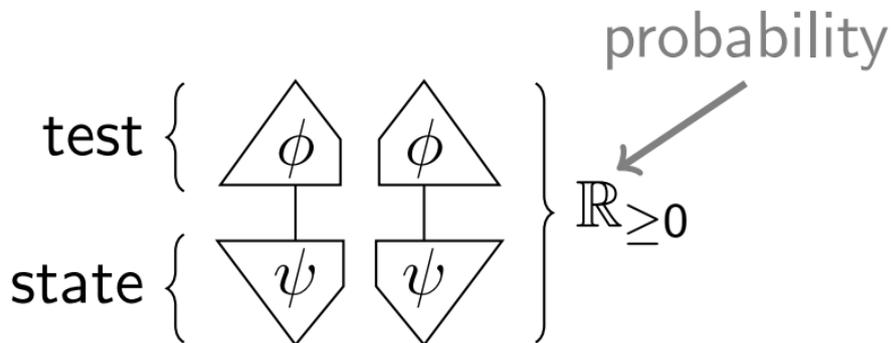
## The Born rule for **relations**



## The Born rule for **linear maps**



## Fixing the problem



## Doubled states and effects

Letting:

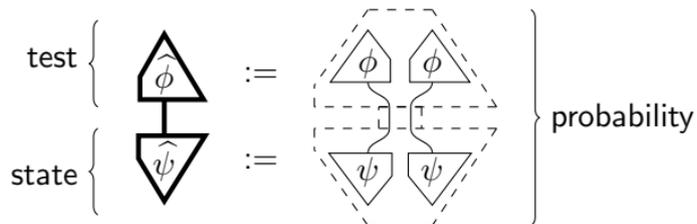


## Doubled states and effects

Letting:



yields...



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$$\left| \hat{A} \right| := \left[ \left[ \right] \right]$$

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- and processes:

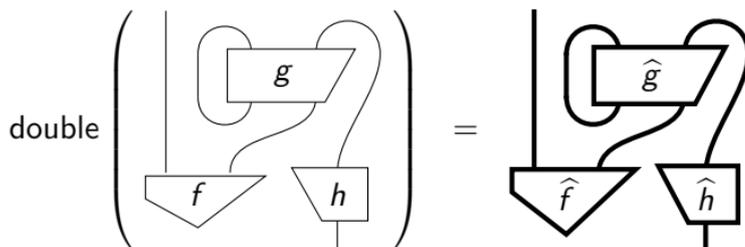
$$\boxed{\hat{f}} = \text{---} \left[ \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} \right] \text{---}$$

The diagram shows a trapezoidal box labeled  $\hat{f}$  with a vertical line entering from the top and exiting from the bottom. This is equal to a dashed trapezoidal box containing two trapezoidal boxes labeled  $f$ . The top and bottom of the dashed box are connected by vertical lines, and the two  $f$  boxes are connected to these lines by curved lines.

for all processes  $f$  from **linear maps**.

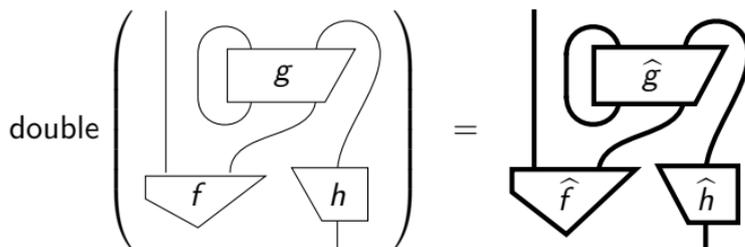
## Embedding the old theory

- **linear maps** embed in **quantum maps**, and this embedding preserves diagrams:



