



Automated Formal Analysis of Side-Channel Attacks on Probabilistic Systems

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Motivation

- **Side-channel attacks**
 - potential leakage of secret data via e.g. time/power info
 - side channel attacks increasingly viable
- **Probabilistic systems**
 - information leakage is naturally expressed probabilistically
 - security systems often employ randomisation or operate in uncertain environments
- **Automated verification techniques**
 - build on recent advances in techniques & tools (probabilistic model checking & PRISM)
 - meaningfully quantify the severity of potential attacks
 - synthesise optimal attacks and analyse defences/trade-offs

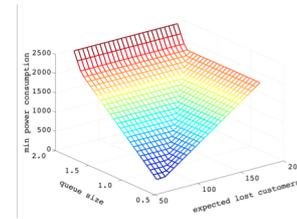
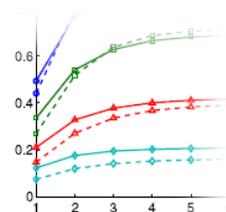
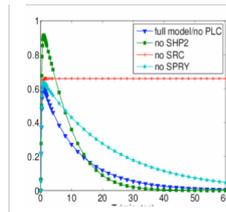
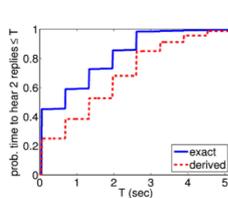
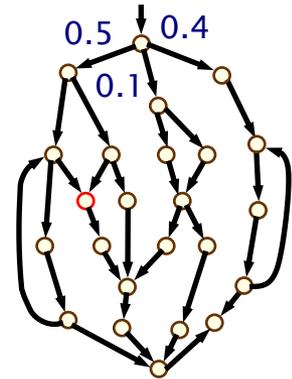
Overview

- Probabilistic model checking
 - discrete-time Markov chains
 - partially observable Markov decision processes (POMDPs)
- The SCH-IMP language
- Automated side channel analysis
 - via probabilistic model checking on POMDPs
- Tool support
- Examples and results

Probabilistic model checking

- Probabilistic model checking

- formal construction/analysis of probabilistic models
- “correctness” properties expressed in temporal logic
- e.g. $\text{trigger} \rightarrow P_{\geq 0.999} [F^{\leq 20} \text{deploy}]$
- mix of exhaustive & numerical/quantitative reasoning

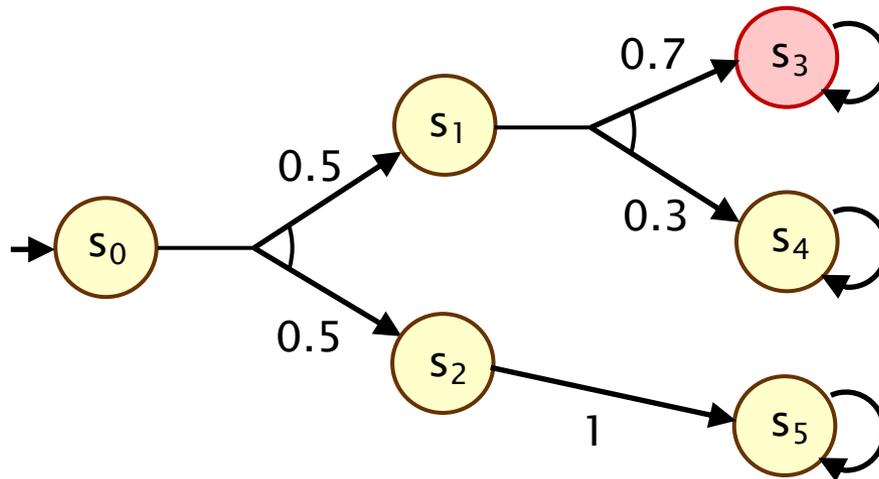


- Trends and advances

- improvement in **scalability** to larger models
- increasingly expressive/powerful **model classes**
- from verification problems to **control/synthesis problems**
- (i.e., synthesis of optimal strategies/policies)

Probabilistic models

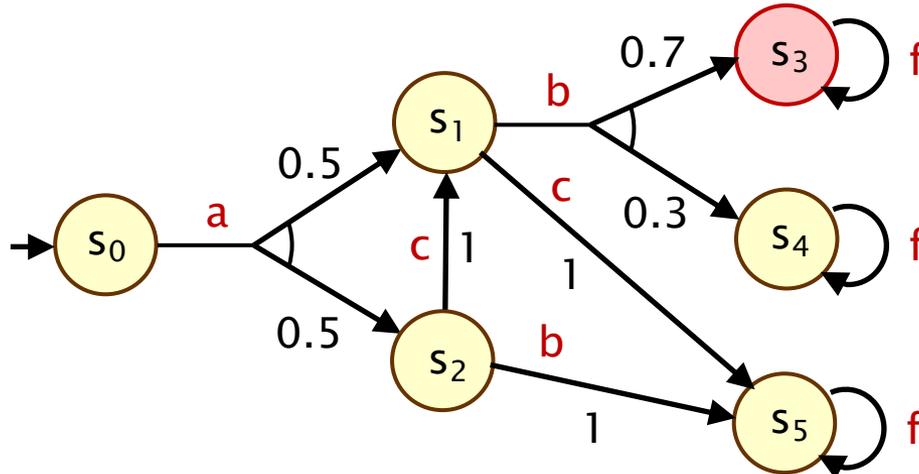
- Models used here:
 - discrete-time Markov chains
 - partially observable Markov decision processes (POMDPs)
- Discrete-time Markov chains



- e.g. what is the probability of reaching s_3 ?

Probabilistic models: MDPs

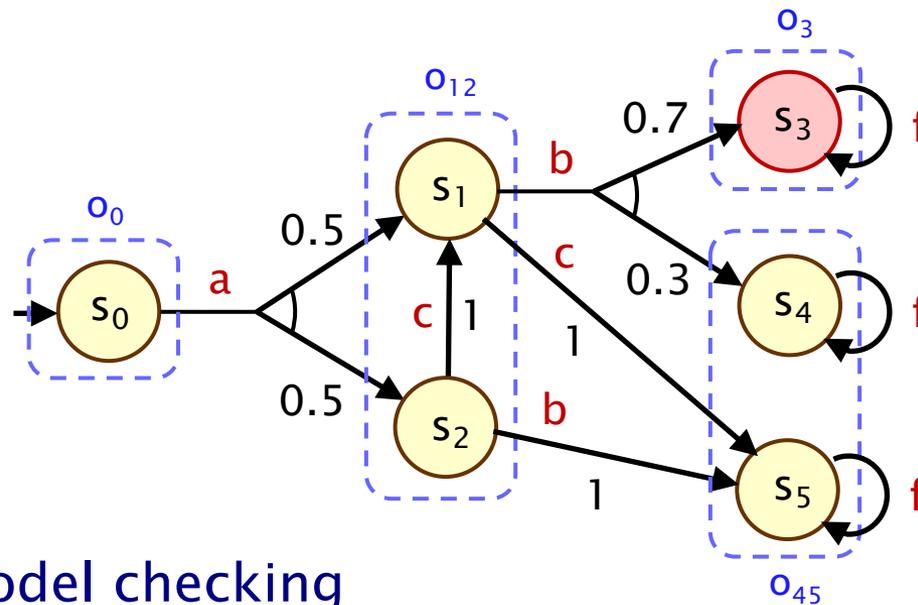
- Markov decision processes (MDPs)
 - mix nondeterministic and probabilistic choice
 - **strategies** (or policies) resolve actions based on history



- MDP model checking
 - e.g. what is the maximum probability of reaching s_3 achievable by any strategy?