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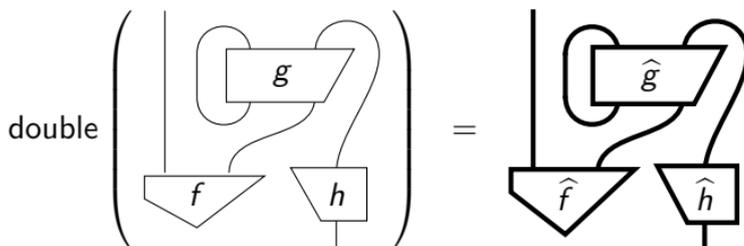
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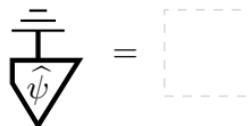
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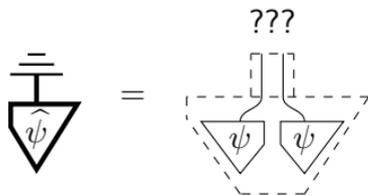
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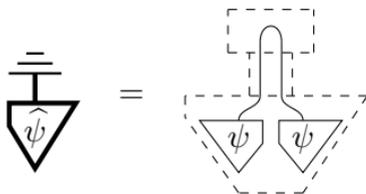
- But now we're in a bigger space, so there is room for something new, *discarding*:



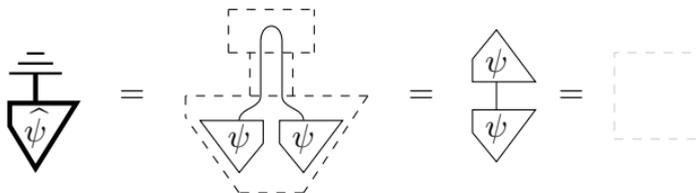
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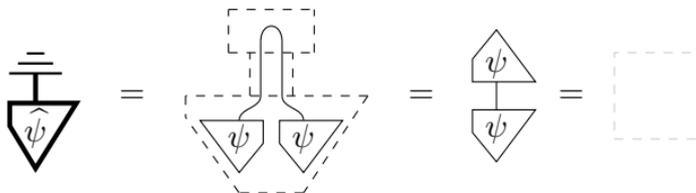
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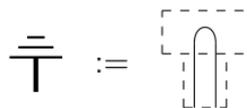
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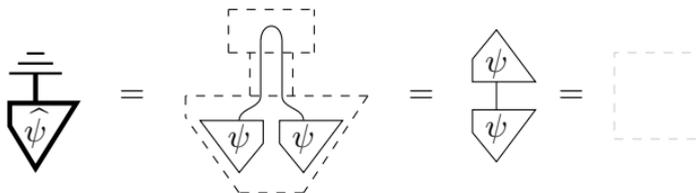
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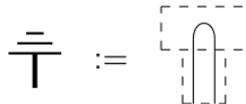
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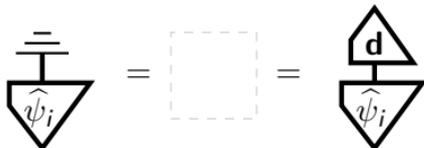
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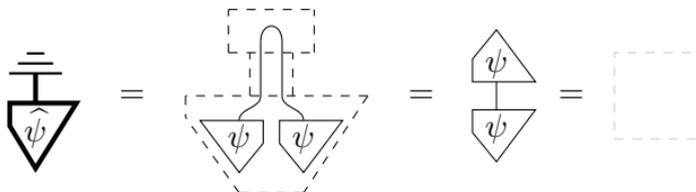
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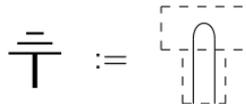
- In fact, this is the unique map with this property. Let  $\{\widehat{\psi}_i\}_i$  be a basis of pure states (e.g.  $z^+, z^-, x^+, y^+$ ), then:



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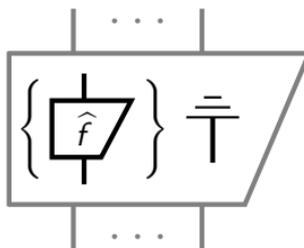
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## quantum maps

### Definition

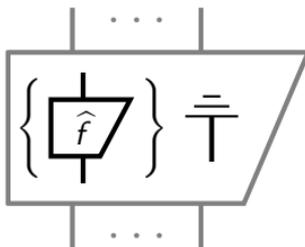
The process theory of **quantum maps** consists of all processes obtained from pure quantum maps and discarding:



## quantum maps

### Definition

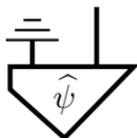
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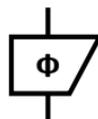
• e.g.