Probabilistic models: POMDPs

- Partially observable Markov decision processes (POMDPs)
 - an **observation function** limits what a strategy can observe
 - strategies make the same choice in equivalent states



- POMDP model checking
 - basic verification problems are undecidable
 - but techniques exist to approximate optimal strategies
 - and tool support is now available, e.g. PRISM-pomdps

The SCH-IMP language

- SCH-IMP: Simple imperative programming language, with:
 - secret variables, probabilistic assignment, resource usage
 - extension of CH-IMP [CSF'13] for information leakage
- Key language components:
 - variables (finite-ranging)
 - assigned via discrete probability distributions
 - either “initial” (secret) or “regular” variables
 - output statements + functions + basic control flow

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 - either “initial” (secret) or “regular” variables
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```
 $\mathbb{P} ::=$  [initial  $V := \rho$ ;*  
  [new  $V := \rho$ ;*  
  [function  $F([V]^*)$  {  $C$ ; [output  $[A]^+$ ;? return }];+  
   $F([A]^*)$ ; end  
  
 $C ::=$  skip | new  $V := \rho$  |  $V := \rho$  |  $F([A]^*)$   
  | if ( $B$ ) {  $C$  } [else {  $C$  }]?  
  | while ( $B$ ) {  $C$  } |  $C$ ;  $C$ 
```

SCH-IMP example

- Simple example program

```
initial i := { 0→ $\frac{1}{4}$ , 1→ $\frac{1}{4}$ , 2→ $\frac{1}{4}$ , 3→ $\frac{1}{4}$  };  
function f(x) {  
    new o := 1;  
    if (x>0) { o := x/x };  
    output o;  
    return  
};  
f(i);  
end
```

- Note:
 - secret/non-secret variables declared with **initial** and **new**
 - variable **o** is always set to 1, so no leakage via **output**

SCH-IMP: Resources

- Resource usage

- we focus here on just two: **time** and **power** consumption
- defined at the level of functions, not individual commands
- a **resource function** gives, for a subset of program functions, a mapping from function arguments to discrete probability distributions over time/power usage

```
initial i := { 0 → 1/4, 1 → 1/4, 2 → 1/4, 3 → 1/4 };  
function f(x) {  
  new o := 1;  
  if (x > 0) { o := x/x };  
  output o;  
  return  
};  
f(i);  
end
```

```
f → {  
  (0) → { (5,7) → 1/2, (6,7) → 1/2 },  
  ( ) → { (6,7) → 1/2, (7,7) → 1/2 }  
}
```

SCH-IMP semantics

- Semantics for execution of a SCH-IMP program
 - defined as a discrete-time Markov chain
 - see the paper for a formal definition
- A SCH-IMP **state** is a tuple (F, I, t, p, Δ)
 - F is the current command stack (+ associated variable values)
 - $I : \text{Var} \rightarrow \text{Val}$ is the initial variables and their values
 - $t : \mathbb{N}$ is the total time elapsed so far
 - $p : \mathbb{N}$ is the power consumed so far
 - $\Delta : \mathbb{N} \rightarrow \mathbb{N} \times \text{seq}(\text{Val})$ is an observation function defining, for certain time points, the cumulative power consumed and values that were output from the program

SCH-IMP semantics: Example

- Initial fragment of semantics for example