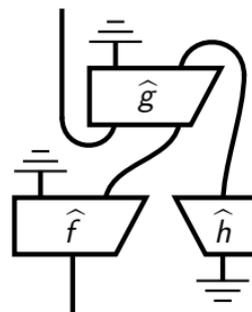
:=



and



:=



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The diagram shows an equality between two quantum process symbols. On the left is a trapezoidal box with a vertical line on the left side and a vertical line on the right side. Inside the box is the Greek letter  $\Phi$ . Above the top-left corner, there is a horizontal line with two short vertical lines extending upwards from it. Below the bottom-right corner, there is a vertical line extending downwards. On the right side of the box is an equals sign. To the right of the equals sign is a symbol consisting of a horizontal line with two short vertical lines extending upwards from it, and a vertical line extending downwards from the center of the horizontal line.



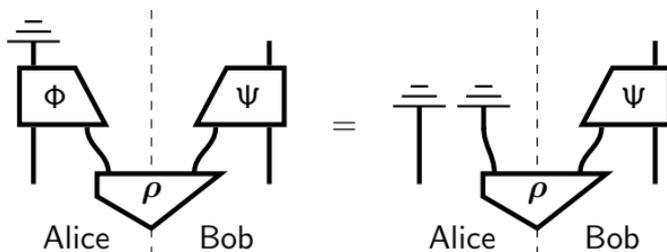


# Causality

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- To get all of the *deterministically realisable* processes, we additionally require *causality*:

$$\begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \square \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \Phi \quad = \quad \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array}$$

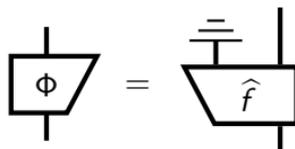
- Causality  $\implies$  no-signalling:





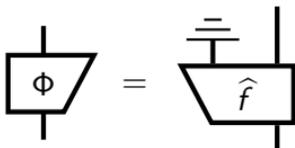
## Purification

- Any quantum map extends to a pure quantum map on an extended system:

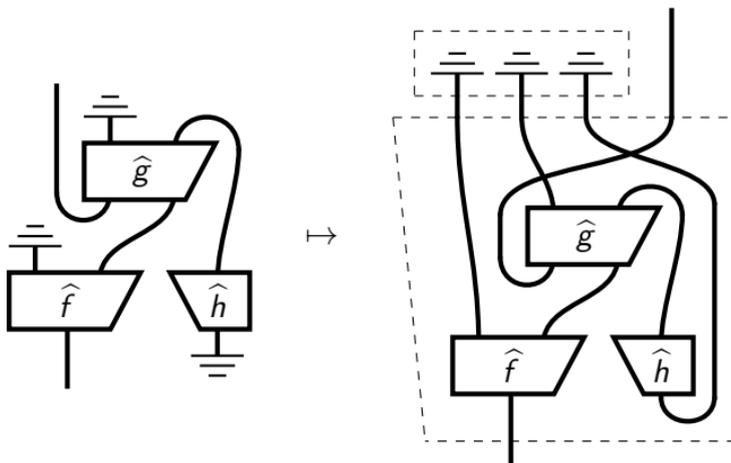


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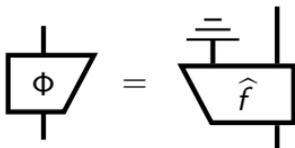


- This is built-in to our definition of quantum maps:

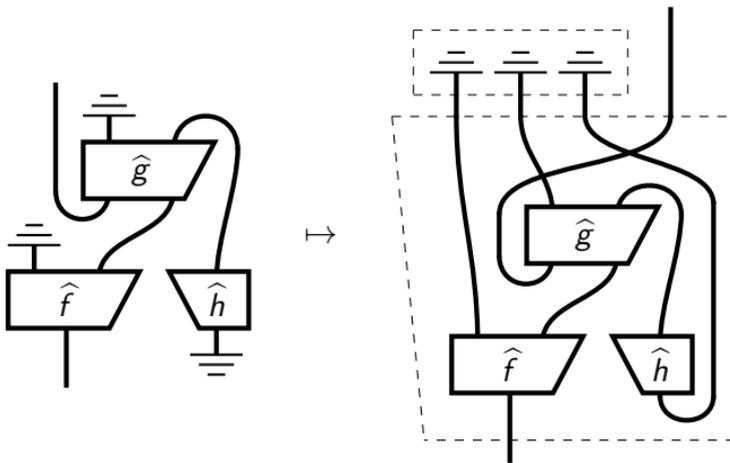


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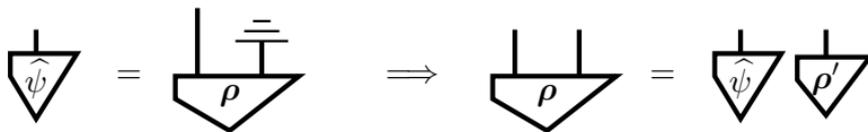


- If  $\Psi$  causal,  $\hat{f}$  must be isometry: *Stinespring dilation*.

## No-broadcasting from pure extension

### Theorem

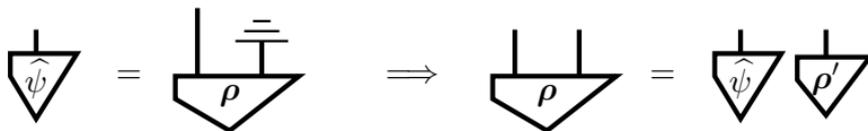
A state is pure if and only if any *extension* separates:



## No-broadcasting from pure extension

### Theorem

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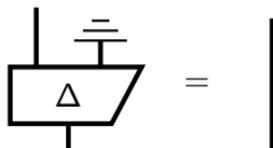
### Corollary

There exists no quantum map  $\Delta$  such that:



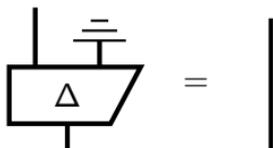
## No-broadcasting from pure extension - proof

Broadcast to the left:

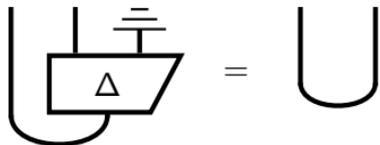


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Broadcast to the left:



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## No-broadcasting from pure extension - proof

Broadcast to the left:

$$\text{[Trapezoidal box with } \Delta \text{ and two top wires, one bottom wire]} = |$$

Bend the wire:

$$\text{[Trapezoidal box with } \Delta \text{ and two top wires, one bottom wire looping left]} = \cup \Rightarrow \text{[Trapezoidal box with } \Delta \text{ and two top wires, one bottom wire looping right]} = \cup \downarrow \rho$$

## No-broadcasting from pure extension - proof

Broadcast to the left:

$$\text{Diagram of } \Delta \text{ with two top wires and one bottom wire} = |$$

Bend the wire:

$$\text{Diagram of } \Delta \text{ with a loop on the bottom wire} = \text{U-shaped wire} \Rightarrow \text{Diagram of } \Delta \text{ with a loop on the bottom wire} = \text{U-shaped wire} \downarrow \rho$$

Unbend the wire and try to broadcast to the right:

$$\text{Diagram of } \Delta \text{ with one bottom wire} = | \downarrow \rho \Rightarrow \text{Diagram of } \Delta \text{ with one bottom wire} = \text{Diagram of } \Delta \text{ with one bottom wire} \downarrow \rho$$