```

1: initial i := { 0 → 1/4, 1 → 1/4, 2 → 1/4, 3 → 1/4 };
   function f(x) {

```

```

   ...
5: };
6: f(i);
7: end

```

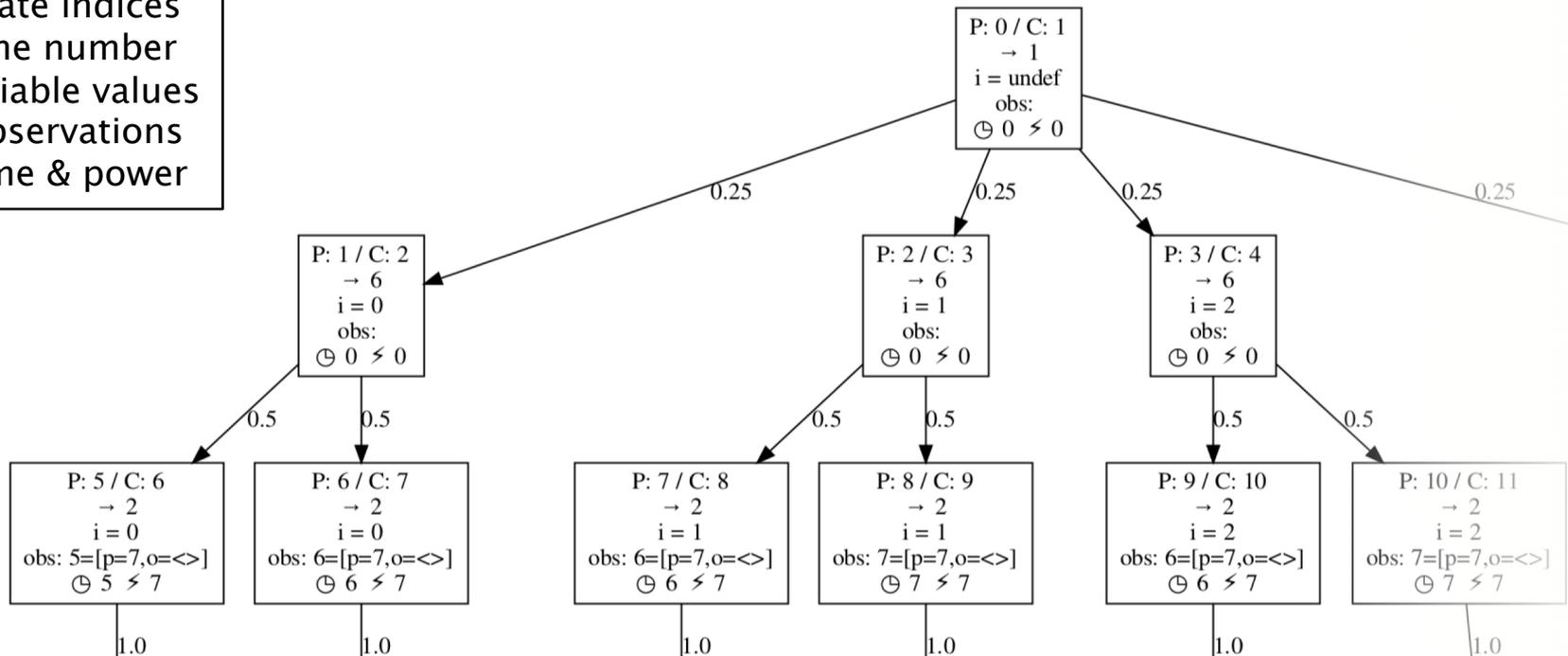
```

f → {
  (0) → { (5,7) → 1/2, (6,7) → 1/2 },
  ( ) → { (6,7) → 1/2, (7,7) → 1/2 }
}

```

Key:

state indices
line number
variable values
observations
time & power



Side channel analysis

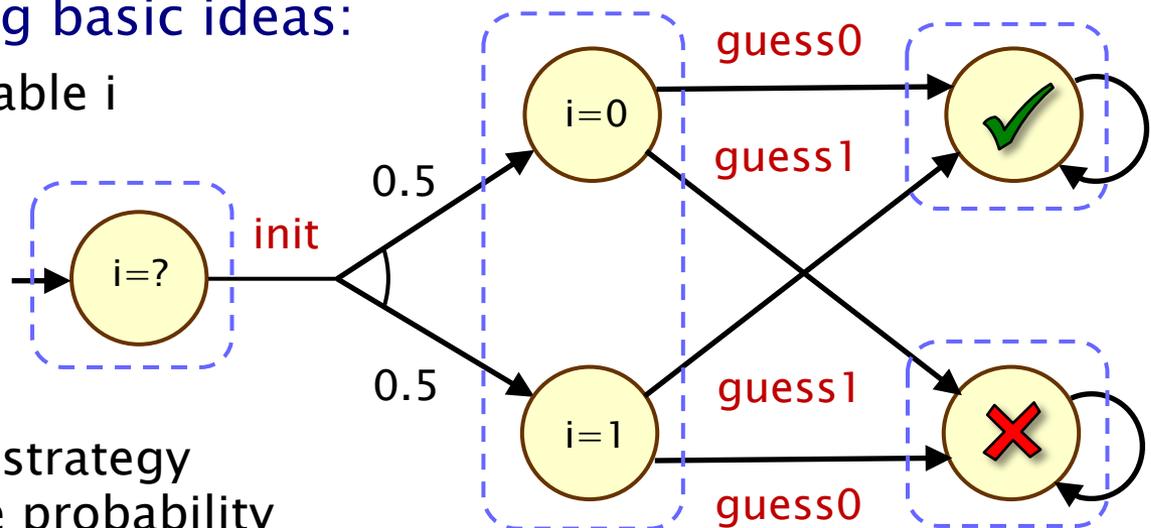
- **Attack model**
 - attacker has full access to program source code
 - sees program outputs & power usage at fixed time points
- **3-phase process**
 - systematic construction of Markov chain model
 - following SCH-IMP semantics (including optimisations)
 - conversion to POMDP model
 - encoding (side channel) attacker knowledge and choices
 - solution of POMDP
 - construct optimal strategy for maximising info leakage

POMDP construction

- POMDP extends Markov chain model of program
 - observation function hides program internals
 - attacker “guess” actions added
 - (for now, these occur on termination)

- Example showing basic ideas:

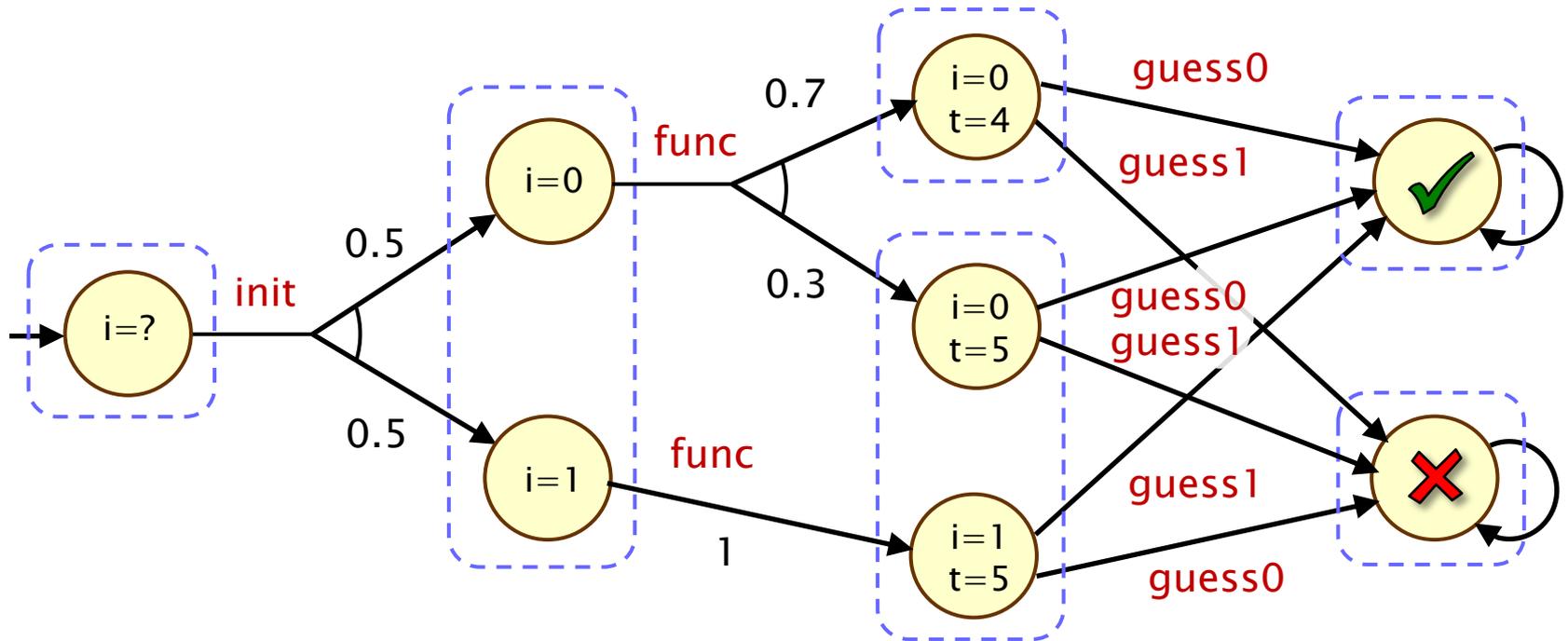
- for secret variable i



- which POMDP strategy maximises the probability of reaching the success state?

POMDP construction

- Extended example
 - incorporating time observations (t)