## Classical systems

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- Processes are *stochastic maps*:

$$\begin{array}{c} \downarrow \\ \boxed{f} \\ \downarrow \end{array} \leftrightarrow \begin{pmatrix} p_1^1 & p_2^1 & \cdots & p_m^1 \\ p_1^2 & p_2^2 & \cdots & p_m^2 \\ \vdots & \vdots & \ddots & \vdots \\ p_1^n & p_2^n & \cdots & p_m^n \end{pmatrix}$$

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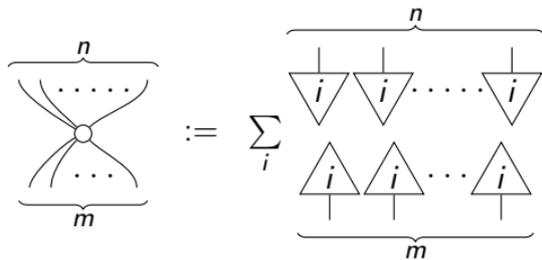
$$\text{○} \text{---} \text{▭}_f = \text{○}$$

- We *can* broadcast classically:

$$\text{○} \text{---} \text{U} = \text{○} \text{---} \text{V} = \text{○} \text{---} \text{W} \quad \text{where} \quad \text{○} \text{---} \text{Y} := \sum_i \text{△}_i \text{---} \text{△}_i$$

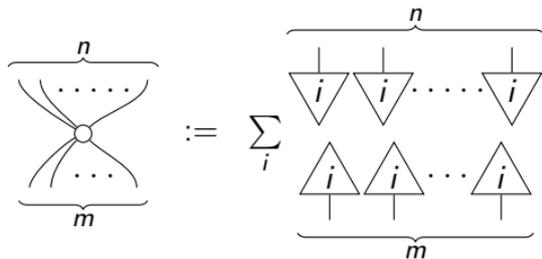
## Generalising to *spiders*

- These generalise to a whole family of maps, called *spiders*:

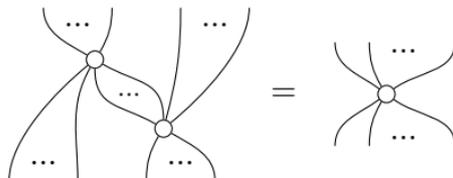


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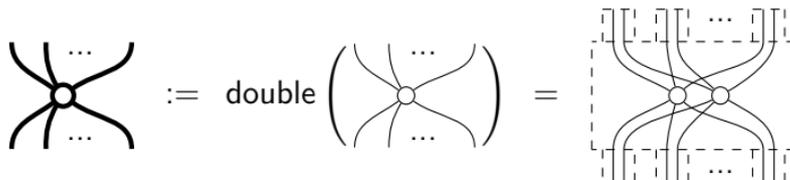


- where the only rule to remember is:



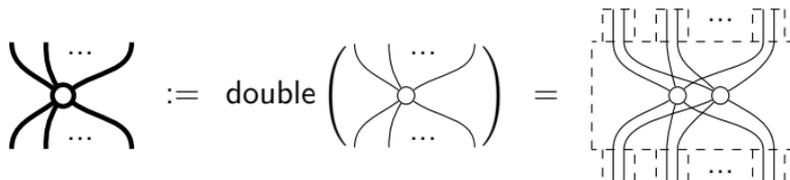
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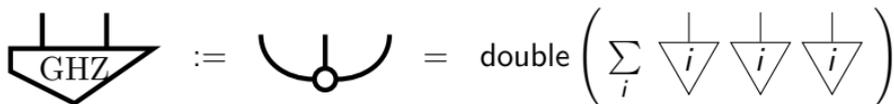


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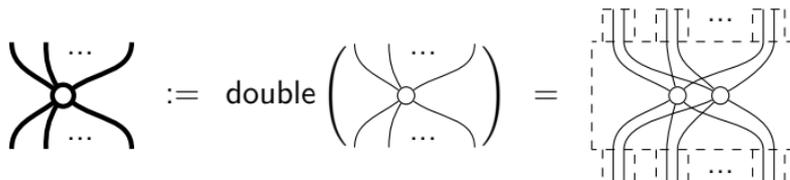


- An example is the GHZ state:

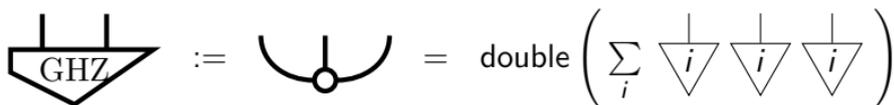


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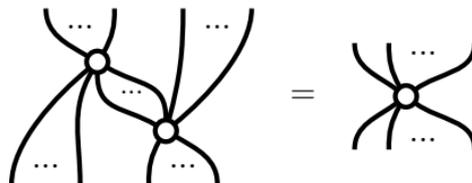
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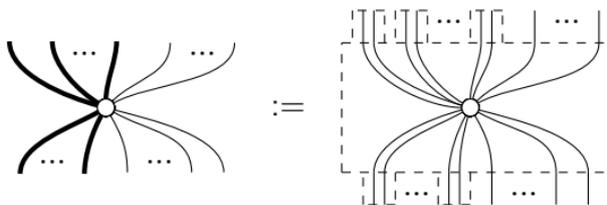


- They also fuse:



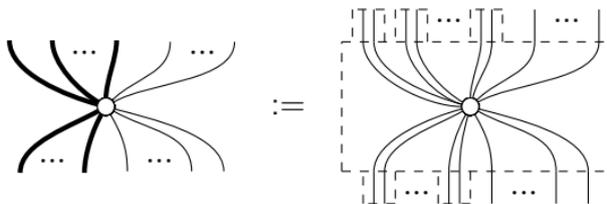
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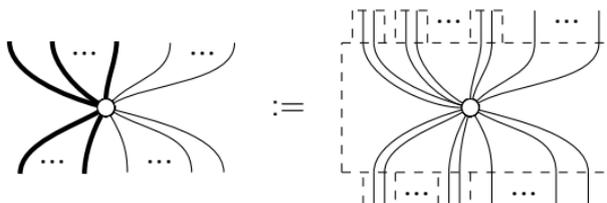
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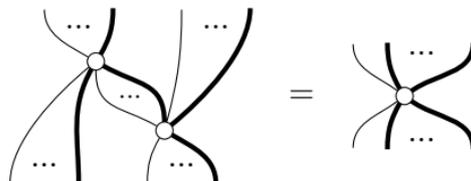
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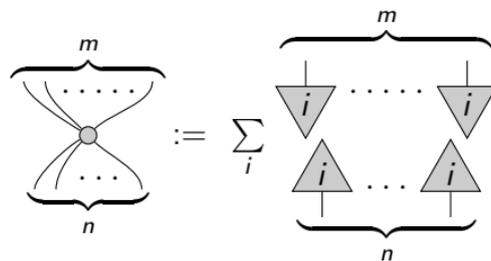
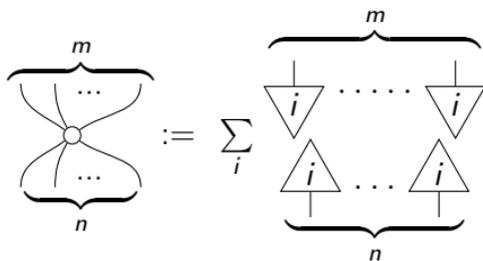
$$\text{meter symbol} \quad :: \quad \text{triangle symbol } i \quad \mapsto \quad \text{triangle symbol } i$$