- optimal strategy can now use time observations

Tool support

- Prototype tool, including
 - language parser and processor
 - model construction via **PRISM** (model generator API)
 - POMDP model checking via **PRISM-pomdps**
 - source/binaries/examples available from:
www.cs.bham.ac.uk/research/projects/schimp/
- Sample of performance results for experiments

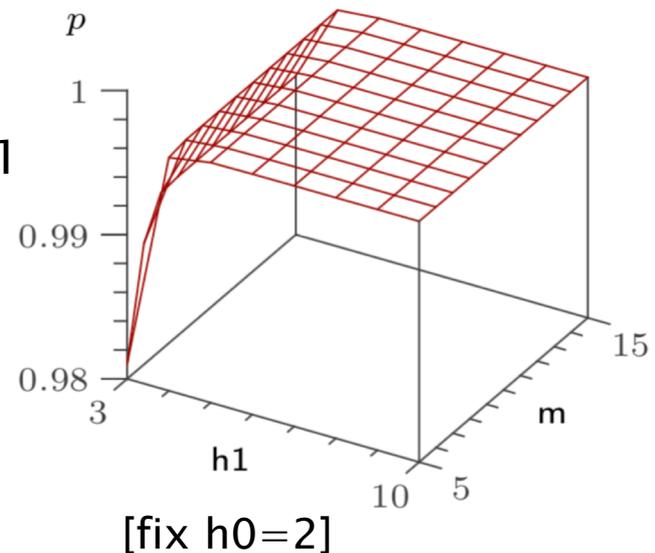
Example	States		Result		Time (min)
	DTMC	POMDP	p	Error	
DC-net	93333	20003	0.527	0.017	63
Uni. network: $h1 = 3, m \leq 15$	142547	36009	0.991	0.000	13
Uni. network: $h1 = 10, m \leq 5$	43461	12403	0.997	0.003	1
Square-and-multiply: naive	60749	27003	0.964	0.000	16

Case studies

- Three case studies considered for evaluation
 - investigate severity/details/trade-offs of known side-channels
- 1. Covert information flow: the NRL pump
 - network messages only sent from “low” to “high” hosts
 - probabilistic network delays inserted to prevent deliberate side-channel via delays in message acknowledgements

Our analysis:

- max m attempts to send a message
- ack delays h_0 , h_1 for secret values 0, 1
- for a given m , we find ack delays h_0 , h_1 that maximise the probability of a successful leak
- study trade-off between network performance and security



Case studies

2. Square and multiply algorithms

- more efficient implementation of modular exponentiation
- known power analysis side channel [Messerges et al.'99] since multiply is more expensive and only needed for some bits
- we measure the increase in the probability of a successful attack for a variety of different power usage schemes
- and verify that several alternative schemes indeed fix this

3. Anonymous communication networks: DC-net

- based on dining cryptographers protocol [Chaum'88]
- protocol successfully preserves anonymity of the sender
- study leakage due to timing of extra ops performed by sender

Conclusions

- **Automated formal analysis of side channel attacks**
 - using an imperative probabilistic language SCH-IMP
 - and building on probabilistic model checking of POMDPs
- **Benefits**
 - meaningful quantification of attack severity (and generation of the details of the worst-case attack)
 - study of trade-offs involved in defending against attacks
- **Limitations**
 - custom modelling language; scalability remains a challenge
- **Future work**
 - optimising POMDP construction and analysis
 - use POMDP to extend attack search (e.g. considering cost)
 - more structured representation of strategies