- Combining these yields more general stuff, e.g. non-demo measurements:

$$\text{meter symbol with three circles} = \text{meter symbol with one circle}$$

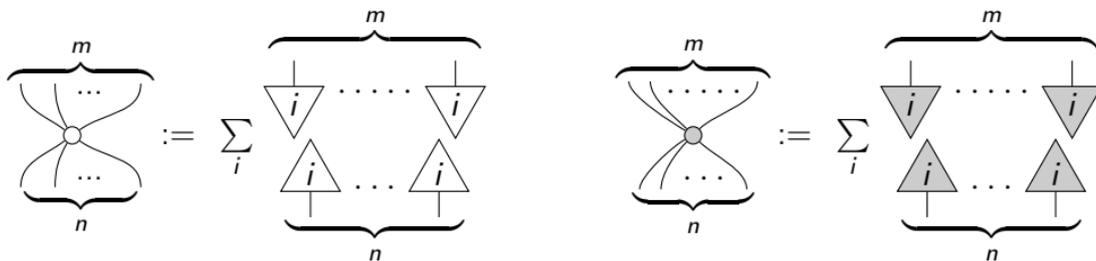
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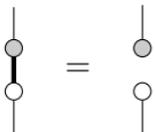


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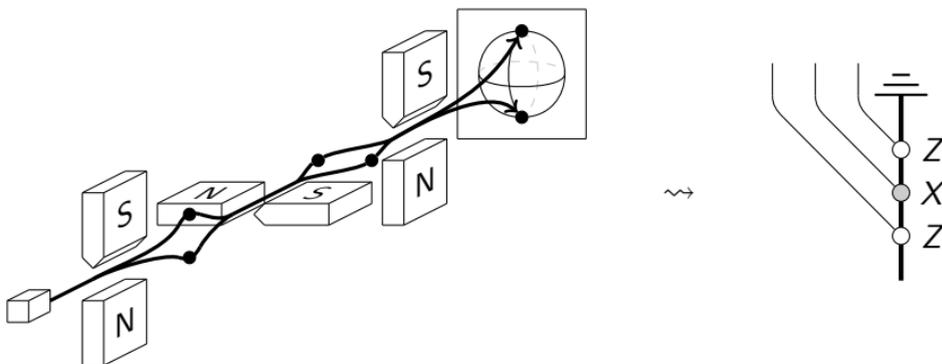
- Two spiders  $\circ$  and  $\bullet$  are *complementary* if:



(encode in  $\circ$ ) + (measure in  $\bullet$ ) = (no data transfer)

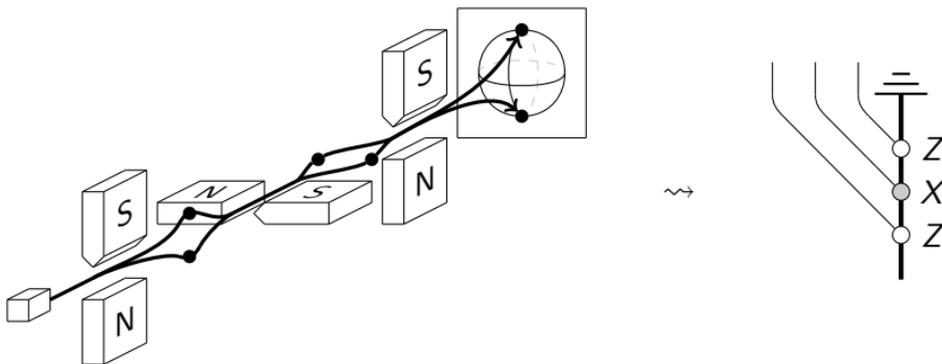
## Complementarity – Stern-Gerlach

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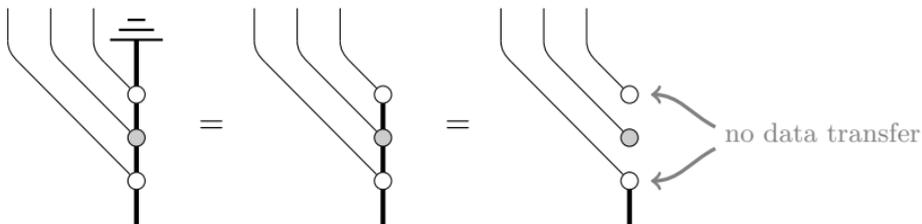


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- which simplifies as:



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the rest...

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Thanks!